# Effect of Three-Nucleon Interactions in $p$-$^3$He Elastic Scattering

M. Viviani,[1] L. Girlanda,[2] A. Kievsky,[1] and L. E. Marcucci[1,3]

[1]Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Largo Bruno Pontecorvo 3, I-56127 Pisa, Italy
[2]Department of Mathematics and Physics, University of Salento, and INFN-Lecce, I-73100 Lecce, Italy
[3]Department of Physics, University of Pisa, Largo Bruno Pontecorvo 3, I-56127 Pisa, Italy

(Received 19 July 2013; published 22 October 2013)

We present a detailed study of the effect of different three-nucleon interactions in $p$-$^3$He elastic scattering at low energies. In particular, two interactions have been considered: one derived from effective field theory at next-to-next-to-leading order and one derived from a more phenomenological point of view—the so-called Illinois model. The four-nucleon scattering observables are calculated by using the Kohn variational principle and the hyperspherical harmonics technique, and the results are compared with available experimental data. We have found that the inclusion of both interactions improves the agreement with the experimental data, in particular, for the proton vector analyzing power.

DOI: 10.1103/PhysRevLett.111.172302 PACS numbers: 13.75.Cs, 21.45.Ff, 25.10.+s, 27.10.+h

Acquiring the complete knowledge of the three-nucleon ($3N$) interaction is one of the open question in nuclear physics nowadays. As is well known, there exist a number of different realistic nucleon-nucleon ($NN$) interactions capable to reproduce almost perfectly the experimental $NN$ scattering data up to energies of 350 MeV. However, with only this component of the nuclear interaction, one encounters several problems in the description of $A \geq 3$ nuclear systems (see, e.g., Refs. [1–3]). To improve that situation, different $3N$ forces have been introduced.

The recent development of $3N$ forces has followed mainly two lines. First, there are $3N$ forces derived within a chiral effective field theory (EFT) approach [4,5]. Interactions derived at next-to-next-to-leading order (N2LO) of the so-called chiral expansion have been used so far. At this particular order, the $3N$ force contains two unknown constants [5] usually determined either by fitting the $3N$ and four-nucleon ($4N$) binding energies [6] or, alternatively, the $3N$ binding energy and the Gamow-Teller matrix element (GTME) in the tritium $\beta$ decay [7,8]. The $3N$ force depends also on a cutoff function, which in general includes a cutoff parameter $\Lambda$. With a particular choice of the cutoff function, a local version of the N2LO $3N$ interaction has been derived [6]. The parameter $\Lambda$ is chosen to be for physical reasons of the order of 500 MeV (for a discussion about the size of the $\Lambda$, see Ref. [9]). The derivation of chiral $3N$ forces at successive orders is now in rapid progress [10–12].

Alternatively, within a more phenomenological approach, the so-called Illinois model for the $3N$ force has been derived [13]. This model has been constructed to include specific two- and three-pion exchange mechanisms between the three nucleons. The model contains a few unknown parameters, which have been determined by fitting the spectra of $A = 4$–12 nuclei.

Clearly, it is very important to test these forces to understand how they describe nuclear dynamics. The $A = 3$ and 4 scattering observables are between the best testing grounds to this aim. However, most of the $A = 3$ scattering observables are not very sensitive to the effect of the $3N$ force [1,3]. It is therefore of relevance to study their effect in $4N$ systems.

In recent years, there has been a rapid advance in solving the $4N$ scattering problem with realistic Hamiltonians. Accurate calculations of four-body scattering observables have been achieved in the framework of the Alt-Grassberger-Sandhas (AGS) equations [14,15], solved in momentum space, where the long-range Coulomb interaction is treated by using the screening-renormalization method [16,17]. Solutions of the Faddeev-Yakubovsky (FY) equations in configuration space [18,19] and several calculations using the resonating group model [20,21] were also reported. In this contribution, the four-body scattering problem is solved by using the Kohn variational method and expanding the internal part of the wave function in terms of the hyperspherical harmonic (HH) functions (for a review, see Ref. [22]). Very recently, the efforts of the various groups have culminated in a benchmark paper [23], where it was shown that $p$-$^3$He and $n$-$^3$H phase shifts calculated by using the AGS, FY, and HH techniques and by using several types of $NN$ potentials are in very close agreement with each other (at the level of or less than 1%).

