Experiments on Active Cloaking and Illusion for Laplace Equation

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In recent years, invisibility cloaks have received a lot of attention and interest. These devices are generally classified into two types: passive and active. The design and realization of passive cloaks have been intensively studied using transformation optics and plasmonic approaches. However, active cloaks are still limited to theory and numerical simulations. Here, we present the first experiment on active cloaking and propose an active illusion for the Laplace equation. We make use of a resistor network to simulate a conducting medium. Then, we surround the central region with controlled sources to protect it from outside detection. We show that by dynamically changing the controlled sources, the protected region can be cloaked or disguised as different objects (illusion). Our measurement results agree very well with numerical simulations. Compared with the passive counterparts, the active cloaking and illusion devices do not need complicated metamaterials. They are flexible, in-line controllable, and adaptable to the environment. In addition to dc electricity, the proposed method can also be used for thermodynamics and other problems governed by the Laplace equation.

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The recent interest in invisibility cloaks is mainly due to two factors. The first is the methodology proposed by Pendry and Leonhardt, i.e., transformation optics (TO), which is based on the invariance of Maxwell's equations under a general coordinate transformation [1,2]. The second is the rapid development of metamaterials, a physical means to control electromagnetic waves by using artificial composites with subwavelength structures [3,4]. Generally speaking, invisibility cloaks can be realized by many methods. The most well-known and well-studied one is the TO method that employs the equivalence between geometry and electromagnetic parameters [5]. However, designs using TO always give rise to cloaks with anisotropic and inhomogeneous materials, and it is very challenging to achieve practical implementation [6–8]. Although various improvements have been achieved to address this problem [9–16], we still have a long way to go before a full-parameter perfect cloak is created. Another approach to realize invisibility cloaks is based on the scattering cancellation [17–19]. Such a methodology heavily depends on a homogeneous and isotropic shell structure, which is used to cancel the electric or magnetic polarization of the concealed object. Compared to its TO counterpart, the shell is relatively easy to implement and the cloaking performance is acceptable. However, such a methodology is restricted to the small size of the cloak, which limits the practical applications. The third scheme is based on anomalous localized resonances [20], in which the cloaking phenomenon occurs when the resonant field generated by a polarizable line or point dipole acts back on the polarizable line or point dipole and effectively cancels the field acting on it from the outside sources. But, this idea is still only in theory and simulations.

In addition to the passive cloaking schemes mentioned above [1–20], the idea of an active invisibility cloak has been recently proposed [21,22]. This methodology makes use of controlled sources around the protected object. When a probing signal is detected, all sources are appropriately excited after quick calculations, whose signals add together so as to actively cancel the probing signal. The advantage of active cloaking is obvious: it does not need complicated materials, and the method is flexible and dynamically controllable. The only problem for this proposal is obtaining prior knowledge of the probing signal, which requires complicated and quick-response devices in practice. Hence, so far, experiments on active cloaking have not yet been reported.

Here, we give the first experimental verification of active cloaking and propose active illusions for the Laplace equation at the zero frequency (dc). Like their time-varying counterparts, TO-based dc cloaking and illusion devices have aroused great interest in recent years [23–28]. In this work, we make use of a resistor network to simulate the dc conducting problem, and design controlled sources around the protected (or cloaking) region. By modulating the controlled sources dynamically, we experimentally show that the cloaking region is rendered invisible and that the protected (illusion) region can be disguised as something else from outside detection. Numerical simulations and measurement data have excellent agreement, verifying the good active cloaking and illusion performance.

The schematic configuration of the dc active cloaking and illusion devices is depicted in Fig. 1. The studied problem is governed by the Laplace equation in a conducting medium. A probing source is implemented to excite the whole system for the detection of a concealed object (something hiding in the protected region A). The measurements around the exterior boundary will produce the conductivity profile in this area and hence detect the concealed object. This fact lays the foundation of well-known electrical impedance tomography technology [23]. To realize active cloaking and illusion devices, we design an array of controlled sources around the protected region, which will help to conceal the object.

Let us first visualize an imaginary reference system, where the studied problem is an empty background medium with the probing source. Then, the electric potential distribution in the background is readily obtained by solving Poisson's equation

$$\nabla^2 \phi = f, \tag{1}$$

in which f is related to the probing source. It is easy to obtain the potential distribution along the controlled boundary, which is represented as V_a. With this in hand, we turn on the controlled sources and modulate their values to ensure that they will also produce V_a along the same controlled boundary in the presence of the probing source.

