Predicted Nucleation of Domain Walls in $p_x + ip_y$ Superconductors by a Z_2 Symmetry-Breaking Transition in External Magnetic Fields

Vasily Vadimov[1] and Mihail Silaev[1,2]

[1]Institute for Physics of Microstructures RAS, 603950 Nizhny Novgorod, Russia
[2]O.V. Lounasmaa Laboratory, P.O. Box 15100, FI-00076 Aalto, Finland

(Received 13 July 2013; published 24 October 2013)

We show that time reversal symmetry-breaking $p_x + ip_y$ wave superconductors undergo several phase transitions subjected to an external magnetic field or supercurrent. In such a system, the discrete Z_2 symmetry can recover before a complete destruction of the order parameter. The domain walls associated with Z_2 symmetry can be created in a controllable way by a magnetic field or current sweep according to the Kibble-Zurek scenario. Such domain wall generation can take place in exotic superconductors like Sr_2RuO_4, thin films of superfluid ^{3}He-A, and some heavy fermion compounds.

DOI: 10.1103/PhysRevLett.111.177001 PACS numbers: 74.20.Rp, 11.27.+d, 74.25.Ha, 74.70.Pq

Topological defect formation in the systems which undergo nonequilibrium phase transitions has become a subject of interdisciplinary research between high energy and condensed matter physics [1–3]. The commonly accepted cosmological model suggests that cosmic strings can form according to the Kibble-Zurek (KZ) scenario through the nonequilibrium phase transition in an expanding Universe [4,5]. The KZ mechanism was tested in experiments with quantized vortices produced by a rapid quench or pressure sweep in a number of condensed matter systems including superfluid ^{3}He [6,7], superconductors [8], and liquid crystals [9]. In these systems, the rate of vortex formation agrees with KZ theory predictions. On the other hand, the latest experiments [10] reveal no evidence of quench induced vortex nucleation in superfluid ^{4}He in contrast with the initial proposals [5,11].

The physics of domain walls (DWs) is less studied and remains a large enigma both in cosmology and condensed matter systems [4,12]. Indeed the observational constraints required to accept the fact that the domain structure of vacuum created during the cooling down and cosmological expansion have disappeared at the early history of the Universe [1]. A plausible explanation involves assumptions of the initial baryon asymmetry or time inversion symmetry violation which finally totally removes the domains of one kind [12]. However, these speculations remain yet unconfirmed, which make theorists rule out the models with discrete symmetry breaking since the mechanism of the DWs' disappearance remains a mystery [4].

One of the few known condensed matter systems which allows us to study quench induced formation of cosmiclike DWs is superfluid ^{3}He [13]. Experimentally, DW generation was detected during cooling into the A phase or warming up from the B phase [14,15]. However, with a rapid temperature sweep one can hardly fine-tune the parameters in order to produce DWs exclusively without producing vortices pinned by DWs [15] and composite defects [16]. Moreover, in a real system quench is always spatially inhomogeneous, which provides important modifications to the physics of defect formation [17–20].

In this Letter, we propose a unique selective mechanism of DW formation during a spatially homogeneous phase transition in exotic superconductors with chiral $p_x + ip_y$ pairing symmetry. This mechanism is likely to be tested in the recently discovered layered-perovskite superconductor Sr_2RuO_4 [21,22]. According to evidence from a number of experiments [22–24] it is assumed to be a chiral $p_x + ip_y$ wave superconductor with Cooper pairs having an effective internal orbital momentum projection on the crystal anisotropy axis $l_z = \pm 1$. Such a superconducting state has a broken time reversal symmetry (BTRS) so the superconducting phase transition is determined by the spontaneous $U(1) \times Z_2$ symmetry violation analogously to the recently proposed $s + is$ BTRS multiband superconductors [25]. The $p_x + ip_y$ order parameter is realized also in a superfluid ^{3}He-A phase confined in a thin slab [15,26–28]. To exclude the Fréedericksz transition [27] of the orbital anisotropy $\hat{\mathbf{l}}$ texture, the slab thickness and coherence length should be of the same order. Such samples have become available in recent experiments [26]. The proposed mechanism can take place in heavy fermion compounds such as UPt_3. It has two superconducting phase transitions in a zero magnetic field and the one at the lowest temperature is believed to be a Z_2 symmetry-breaking one [29]. In this case, one can expect additional interesting effects to appear due to the interplay of antiferromagnetic and superconducting order parameters.