Since $4N$ scattering observables can be calculated with high accuracy, it is timely to investigate the effect of the $3N$ force in these systems. It is important to note that the $4N$ studies performed so far have revealed the presence of several discrepancies between theoretical predictions and experimental data. In $p$-$^3$He elastic scattering, several accurate measurements exist for the unpolarized cross section [24–26], the proton analyzing power $A_y$ [26–28], and other polarization observables [29]. The calculations performed with a variety of $NN$ interactions have shown a glaring discrepancy between theory and experiment for $A_y$ [14,20,26,28,30]. This discrepancy is very similar to the well known "$A_y$ puzzle" in $N$-$d$ scattering. This is a fairly old problem, already reported about 20 years ago [31,32] in the case of $n$-$d$ and later confirmed also in the $p$-$d$ case [33]. For other $p$-$^3$He observables, such as the $^3$He analyzing power $A_{0y}$ and some spin correlation observables, discrepancies have been also observed. Recently [29], at the Triangle University National Laboratory (TUNL) there has been a new set of accurate measurements (at $E_p =$ 1.60, 2.25, 4, and 5.54 MeV) of various spin correlation coefficients, which has allowed for a phase-shift analysis (PSA).

In this Letter, we report a study of the effect of 3$N$ forces in $p$-$^3$He elastic scattering in order to see whether their inclusion allows one to reduce the above-mentioned discrepancies. Clearly, it is important to specify which $NN$ potential is used together with a particular version of 3$N$ interaction. The N2LO 3$N$ force derived from EFT has been used together with the $NN$ potentials constructed within the same approach, in particular, the next-to-next-to-next-to-leading order (N3LO) interaction derived by Entem and Machleidt [9,34]. We have considered the N3LO500 and N3LO600 versions of this $NN$ force, corresponding to cutoff parameters $\Lambda = 500$ MeV and $\Lambda =$ 600 MeV, respectively. Correspondingly, we have to fix the two parameters $c_D$ and $c_E$ present in the N2LO 3$N$ force. Together with the N3LO500 $NN$ interaction, we have considered two versions of the N2LO 3$N$ force; in the first one, labeled 3N-N2LO500*, $c_D$ and $c_E$ have been chosen so as to reproduce the $A = 3, 4$ binding energies as in Ref. [6]. In the second one, labeled 3N-N2LO500, the two parameters have been fixed reproducing the 3$N$ binding energy and the tritium GTME [8]. These two versions have been used to explore the dependence of the results on $c_D$ and $c_E$.

With the N3LO600 $NN$ interaction, we have considered the 3$N$ N2LO force labeled 3N-N2LO600 with $c_D$ and $c_E$ fixed to reproduce the 3$N$ binding energy and the tritium GTME [8]. In this way, we can explore the dependence on $\Lambda$ of the 4$N$ observables. The specific values of the parameters $c_D$ and $c_E$ are summarized in Table I.

The Illinois 3$N$ model has been used in conjunction with the Argonne $v_{18}$ (AV18) $NN$ potential [35]. Between the different Illinois models, we have considered the most recent one, the so-called Illinois-7 model (IL7) [36]. In Table I, we have also reported the corresponding $^4$He binding energy, which results rather close to the experimental value of 28.30 MeV. Therefore, eventual 4$N$ forces should be rather tiny, and their effect in $p$-$^3$He scattering at low energy can be safely neglected.

TABLE I. $NN$-3$N$ interactions used in this work. In columns 2–4, the values of the cutoff parameter $\Lambda$ and the coefficients $c_D$ and $c_E$ entering the EFT forces are reported (the coefficients are adimensional). In the last column, we have reported the corresponding $^4$He binding energy.

| $NN$ + 3$N$ interaction | $\Lambda$ (MeV) | $c_D$ | $c_E$ | $B(^4\text{He})$ (MeV) |
|---|---|---|---|---|
| N3LO500–3N-N2LO500* | 500 | 1.0 | −0.029 | 28.36 |
| N3LO500–3N-N2LO500 | 500 | −0.12 | −0.196 | 28.49 |
| N3LO600–3N-N2LO600 | 600 | −0.26 | −0.846 | 28.64 |
| AV18-IL7 | | | | 28.44 |

For this study we have focused our attention to the effect of the 3$N$ interaction. For this reason we have restricted the electromagnetic interaction between the nucleons to just the point Coulomb interaction between the protons. To be noticed that with the AV18 potential one should include the full electromagnetic interaction, including two-photon exchange, a Darwin-Foldy term, vacuum polarization, and magnetic moment interactions as discussed in Ref. [35]. The effect of these additional terms for $N$-$d$ scattering was studied in Refs. [37,38] and found to have a sizable effect for some polarization observables. Regarding the N3LO500 and N3LO600 $NN$ interactions, one should include only the effect of the two-photon exchange, a Darwin-Foldy term, and vacuum polarization interactions in the $^1S_0$ partial wave [9,39]. Again, we have disregarded them in this work. The effect of these additional electromagnetic interactions will be the subject of a forthcoming paper [40].