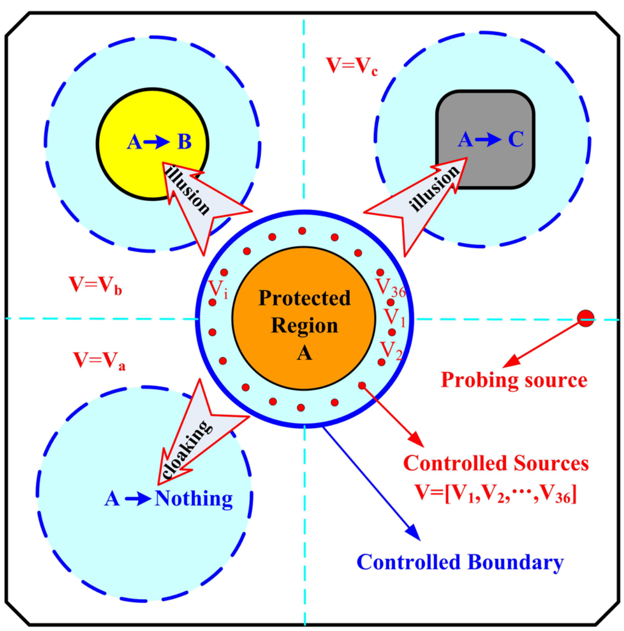

FIG. 1 (color online). Schematic diagram of the active cloaking and illusions. By dynamically modulating the controlled sources to V_a, V_b, and V_c, the protected region A can be either rendered invisible (V_a) or disguised as something else (V_b for B and V_c for C). These voltage distributions V_a, V_b, and V_c can be determined through an analytical method or numerical simulations.

The principle of uniqueness theorem for the Laplace equation suggests that the protected region looks exactly the same as the empty case; i.e., the central region is rendered invisible to the probing signal (represented as $A \rightarrow$ Nothing in Fig. 1). In our design, we make the controlled boundary coincide with the controlled sources, and hence the controlled voltages (V_a) can be immediately determined. This is different from the theoretical design proposed by Guevara Vasquez and co-workers [21,22].

Similarly to the active cloaking, we also propose an active illusion design to make the protected region look like a different object. To this end, the potential distribution along the controlled boundary is calculated through a prespecified reference system (e.g., a perfectly conducting plate inside the conducting medium, represented as $A \rightarrow B$ in the figure) excited by the probing source. By solving Poisson's equation analytically or numerically, the voltage profile along the controlled boundary can be obtained, which is represented as V_b. Then, the turn-on of the controlled sources will produce the same voltage V_b, making the protected region look like a perfectly conducting plate in the background, in which the controlled boundary again coincides with the source positions. Hence, we realize the dc illusion through an active scheme. Other illusions with voltage profiles V_c are also possible (represented as $A \rightarrow C$ in the figure), as demonstrated in Fig. 1.

Based on the above analyses, we design a two-dimensional dc experiment to test the functionality of active cloaking and illusions for the Laplace equation. However, this scheme can be extended to the three-dimensional situation [27]. The studied area is a circular conducting plate whose radius is 21 cm with the conductivity (σ) of 1 S/m. The thickness of the plate h is not important and is chosen as 1 cm in the experiment. The plate is first discretized into small sectors using polar grids (21 in the radial direction and 36 in the tangential direction), which are then emulated with radial and circumferential resistors through the following relations [27]:

$$R_\rho = \frac{\Delta\rho}{\sigma_\rho \cdot \rho\Delta\varphi \cdot h}, \qquad R_\varphi = \frac{\rho\Delta\varphi}{\sigma_\varphi \cdot \Delta\rho \cdot h}, \tag{2}$$

where $\triangle\rho$ and $\triangle\varphi$ are step lengths in the radial and tangential directions, respectively. We remark that the background medium is homogeneous and isotropic: $\sigma_\rho = \sigma_\varphi = \sigma$. The resultant resistor networks are connected to the ground at the exterior boundary using matching resistors to emulate an infinitely large conducting medium. The detailed derivation of the matching resistors has been given in the Supplemental Material of Ref. [27]. The ideal and actual resistors used in simulations and experiments are listed in the Supplemental Material of this Letter for the readers' reference [29].