The two different BTRS vacuum states can be separated by DWs which are known to support spontaneous supercurrent generating magnetic fields [30]. However, high resolution magnetic imaging microscopy experiments detected no stray fields which should be generated by DWs above the surface of superconducting Sr_2RuO_4 [31–33]. Moreover, polar Kerr effect measurements [34] also did not reveal chiral domains. Thus, up until now no direct observation of DWs in Sr_2RuO_4 was obtained although phase-sensitive Josephson spectroscopy experiments [35] have shown some evidence of a dynamical domain structure. It has been suggested that in some cases the DW generates only a very weak stray fields [36]. The stray fields' suppression can result also from the multiband superconductivity [37] which, on the other hand, can stimulate the proposed unconventional mixed state with a vortex coalescence in Sr_2RuO_4 [38].

In addition, it is possible that DWs disappear at some stage of the superconducting transition in Sr_2RuO_4. Therefore, the proposed method to create in a controllable way an arbitrary initial concentration of DWs in Sr_2RuO_4 can prompt experimental identification of these defects. Moreover, it can shed a new light on the fate of cosmic DWs during the early history of the Universe.

To describe DWs separating different $l_z = \pm 1$ vacuum states we use the Ginzburg-Landau (GL) model of a chiral spin triplet superconducting state in Sr_2RuO_4. This material belongs to the tetragonal crystallographic symmetry group D_{4h} and has a strong crystal anisotropy which is assumed to keep the orbital momentum of the Cooper pairs parallel to the $\hat{\mathbf{c}}$ axis [22]. Moreover, recent experiments reveal no Knight shift change during the superconducting phase transition [39]. Hence, we assume that the spin $\hat{\mathbf{d}}$ vector direction is fixed.

The coordinate system is chosen so that the crystal anisotropy axis is $\hat{\mathbf{c}} \parallel \hat{\mathbf{z}}$. Then the $p_x + ip_y$ state corresponds to the two-dimensional representation $\Gamma_5^- = (k_x\hat{\mathbf{z}}, k_y\hat{\mathbf{z}})$ and the order parameter is described by a complex two-dimensional vector $\eta = (\eta_X, \eta_Y)$ [22,29,40]. Thus, introducing chiral order parameter components $\eta_\pm = \eta_X \pm i\eta_Y$ we consider a GL free energy density in the usual dimensionless units:

$$f = -|\eta_+|^2 - |\eta_-|^2 + (|\eta_+|^4 + |\eta_-|^4)/2 + 2|\eta_+\eta_-|^2 + [\nu_1(\eta_-\eta_+^*)^2 + \text{c.c.}]/2 + |\mathbf{D}\eta_+|^2 + |\mathbf{D}\eta_-|^2 + [(D_-\eta_+)^*(D_+\eta_-) + \nu_2(D_+\eta_+)^*(D_-\eta_-) + \text{c.c.}]/2, \quad (1)$$

where $\mathbf{D} = -i\nabla + \mathbf{A}$ and $D_\pm = D_x \pm iD_y$. The length is normalized to the coherence length ξ and the vector potential $\mathbf{A}$ to the value $\phi_0/2\pi\xi$ where ϕ_0 is superconducting flux quantum. Coefficients $\nu_{1,2}$ determine the anisotropy in the xy plane induced by tetragonal distortions. In the case of $\nu_1 = \nu_2$, the free energy, Eq. (1), was obtained from the weak coupling microscopic theory [41]. The GL model, Eq. (1), yields two degenerate ground states $(\eta_+, \eta_-) = (0, 1)$ and $(1, 0)$, which can be separated by DWs [30,36,42].