In the energy range considered here ($E_p \leq 6$ MeV), the various $p$-$^3$He observables are dominated by $S$-wave and $P$-wave phase shifts ($D$-wave phase shifts give only a marginal contribution, and more peripheral phase shifts are negligible). A comparison of a selected set of calculated phase shifts and mixing parameters with those obtained by the recent PSA [29] reveals that, by using interactions including a $NN$ force only, both $S$- and $P$-wave phase shifts result to be at variance with the PSA. Including the 3$N$ force, we observe a general improvement of the description of the $S$- and $P$-wave phase shifts and mixing parameters. A detailed comparison between the calculated phase shifts and those obtained from the PSA has been reported in Ref. [41].

Let us compare the theoretical results directly with a selected set of available experimental data. To see the effect of the 3$N$ interaction, we have reported in Fig. 1 two bands: one collecting the results obtained by using only $NN$ interactions and one obtained by including also a 3$N$ interaction. We have reported the results for the $p$-$^3$He unpolarized differential cross section, two analyzing power observables, and some spin correlation observables. We note that the differential cross section, the $^3$He analyzing power $A_{y0}$, and the spin correlation coefficients are not particularly sensitive to the adopted interactions, and in general we observe a good agreement with the experimental values in all considered cases.

In contrast, for the proton analyzing power $A_y$, shown in the upper right panel, we note a large sensitivity to the inclusion of the 3$N$ interaction. The calculations performed by using N3LO500 and AV18, in fact, largely

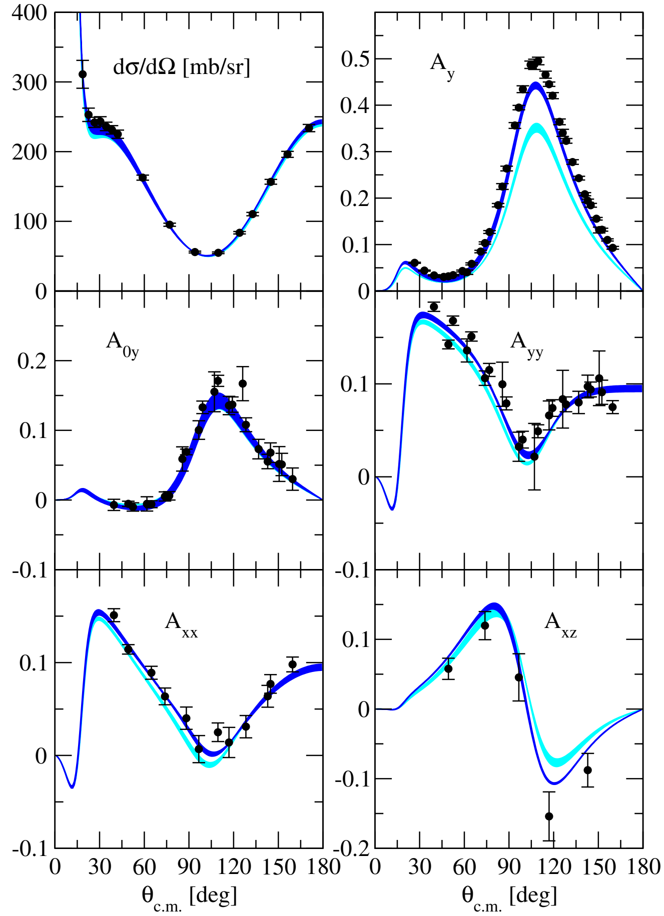


FIG. 1 (color online). $p$-$^3$He differential cross section, analyzing powers, and various spin correlation coefficients at $E_p = 5.54$ MeV calculated with only the $NN$ potential (light cyan band) or including also the $3N$ interaction (darker blue band). The experimental data are from Refs. [26–28].

underpredict the experimental points, a fact already observed before [23,26,28]. A sizable improvement is found by including the $3N$ interaction. The underprediction of the experimental data is now around 8%–10%.