In our design, the protected region is arbitrarily chosen as a small circular area in the center with radius of 6 cm, which is actually a grounded perfectly electrical

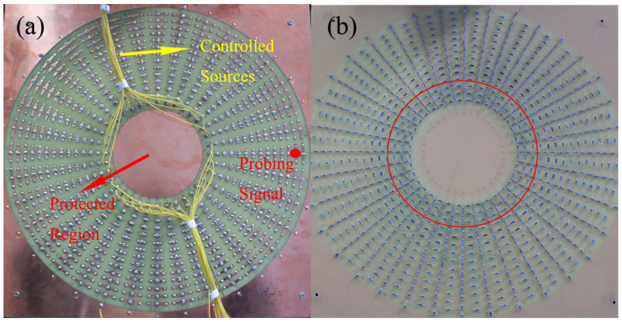

FIG. 2 (color online). Photographs of the fabricated device. (a) The bottom view. (b) The top view. The red circle in (b) indicates $r = 10$ cm.

conducting (PEC) plate. We remark that anything can be put into this region and that it has no effect on the cloaking or illusion performance. The controlled sources are arrayed around a circle with a radius of 7 cm (i.e., the controlled boundary). Figure 2 shows the photograph of the fabricated device. Four cases are studied in the experiment: one is for cloaking (case *A*), and the other three are for illusions (cases *B*, *C*, and *D*). Corresponding to the four cases, we make four control simulations using commercial software, Agilent's Advanced Design System (ADS), to make comparisons later and to calculate the voltage profiles for the controlled sources.

In case *A*, the protected region is imaginarily replaced with resistors that are consistent with those in the background to emulate an empty background medium. The ADS simulation result is shown in Fig. 3(a), in which the equipotential lines are observed as concentric circles, revealing the correct physics. The calculated potential distribution (V_a) around the controlled boundary with $r = 7^+$ cm is also determined, as listed in the Supplemental Material [29]. The exported V_a is then used to modulate the controlled sources in the actual device. As has been discussed earlier, this configuration, along with the probing signal, will reproduce the same voltage and current distributions as those in the background due to the principle of uniqueness. We will show later that it renders the protected region invisible to the probing signal.

In case *B*, the protected region is disguised as a grounded PEC plate with the same radius. Please note that different geometries with different sizes can be chosen. The considered case here actually demonstrates the active version of the "anticloak," a device used to cancel the function of a cloak and make the original hidden object reappear [30]. The ADS simulation for this case is also made to determine the voltage profile (V_b) along the controlled boundary. The simulation result for the potential distribution is illustrated in Fig. 3(b), in which the equipotential lines are significantly distorted from those in Fig. 3(a). Similarly, cases *C* and *D* are simulated for the illusion demonstration, which can disguise the protected region as a hole (with zero conductivity) or an isolated PEC

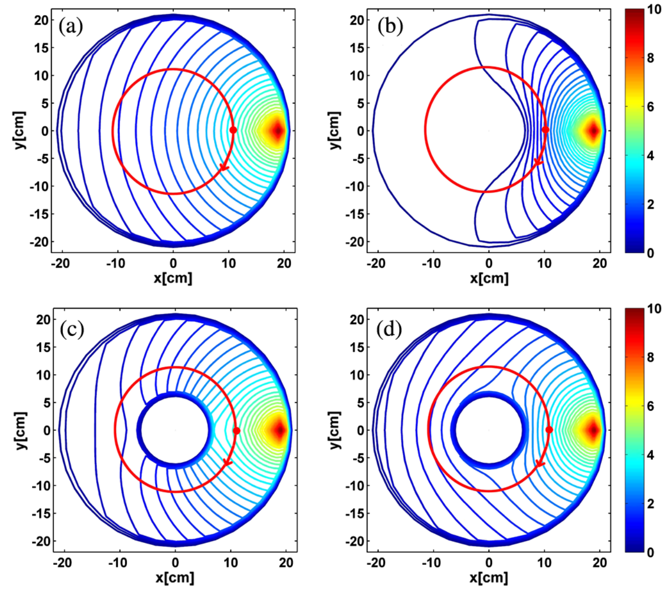

FIG. 3 (color online). The ADS simulation results of potential distributions for different cases, which are used to determine the controlled sources and make comparisons with the experiments. (a) The potential distribution in a resistor network, for the emulation of a homogeneous background medium excited by the probing source. (b) The potential distribution when the central region ($r = 6$ cm) is a grounded PEC plate. (c) The potential distribution when the central region is punched out (i.e., a hole). (d) The potential distribution when the central region is an isolated PEC plate. The red circle in each panel indicates $r = 10$ cm, the red dot represents node 1, and the arrow indicates the direction from nodes 1 to 36.

plate (with infinite conductivity) in the homogeneous background medium, respectively. The related simulation results are presented in Figs. 3(c) and 3(d).