Let us now consider the $p_x + ip_y$ superconducting film in the xy plane. The film is supposed to be thin $d \ll \xi$, λ, where λ is the London penetration length so that we can use the standard approximation when the magnetic field and order parameter are homogeneous along the z axis inside the film. First we assume that the film is subjected to the magnetic field parallel to the film plane $\mathbf{H} = H\hat{\mathbf{y}}$ as shown in Fig. 1(a). In a thin film of a conventional superconductor, the $U(1)$ symmetry-breaking phase transition is known to be of the second order and the critical field is [43] $H_c = \sqrt{6}H_{\text{cm}}\lambda/d$ where H_{cm} is a thermodynamic critical field. However, in the $U(1) \times Z_2$ superconductor one can expect qualitatively new features. Indeed the in-plane current couples the $l_z = \pm 1$ order parameter components which leads to the formation of a $\alpha p_x \pm ip_y$ state with $0 < \alpha \leq 1$. The broken discrete symmetry subgroup is $Z_2 = TU_{\pi z}e^{i\pi}$, where T is the time reversal and $U_{\pi z}$ is the π rotation by around the z axis. Thus, at a certain critical field $H = H_{Z_2}$ the coupling can be so strong to put $\alpha = 0$ and remove the Z_2 degeneracy of the superconducting state. Such symmetry restoration occurs via the second-order phase transition which is determined by the coherence length ξ_{Z_2} which is naturally connected with the size of the DW between different chiral domains. At the point of the Z_2 phase transition, the DW width ξ_{Z_2} diverges and chiral domains disappear. The two transitions were shown to exist in the vortex phase of Sr_2RuO_4 under the action of

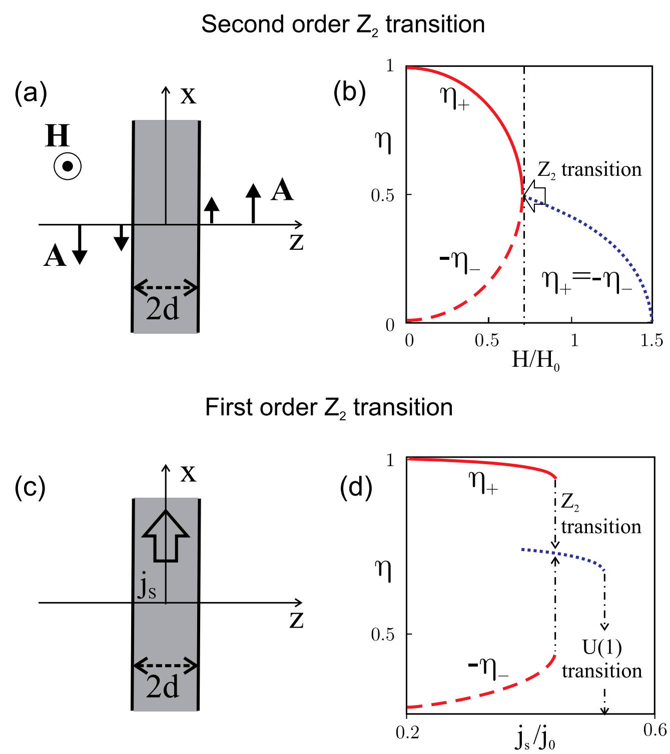

FIG. 1 (color online). Phase transitions in a thin film of the $p_x + ip_y$ superconductor. (a),(b) Second-order Z_2 and $U(1)$ transitions under the action of an external magnetic field and (c),(d) first order transitions in an external current. By red solid and dashed lines, the order parameter amplitudes η_+ and $-\eta_-$ are shown in the $U(1) \times Z_2$ phase. The energetically equivalent state is obtained by interchanging values of η_+ and η_-. The dotted blue line corresponds to the nondegenerate $U(1)$ phase with order parameter components $\eta_+ = -\eta_-$. The magnetic field and current are normalized to $H_0 = H_{cm}d/(2\sqrt{3}\lambda)$ and $j_0 = (c/4\pi)H_{cm}/\sqrt{2}\lambda$, correspondingly.

the magnetic field in the basal plane [41]. Here we will focus on the structure of DWs and the physics of the nonequilibrium Z_2 phase transition in Sr_2RuO_4 under the action of the external magnetic field and transport current.