To better point out the sensitivity to the particular interaction, in Fig. 2, an enlargement of $A_y$ and $A_{0y}$ in the peak region is shown. From the inspection of the figure, we can see that the results obtained by using the N3LO500–3N-N2LO500* and N3LO500–3N-N2LO500 interactions are very similar, showing that there is not much sensitivity to the parameters $c_D$ and $c_E$. The observables are more sensitive to the choice of the cutoff $\Lambda$; in particular, $A_y$ calculated with the $\Lambda = 600$ MeV interaction is slightly closer to the experimental data. Finally, the $A_y$ calculated with AV18 and IL7 is very similar to those obtained with the chiral forces, while $A_{0y}$ is in better agreement with the data (however, for this observable the experimental uncertainties are rather large).

The previously observed large underprediction of the $p$-$^3$He $A_y$ observable was considered to be due to some

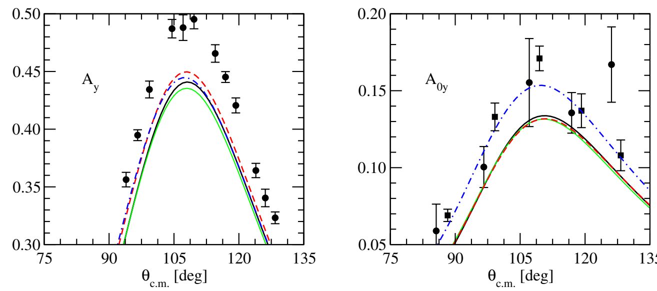


FIG. 2 (color online). $p$-$^3$He observables at $E_p = 5.54$ MeV calculated with the N3LO500–3N-N2LO500* (thick black solid lines), N3LO500–3N-N2LO500 (thin green solid lines), N3LO600–3N-N2LO600 (dashed red lines), and AV18-IL7 (dot-dashed blue lines) interactions. The experimental data are from Refs. [26–28].

deficiencies of the interaction in $P$ waves [28,30], as, for example, due to the appearance of a unconventional "spin-orbit" interaction in $A > 2$ systems [42]. The IL7 force has been fitted to reproduce the $P$-shell nuclei spectra and, in particular, the two low-lying states in $^7$Li. This may explain the improvement in the description of the $p$-$^3$He $A_y$ obtained with this interaction model with respect to that found in Ref. [43] with other $3N$ interactions, as the Urbana IX model [44]. Regarding the N2LO $3N$ forces, its two parameters have been fitted either to the $A = 3$ and 4 binding energies or to reproduce the $3N$ binding energy and the tritium GTME, quantities which are more sensitive to $S$ waves. Therefore, its capability to improve the description of the $p$-$^3$He $A_y$ observable is not imposed, but it is somewhat built-in. We note that, by using the N3LO500–3N-N2LO* interaction, a good reproduction of the experimental $P$-shell nuclei energy levels has been found [45]. Therefore, it seems that, with the interactions which provide a good description of the $P$-shell nuclei energy levels, an improvements of the description of the $p$-$^3$He $A_y$ is found.

It is interesting to examine the effect of the same interactions in $p$-$d$ scattering. To this aim, we report in Fig. 3

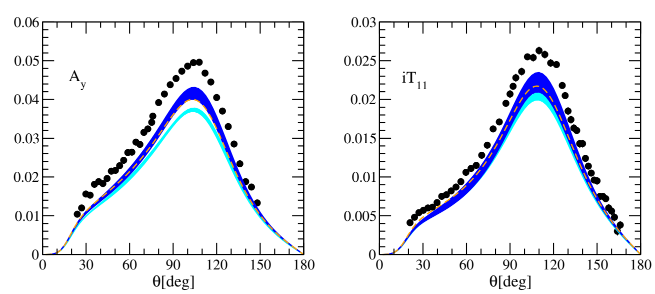


FIG. 3 (color online). $p$-$d$ vector polarization observables at $E_p = 3$ MeV calculated with only the $NN$ potentials (light cyan band) or including also the $3N$ interactions (dark blue band) obtained within EFT. The results obtained with the AV18-IL7 interaction model are reported as the dashed (orange) lines. The experimental data are from Ref. [47].