Figure 4 demonstrates the experimental results for the active cloaking. In Fig. 4(a), we first give the ADS

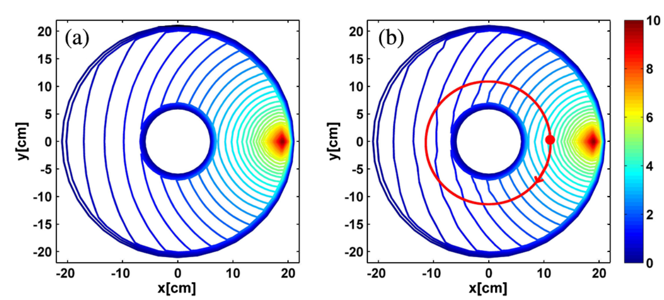

FIG. 4 (color online). The simulated and measured potential distributions for the cloaking case. (a) The ADS simulation result when the controlled sources and probing source are all turned on in the isotropic and homogeneous background, in which the concentric circles represent the equipotential lines. (b) The measurement result for this configuration, which has very good agreement with the simulation. The red circle in panel (b) indicates $r = 10$ cm, the red dot represents node 1, and the arrow indicates the direction from nodes 1 to 36.

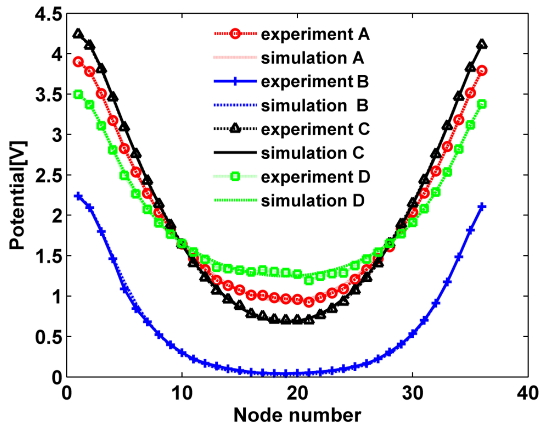

FIG. 5 (color online). The simulated and measured potential distributions along the measurement circle ($r = 10$ cm) for different cases. Case A, the active cloaking; cases B, C, and D, the active illusions, in which the protected region is disguised as a grounded PEC plate, a hole ($\sigma = 0$), and an isolated PEC plate ($\sigma = \infty$), respectively. Excellent agreement is observed in all cases.

simulation result for comparison, in which the controlled sources are included with the voltage distribution V_a, showing an excellent cloaking effect with active sources. The measured potential profile is given in Fig. 4(b), which has very good agreement with the simulation result. The quantitative comparison of the measurement and simulation along a circle with $r = 10$ cm (the red circles in Figs. 3 and 4) is illustrated in Fig. 5. Such results firmly validate the excellent active cloaking performance. In the experiment, the probing source is a dc power supply with a 10 V magnitude. The controlled sources are realized with 36 power supplies of the same type. The voltage at each node is manually measured with a four-and-a-half multimeter, and the final data are processed in the MATLAB environment.

For cases B, C, and D, the experimental results along the measurement circle ($r = 10$ cm) with the active controlled sources are also shown in Fig. 5, which again have very good agreements with the earlier ADS simulations for the grounded PEC plate, the hole region ($\sigma = 0$), and the isolated PEC plate ($\sigma = \infty$), illustrated in Figs. 3(b)–3(d). The good agreement along the measurement circle will ensure the overall agreement in the whole background medium, as dictated by the principle of uniqueness. This fact implies that the active illusion performance is well demonstrated. From the quantitative comparisons illustrated in Fig. 5, we find small discrepancies between experiments and simulations, which are mainly attributed to the following factors: (1) the difference between ideal and actually available resistors, (2) nonideal power supplies, whose internal resistance is not zero, and (3) the manual welding process.

The active cloaking and illusion scheme for Laplace or Helmholtz equations also has its weakness: this scheme depends on knowledge of the probing signal, which is unknown most of the time. This fact implies that one has to use sophisticated, fast, and quick-response devices for the detection of the incoming signals in practice, rendering the whole system complicated. For detailed hardware configuration and source calculation through measurements, the readers are kindly referred to Ref. [31] for more information. As the working frequency increases, this task will be more and more challenging. Furthermore, even if one has obtained the related information, causality imposes the fact that there will be time delays in the regenerated signals, making perfect cloaking and illusion impossible. However, as the development of technology, these problems may be appropriately dealt with. At the present stage, lower frequencies (e.g., dc and acoustic) will be more realizable.

In summary, we have presented the first experimental demonstration of active cloaking and active illusions for the Laplace equation. By using the controllable sources around a protected region, we can make the region totally invisible or make it look significantly different from outside detection. We remark that the proposed method is not only valid for dc electricity but also for thermodynamics and other problems governed by the Laplace equation. In fact, in the current design, the electric currents flowing on the resistor network directly produce heat ($P = I^2R$), mimicking the properties of thermodynamics. Hence, we can use the proposed method to study the control of heat transfers.

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