The proposed scenario can indeed be confirmed by a straightforward calculation. At first we consider an auxiliary problem. Suppose the Cooper pairs have constant velocity directed along the x axis. Then the order parameters can be represented as $\eta_\pm = \psi_\pm e^{ikx}$ where k is a dimensionless Cooper pair velocity. Minimizing the free energy, Eq. (1), by the amplitudes $\psi_\pm$ at fixed k, we obtain two stable branches of the order parameter. (i) On the first branch the magnitude of the order parameter components is different $|\psi_+| \neq |\psi_-|$ and they have opposite signs

$$|\psi_\pm|^2 = \frac{1-k^2}{2} \pm \frac{\sqrt{(1-k^2)^2 - k^4[(1+\nu_2)/(1+\nu_1)]^2}}{2}.$$

Because of the invariance of GL theory, Eq. (1), with respect to the replacement of ψ_+ to ψ_-, and vice versa, the found solution is twice degenerate and corresponds to the superconducting $U(1)\times Z_2$ phase. This solution is stable if the velocity of Cooper pairs is smaller than the critical value $|k| < k_{Z_2} = \sqrt{(1+\nu_1)/(2+\nu_1+\nu_2)}$. Note that $k_{Z_2} < k_c = \sqrt{2/(1-\nu_2)}$ where k_c is the depairing superfluid velocity which destroys the superconducting state completely. (ii) On the second branch, the magnitudes of the order parameter components are the same $\psi_+ = -\psi_-$, where $|\psi_\pm|^2 = [1 - k^2(1-\nu_2)/2]/(3+\nu_1)$. Unlike the previous case, this solution is nondegenerate. Therefore it corresponds to usual $U(1)$ superconducting state. This phase is stable in the interval $k_{Z_2} < |k| < k_c$.

That is, we obtain an additional phase transition at $k = k_{Z_2}$ when the ground state double degeneracy is removed and the corresponding discrete Z_2 symmetry is restored. The order parameter components change continuously while we shift the k value through the Z_2 critical point; therefore, this is a second-order phase transition.

The solution of an auxiliary problem considered above can be applied to find the critical fields of a thin $p_x + ip_y$ superconducting film. Indeed, we choose Landau gauge $A_x = B_y z$ [see Fig. 1(a)] and use a standard thin film approximation assuming $\eta_\pm$ to be constant along the z coordinate. Taking the z average of the free energy yields an effective superfluid velocity $k = \sqrt{\langle A_x^2\rangle} = dH/\sqrt{6}$. Then one immediately finds the critical fields values:

$$H_{Z_2} = \left(2\sqrt{3}\lambda/d\right)k_{Z_2}H_{cm}, \quad (2)$$

$$H_c = \left(2\sqrt{3}\lambda/d\right)k_c H_{cm}. \quad (3)$$

The critical field H_{Z_2}, Eq. (2), restores the discrete Z_2 symmetry and the field H_c, Eq. (3), is a standard critical field of a thin superconducting film which suppresses superconductivity completely. The evolution of order parameter components as functions of an applied magnetic field is shown in Fig. 1(b). In this case, both Z_2 and $U(1)$ phase transitions are of the second order and characterized by vanishing order parameters and divergent coherence lengths.

Naturally, the order parameter of a Z_2 phase transition can be chosen in the form $\eta_X = (\eta_+ + \eta_-)/2$. Indeed, η_X vanishes near H_{Z_2} in the first phase and is identical to zero in the second phase. To reveal the physical origin of the Z_2 coherence length let us consider the structure of a DW in the vicinity of a critical point. Here we can derive an equation for the order parameter η_X taking the other component $\eta_Y = (\eta_+ - \eta_-)/2i$ to be constant $\eta_Y = \eta_Y(H = H_{Z_2})$. In this way we assume the order parameter amplitude to be a slowly varying real valued function $\eta_X = \eta_X(x, y)$ and obtain the single component GL equation:

$$-D\nabla_r^2\eta_X + a\eta_X/2 + b\eta_X^3 = 0 \quad (4)$$

with coefficients $D = (3+\nu_2)$, $a = (1+\nu_2/3)\times(H^2 - H_{Z_2}^2)d^2$, and $b = 2(3+\nu_1)$. We can find a DW structure as the topological soliton in Eq. (4) $\eta_X = \sqrt{a/b}\tanh(\sqrt{a/D}x)$. Since $a \sim (H_{Z_2} - H)$, we see that the DW dissolves near the critical field H_{Z_2} and the size of the DW is proportional to $\xi_{Z_2} \sim (H_{Z_2} - H)^{-1/2}$.