two vector polarization observables at $E_p = 3$ MeV. In this figure, the light (cyan) band has been obtained by using the *NN* chiral interaction only (in this case, the N3LO500 and N3LO600 forces). The dark (blue) band has been obtained by adding the corresponding N2LO 3*N* interaction. In this figure, the results obtained with AV18 and IL7 are shown by the dashed (orange) lines (in this case, we have included the effect of the magnetic moment interactions since here it is sizable [37]). As can be seen, with the inclusion of 3*N* forces, the underprediction of both observables is reduced; however, it is still of the order of 18%–20%, somewhat larger than for the $p$-$^3$He $A_y$ observable. It should be noticed that the two $p$-$d$ asymmetries, though rather tiny, show a large sensitivity to the *P*-wave phase-shift splitting [1,33,46]. Accordingly, they can be used to fine-tune the strength of the subleading 3*N* spin orbit appearing at next-to-next-to-next-to-next-to-leading order (N4LO) [12].

In conclusion, we have presented for the first time an analysis of $p$-$^3$He elastic scattering observables including the effect of different 3*N* forces. The results obtained have been compared with the available experimental data. We have found that the phase shifts obtained with both the chiral and AV18-IL7 interactions are very close [41] with those derived from the recent PSA performed at TUNL [29]. The direct comparison of the theoretical results with the experimental data has shown that there are still some discrepancies, but the $A_y$ problem is noticeably reduced. In fact, we observe that now the discrepancy is reduced to be of the order of 10% at the peak, much less than before. We have also found that the results obtained with the N3LO-N2LO and AV18-IL7 interactions are always rather close to each other (except for $A_{0y}$). Since the frameworks used to derive these 3*N* forces are rather different, this outcome is somewhat surprising. Finally, it will be certainly very interesting to test the effect of the inclusion of the N3LO and N4LO 3*N* forces derived from EFT. Work in this direction is in progress.

The authors acknowledge the assistance and help of the staff of the computer center of INFN-Pisa, where all the calculations presented in this Letter were performed.