The obtained Z_2 transition provides a unique possibility to create an arbitrary concentration of DWs in the $p_x + ip_y$ superconductor. We employ a generalization of the KZ defect formation mechanism [4,5] to explore the DW appearance during the nonequilibrium Z_2 phase transition. Let us assume that the external field decreases with the constant rate τ_H so that $H(t) = (1 - t/\tau_H)H_{Z_2}$. Just below the Z_2 critical point $H < H_{Z_2}$ the growth of Z_2 order parameter fluctuations can be described by a linearized time dependent GL equation [19,44]:

$$\tau\eta'_{Xt} = [H_{Z_2}^2 - H^2(t)]\eta_X + \nabla_r^2\eta_X, \quad (5)$$

where we put $\nu_2 = 0$ for simplicity and normalize the magnetic field by H_c given by Eq. (3). There are two competing effects described by Eq. (5): exponential growth and diffusive spreading due to the last term in the rhs. Comparing the characteristic times of these processes we can obtain a distance between defects just after the phase transition as the minimal length scale which can grow. The characteristic growth time is $t_Z \sim \sqrt{\tau\tau_H}/H_{Z_2}$, which is also known as Zurek time [5,44]. This time should be much less than the diffusive time $L^2\tau$, where L is the characteristic length scale. So, we obtain the distance between defects $L \sim (\tau_H/\tau)^{1/4}$. Thus, varying the rate τ_H it is possible to create an arbitrary concentration of DWs.

Applying an external transport current $\mathbf{j}_s$ along the film plane [see Fig. 1(c)] it is possible to obtain the Z_2 transition of the first order. In this case a stable state can be found only numerically. An example of a phase diagram where the $U(1) \times Z_2$ and $U(1)$ phases coexist is shown in Fig. 1(d). To realize the first order Z_2 transition between them the critical current of the $U(1) \times Z_2$ state should be smaller than that of the $U(1)$ state. Otherwise the system will fall into a normal phase directly from the $U(1) \times Z_2$ state. We obtain that such a regime is realized when $2 + (1 + \nu_2)^2/(1 + \nu_1) > (3 + \nu_1)^2/(1 - \nu_2)$. Therefore, in a weak coupling model [41] with $\nu_1 = \nu_2$ there is no first order Z_2 phase transition in the external current.

As usual the first order Z_2 phase transition discussed above occurs through the growth of the nuclei with sizes larger than the critical one R_c [45]. It can be estimated as $R_c \sim \tilde{f}_s/\Delta\tilde{f}_b$, where $\tilde{f}_s$ is the surface free energy density and $\Delta\tilde{f}_b$ is the difference of bulk free energy densities in two phases. Thus, the critical size is determined by the external current $R_c = R_c(j_s)$ due to the bulk energy dependence $\Delta\tilde{f}_b = \Delta\tilde{f}_b(j_s)$. The distance between DWs after the first order transition should be determined by R_c, which can vary from 0 to ∞ by setting the value of j_s.

After the fast Z_2 transition we expect that the volumes occupied by domains of different chirality should be almost the same. The DWs can be stabilized by geometrical confinement in mesoscopic samples [46], pinning on vortices and defects [42,47,48] preventing the formation of a monodomain state. However, the maximal concentration of DWs is determined by the rate of a magnetic field sweep through the Z_2 transition. Hence it is possible to create a domain structure with domains of controllable size. Such a mechanism provides a possibility of systematic studies of DWs density in Sr_2RuO_4. It can help to resolve a controversy between different experiments which are currently incompatible with each other in the assumptions about DW density [32,33].

Finally, let us discuss a way to test the evolution of the domain structure during the transient processes following the nonequilibrium Z_2 phase transition. We suggest employing transport measurements in the mixed state produced by magnetic field $\mathbf{H} \parallel \hat{\mathbf{c}}$. The proposed method is based on the observation that such a field creates Abrikosov vortices which are known to remove the Z_2 degeneracy of the superconducting vacuum in the $p_x + ip_y$ superconductor. That is, vortices have different core structures in the chiral domains with $(\mathbf{H}\hat{\mathbf{l}}) > (<) 0$ [28,49–51], where $\hat{\mathbf{l}}$ denotes the direction of the internal orbital momentum of Cooper pairs which, in our case, is $\hat{\mathbf{l}} \parallel \hat{\mathbf{c}}$. We denote these vortex structures N_+ and N_- vortices, correspondingly.

In the isotropic case $\nu_1 = \nu_2 = 0$ the order parameter in axially symmetric vortices has the form $\eta_\pm = |\eta_\pm|(r)e^{im_\pm\theta}$, where (r, θ) are polar coordinates with the origin at the vortex center. Axial symmetry is preserved provided the winding numbers are $m_+ = 1$, $m_- = 3$ for N_+ and $m_+ = 1$, $m_- = -1$ for N_- vortices. Here we note that N_+ and N_- vortices have different viscosities due to the difference in their core structures. Hence, the flux flow conductivity has a chirality sensitive contribution $\sigma = \sigma_0 + \sigma_1(\mathbf{H}\hat{\mathbf{l}})$. The flux flow conductivity can be calculated within the framework of time dependent GL theory [52]. In this way we obtain

$$\sigma/\tilde{\sigma} = \int_0^\infty \sum_{\alpha=\pm} [\rho|\eta_\alpha|_\rho^2 + |\eta_\alpha|^2(m_\alpha^2 + \rho\mu_0)] d\rho. \quad (6)$$

Here we normalize conductivity by $\tilde{\sigma} = \sigma_n(\xi/L_E)^2 H_{c2}/2H$, where σ_n is a normal metal conductivity, $L_E = \sqrt{\sigma_n/2\tau}\Phi_0/2\pi c$ is electric field penetration length, and $\rho = r/L_E$. The function $\mu_0 = \mu_0(r)$ satisfies the Poisson equation

$$(\nabla_\rho^2 - \rho^{-2} - |\eta_+|^2 - |\eta_-|^2)\mu_0 = \sum_{\alpha=\pm} \rho^{-1} m_\alpha |\eta_\alpha|^2. \quad (7)$$

It determines electrostatic potential around the moving vortex $\varphi = \mu_0(r)(\mathbf{e_r}[\mathbf{v}, \mathbf{z}_0])$, where $\mathbf{v}$ is the vortex velocity and $\mathbf{e_r} = \mathbf{r}/r$.

For example, taking $\xi/L_e = \sqrt{6}$ we obtain that the chirality sensitive part of conductivity is $\sigma_1 = (\sigma_+ - \sigma_-)/2 = 0.018\sigma_0$. One can see that $\sigma_1 \ll \sigma_0$ and in this case the averaged over the sample flux flow conductivity is $\bar{\sigma} = \sigma_+ S_+ + \sigma_- S_-$, where $S_\pm$ are the parts of the volume occupied by domains of positive and negative chiralities.

During the transient process after the Z_2 transition the balance between S_+ and S_- can be broken which will make one chirality dominant. The weak Z_2 asymmetry required for such a scenario is provided by the presence of vortices which were shown recently to stabilize the monodomain chiral state in a ^{3}He-A slab [28]. We expect the transient evolution to be rather slow, allowing time-resolved measurements of flux flow conductivity. The possible observation of its time evolution after the fast Z_2 transition in Sr_2RuO_4 can signal a disappearance of domains of one sign, the scenario which was initially proposed in cosmology to eliminate cosmic DWs created by quench in cosmological expansion [12].

To conclude, we have found a Z_2 symmetry-breaking phase transition in $p_x + ip_y$ superconductors. The transition can be of the first order if driven by an external current and of the second order under the action of an external field. That is, applying an in-plane magnetic field to the thin superconducting film, one can drive it continuously from $U(1) \times Z_2$ to the simple $U(1)$ state. Such Z_2 symmetry restoration is marked by the dissolution of DWs. Decreasing the field through the Z_2 critical point at a constant rate, one can create a particular concentration of DWs according to the KZ scenario. This possibility can facilitate experimental identification of superconducting DWs.