[1] W. Glöckle, H. Witała, D. Hüber, H. Kamada, and J. Golak, Phys. Rep. **274**, 107 (1996).
[2] J. Carlson and R. Schiavilla, Rev. Mod. Phys. **70**, 743 (1998).
[3] N. Kalantar-Nayestanaki, N. Kalantar-Nayestanaki, E. Epelbaum, J. G. Messchendorp, and A. Nogga, Rep. Prog. Phys. **75**, 016301 (2012).
[4] U. van Kolck, Phys. Rev. C **49**, 2932 (1994).
[5] E. Epelbaum, A. Nogga, W. Glöckle, H. Kamada, U.-G. Meißner, and H. Witała, Phys. Rev. C **66**, 064001 (2002).
[6] P. Navrátil, Few-Body Syst. **41**, 117 (2007).
[7] A. Gardestig and D. R. Phillips, Phys. Rev. Lett. **96**, 232301 (2006); D. Gazit, S. Quaglioni, and P. Navrátil, *ibid.* **103**, 102502 (2009).
[8] L. E. Marcucci, A. Kievsky, S. Rosati, R. Schiavilla, and M. Viviani, Phys. Rev. Lett. **108**, 052502 (2012).
[9] R. Machleidt and D. R. Entem, Phys. Rep. **503**, 1 (2011).
[10] V. Bernard, E. Epelbaum, H. Krebs, and U.-G. Meissner, Phys. Rev. C **84**, 054001 (2011); **77**, 064004 (2008).
[11] H. Krebs, A. Gasparyan, and E. Epelbaum, Phys. Rev. C **85**, 054006 (2012); **87**, 054007 (2013).
[12] L. Girlanda, A. Kievsky, and M. Viviani, Phys. Rev. C **84**, 014001 (2011).
[13] S. C. Pieper, V. R. Pandharipande, R. B. Wiringa, and J. Carlson, Phys. Rev. C **64**, 014001 (2001).
[14] A. Deltuva and A. C. Fonseca, Phys. Rev. C **75**, 014005 (2007).
[15] A. Deltuva and A. C. Fonseca, Phys. Rev. Lett. **98**, 162502 (2007); Phys. Rev. C **76**, 021001 (2007).
[16] E. O. Alt, W. Sandhas, and H. Ziegelmann, Phys. Rev. C **17**, 1981 (1978); E. O. Alt and W. Sandhas, Phys. Rev. C **21**, 1733 (1980).
[17] A. Deltuva, A. C. Fonseca, and P. U. Sauer, Phys. Rev. C **71**, 054005 (2005); **72**, 054004 (2005).
[18] F. Ciesielski and J. Carbonell, Phys. Rev. C **58**, 58 (1998); F. Ciesielski, J. Carbonell, and C. Gignoux, Phys. Lett. B **447**, 199 (1999).
[19] R. Lazauskas, J. Carbonell, A. Fonseca, M. Viviani, A. Kievsky, and S. Rosati, Phys. Rev. C **71**, 034004 (2005).
[20] H. M. Hofmann and G. M. Hale, Phys. Rev. C **68**, 021002 (2003); **77**, 044002 (2008).
[21] S. Quaglioni and P. Navrátil, Phys. Rev. Lett. **101**, 092501 (2008).
[22] A. Kievsky, S. Rosati, M. Viviani, L. E. Marcucci, and L. Girlanda, J. Phys. G **35**, 063101 (2008).
[23] M. Viviani, A. Deltuva, R. Lazauskas, J. Carbonell, A. C. Fonseca, A. Kievsky, L. E. Marcucci, and S. Rosati, Phys. Rev. C **84**, 054010 (2011).
[24] K. F. Famularo, Phys. Rev. **93**, 909 (1954).
[25] D. G. McDonald, W. Haberli, and L. W. Morrow, Phys. Rev. B **133**, B1178 (1964).
[26] B. M. Fisher, C. Brune, H. Karwowski, D. Leonard, E. Ludwig, T. Black, M. Viviani, A. Kievsky, and S. Rosati, Phys. Rev. C **74**, 034001 (2006).
[27] M. T. Alley and L. D. Knutson, Phys. Rev. C **48**, 1890 (1993).
[28] M. Viviani, A. Kievsky, S. Rosati, E. A. George, and L. D. Knutson, Phys. Rev. Lett. **86**, 3739 (2001).
[29] T. V. Daniels, C. W. Arnold, J. M. Cesaratto, T. B. Clegg, A. H. Couture, H. J. Karwowski, and T. Katabuchi, Phys. Rev. C **82**, 034002 (2010).
[30] A. C. Fonseca, Phys. Rev. Lett. **83**, 4021 (1999).
[31] Y. Koike and J. Haidenbauer, Nucl. Phys. **A463**, 365 (1987).
[32] H. Witala, W. Glöckle, and T. Cornelius, Nucl. Phys. **A491**, 157 (1989).
[33] A. Kievsky, S. Rosati, W. Tornow, and M. Viviani, Nucl. Phys. **A607**, 402 (1996); A. Kievsky, M. Viviani, and S. Rosati, Phys. Rev. C **64**, 024002 (2001).
[34] D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003).
[35] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).

[36] S. C. Pieper, AIP Conf. Proc. **1011**, 143 (2008).
[37] A. Kievsky, M. Viviani, and L. E. Marcucci, Phys. Rev. C **69**, 014002 (2004).
[38] L. E. Marcucci, A. Kievsky, L. Girlanda, S. Rosati, and M. Viviani, Phys. Rev. C **80**, 034003 (2009).
[39] L. E. Marcucci, R. Schiavilla, and M. Viviani, Phys. Rev. Lett. **110**, 192503 (2013).
[40] M. Viviani *et al.* (to be published).
[41] M. Viviani *et al.*, arXiv:1210.5890.
[42] A. Kievsky, Phys. Rev. C **60**, 034001 (1999).
[43] M. Viviani, L. Girlanda, A. Kievsky, L. E. Marcucci, and S. Rosati, Eur. Phys. J. Web Conf. **3**, 05011 (2010).
[44] B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, Phys. Rev. C **56**, 1720 (1997).
[45] P. Maris, J. P. Vary, and P. Navratil, Phys. Rev. C **87**, 014327 (2013); E. D. Jurgenson, P. Maris, R. J. Furnstahl, P. Navrátil, W. E. Ormand, and J. P. Vary, Phys. Rev. C **87**, 054312 (2013).
[46] A. Kievsky, M. Viviani, L. Girlanda, and L. E. Marcucci, Phys. Rev. C **81**, 044003 (2010).
[47] S. Shimizu, K. Sagara, H. Nakamura, K. Maeda, T. Miwa, N. Nishimori, S. Ueno, T. Nakashima, and S. Morinobu, Phys. Rev. C **52**, 1193 (1995).