We thank Professors Alexander Mel'nikov, Grigory Volovik, Edouard Sonin, Egor Babaev, and Andrei Golov for stimulating discussions and correspondence. This work was supported by the Russian Foundation for Basic Research Grants No. 11-02-00891 and No. 13-02-97126. M. S. was supported by a Russian President scholarship (SP- 6811.2013.5) and European Research Council (Grant No. 240362-Heattronics).

[1] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defect* (Cambridge University Press, Cambridge, England, 1994)
[2] G. E. Volovik, *The Universe in a Helium Droplet* (Clarendon Press, Oxford, 2003)
[3] M. B. Hindmarsh and T. W. B. Kibble, Rep. Prog. Phys. **58**, 477 (1995); W. H. Zurek, Phys. Rep. **276**, 177 (1996).
[4] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976)
[5] W. H. Zurek, Nature (London) **317**, 505 (1985)
[6] V. B. Eltsov, M. Krusius, and G. E. Volovik, Prog. Low Temp. Phys. **15**, 1 (2005).
[7] V. M. H. Ruutu, V. B. Eltsov, A. J. Gill, T. W. B. Kibble, M. Krusius, Yu. G. Makhlin, B. Plaçais, G. E. Volovik, and W. Xu, Nature (London) **382**, 334 (1996); C. Bäuerle, Yu. M. Bunkov, S. N. Fisher, H. Godfrin, and G. R. Pickett, Nature (London) **382**, 332 (1996).
[8] R. Monaco, J. Mygind, and R. J. Rivers, Phys. Rev. Lett. **89**, 080603 (2002); Phys. Rev. B **67**, 104506 (2003).
[9] M. J. Bowick, L. Chandar, E. A. Schiff, and A. M. Srivastava, Science **263**, 943 (1994).
[10] M. E. Dodd, P. C. Hendry, N. S. Lawson, P. V. E. McClintock, and C. D. G. Williams, Phys. Rev. Lett. **81**, 3703 (1998); V. Efimov, O. Griffiths, P. Hendry, G. Kolmakov, P. McClintock, and L. Skrbek, Phys. Rev. E **74**, 056305 (2006).
[11] P. C. Hendry, N. S. Lawson, R. A. M. Lee, P. V. E. McClintock, and C. D. H. Williams, Nature (London) **368**, 315 (1994).
[12] Ya. B. Zel'dovich, I. Yu. Kobzarev and L. B. Okun', Sov. Phys. JETP **40**, 1 (1975).
[13] M. M. Salomaa and G. E. Volovik, Phys. Rev. B **37**, 9298 (1988).
[14] Ü. Parts, V. M. H. Ruutu, J. H. Koivuniemi, M. Krusius, E. V. Thuneberg, and G. E. Volovik, Physica (Amsterdam) B **210**, 311 (1995).
[15] P. M. Walmsley, I. J. White, and A. I. Golov, Phys. Rev. Lett. **93**, 195301 (2004).
[16] V. B. Eltsov, T. W. B. Kibble, M. Krusius, V. M. H. Ruutu, and G. E. Volovik, Phys. Rev. Lett. **85**, 4739 (2000).
[17] T. W. B. Kibble and G. E. Volovik, JETP Lett. **65**, 102 (1997).
[18] J. Dziarmaga, P. Laguna, and W. H. Zurek, Phys. Rev. Lett. **82**, 4749 (1999).
[19] N. B. Kopnin and E. V. Thuneberg, Phys. Rev. Lett. **83**, 116 (1999).
[20] I. S. Aranson, N. B. Kopnin, and V. M. Vinokur, Phys. Rev. Lett. **83**, 2600 (1999)
[21] Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J. G. Bednorz, and F. Lichtenberg, Nature (London) **372**, 532 (1994).
[22] A. P. Mackenzie and Y. Maeno, Rev. Mod. Phys. **75**, 657 (2003).
[23] K. D. Nelson, Z. Q. Mao, Y. Maeno, and Y. Liu, Science **306**, 1151 (2004).
[24] J. Xia, Y. Maeno, P. T. Beyersdorf, M. M. Fejer, and A. Kapitulnik, Phys. Rev. Lett. **97**, 167002 (2006).
[25] J. Carlstrom, J. Garaud, and E. Babaev, Phys. Rev. B **84**, 134518 (2011).
[26] L. V. Levitin, R. G. Bennett, A. Casey, B. Cowan, J. Saunders, D. Drung, T. Schurig, and J. M. Parpia, Science **340**, 841 (2013).
[27] P. M. Walmsley, D. J. Cousins, and A. I. Golov, Phys. Rev. Lett. **91**, 225301 (2003).
[28] P. M. Walmsley and A. I. Golov, Phys. Rev. Lett. **109**, 215301 (2012).
[29] R. Joynt and L. Taillefer, Rev. Mod. Phys. **74**, 235 (2002).
[30] G. E. Volovik and L. P. Gor'kov, Sov. Phys. JETP **61**, 843 (1985); M. Sigrist, T. M. Rice, and K. Ueda, Phys. Rev. Lett. **63**, 1727 (1989); M. Matsumoto and M. Sigrist, J. Phys. Soc. Jpn. **68**, 994 (1999); **68**, 3120(E) (1999).
[31] J. Kirtley, C. Kallin, C. Hicks, E.-A. Kim, Y. Liu, K. Moler, Y. Maeno, and K. Nelson, Phys. Rev. B **76**, 014526 (2007).
[32] C. W. Hicks, J. R. Kirtley, T. M. Lippman, N. C. Koshnick, M. E. Huber, Y. Maeno, W. M. Yuhasz, M. B. Maple, and K. A. Moler, Phys. Rev. B **81**, 214501 (2010).
[33] C. Kallin and A. J. Berlinsky, J. Phys. Condens. Matter **21**, 164210 (2009).
[34] J. Xia, Y. Maeno, P. T. Beyersdorf, M. M. Fejer, and A. Kapitulnik, Phys. Rev. Lett. **97**, 167002 (2006).
[35] F. Kidwingira, J. D. Strand, D. J. van Harlingen, and Y. Maeno, Science **314**, 1267 (2006).
[36] N. A. Logoboy and E. B. Sonin, Phys. Rev. B **79**, 094511 (2009).
[37] S. Raghu, A. Kapitulnik, and S. A. Kivelson, Phys. Rev. Lett. **105**, 136401 (2010); Y. Imai, K. Wakabayashi, and M. Sigrist, Phys. Rev. B **85**, 174532 (2012).
[38] J. Garaud, D. F. Agterberg, and E. Babaev, Phys. Rev. B **86**, 060513 (2012).
[39] Y. Maeno, S. Kittaka, T. Nomura, S. Yonezawa, and K. Ishida, J. Phys. Soc. Jpn. **81**, 011009 (2012).
[40] M. Sigrist and K. Ueda, Rev. Mod. Phys. **63**, 239 (1991).
[41] D. F. Agterberg, Phys. Rev. Lett. **80**, 5184 (1998); Phys. Rev. B **58**, 14 484 (1998).
[42] M. Sigrist and D. F. Agterberg, Prog. Theor. Phys. **102**, 965 (1999).
[43] V. V. Schmidt, *The Physics of Superconductors* (Springer, New York, 1997).
[44] G. E. Volovik, Physica (Amsterdam) B **280**, 122 (2000).
[45] E. M. Lifshitz and L. I. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981).
[46] V. Vakaryuk, Phys. Rev. B **84**, 214524 (2011)
[47] M. Ichioka, Y. Matsunaga, and K. Machida, Phys. Rev. B **71**, 172510 (2005).
[48] D. G. Ferguson and P. M. Goldbart, Phys. Rev. B **84**, 014523 (2011).
[49] Yu. S. Barash and A. S. Mel'nikov, Sov. Phys. JETP **73**, 170 (1991).
[50] R. Heeb and D. F. Agterberg, Phys. Rev. B **59**, 7076 (1999).
[51] M. Takigawa, M. Ichioka, K. Machida, and M. Sigrist, Phys. Rev. B **65**, 014508 (2001).
[52] N. B. Kopnin, *Theory of Nonequilibrium Superconductivity* (Oxford University Press, New York, 2001).