

Next-to-Leading-Order QCD Corrections to Higgs Boson Production in Association with a Top Quark Pair and a Jet

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We present the calculation of the cross section for Higgs boson production in association with a top quark pair plus one jet, at next-to-leading-order accuracy in QCD. All mass dependence is retained without recurring to any approximation. After including the complete next-to-leading-order QCD corrections, we observe a strong reduction in the scale dependence of the result. We also show distributions for the invariant mass of the top quark pair, with and without the additional jet, and for the transverse momentum and the pseudorapidity of the Higgs boson. Results for the virtual contributions are obtained with a novel reduction approach based on integrand decomposition via the Laurent expansion, as implemented in the library, NINJA. Cross sections and differential distributions are obtained with an automated setup which combines the GOSAM and SHERPA frameworks.

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Introduction.—The evidence of the existence of a new particle of mass between 125 and 126 GeV, initially reported about one year ago by the ATLAS and CMS Collaborations [1,2], has been confirmed with a very high confidence level by more recent analyses, thus providing more stringent arguments in favor of the validity of the electroweak symmetry breaking mechanism. It is interesting to observe that all the analyses performed so far are in good agreement with the hypothesis that the new particle is the Higgs boson predicted by the standard model (SM). Indeed, rates and distributions are compatible with the assumption that the new particle is a scalar that couples to other SM particles with a strength proportional to their mass [3–5]. Accurate predictions are necessary and will play a crucial role for the complete determination of the nature of the Higgs boson [6], in particular, to shed light on the structure of its couplings to the other particles.

The production rate for a Higgs boson associated with a top quark pair ($t\bar{t}H$) is particularly interesting in this context, since it is directly proportional to the SM Yukawa coupling of the Higgs boson to the top quark. The study of differential observables and distributions will bring information on the coupling structure and on the parity of the Higgs particle [7,8].

The difficulties related to the analysis of the $t\bar{t}H$ channel are well known. The combined production of three heavy particles requires a large center-of-mass energy for the initial partons, which is strongly suppressed by parton-distribution functions. Furthermore, additional difficulties are represented by the presence of various challenging backgrounds and by the complexity of the final

state, which make its kinematic reconstruction far from straightforward [9].

At the parton level, the $t\bar{t}H$ production at next-to-leading order (NLO) in QCD has been known for some time [10–14]. More recently, this process has been employed in a number of studies, motivated by the new analyses performed at the LHC [7–9,15].

In this Letter, we present the complete NLO QCD corrections to the process $pp \rightarrow t\bar{t}H + 1 \text{ jet } (t\bar{t}Hj)$ at the LHC. Examples of contributing one-loop diagrams are depicted in Fig. 1. We illustrate the outcome of our calculation by showing the total cross section, and a selection of differential distributions.

The goal of the considered calculation is twofold. On the one hand, it is important for the phenomenological analyses at the LHC, in particular for the high- p_T region, where the presence of the additional jet can be sensibly relevant. On the other hand, $t\bar{t}Hj$ constitutes the first application of a novel reduction algorithm for the evaluation of one-loop amplitudes, which strengthens the performances of the integrand decomposition [16], in particular, in the presence of massive particles.

Computational setup.—In perturbation theory, computations at the NLO accuracy require, aside from the evaluation of leading-order (LO) contributions, the calculation

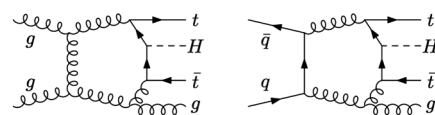


FIG. 1. Sample of one-loop diagrams contributing to the NLO corrections to $gg \rightarrow t\bar{t}Hg$ and $q\bar{q} \rightarrow t\bar{t}Hg$.

of both virtual and real-emission corrections. The Born and the real-emission matrix elements are computed using SHERPA [17] and the library AMEGIC [18], which implements the Catani-Seymour dipole formalism [19]. SHERPA also performs the integration over the phase space and the analysis. The virtual corrections are generated with the GOSAM package [20], which combines automated diagram generation and algebraic manipulation [21–25] with d -dimensional integrand-level reduction techniques [26–32]. The master integrals (MIs) are computed using ONELOOP [33]. The code generated by GOSAM is linked to SHERPA by means of the Binoth Les Houches Accord [34] interface, which uses a system of *order* and *contract* files and allows for a direct communication between the two codes at a running time. The same setup has been recently employed for the computation of NLO QCD corrections to $pp \rightarrow Hjj$ [35] and $pp \rightarrow Hjjj$ [36] (for the latter, in combination with MADDIPOLE-MADEVENT [37–39]) and also for the analysis of the $t\bar{t}$ forward-backward asymmetry [40].

For $t\bar{t}Hj$ production, the basic partonic processes identified by the SHERPA-GOSAM contract file are

$$q\bar{q} \rightarrow t\bar{t}Hg, \quad gg \rightarrow t\bar{t}Hg, \quad (1)$$

while the remaining subprocesses can be obtained by proper crossings. The ultraviolet, the infrared, and the collinear singularities are regularized using dimensional reduction. The renormalization conditions are fixed along the lines of [11,13], where the top mass is renormalized on shell, while the strong coupling is renormalized in the $\overline{\text{MS}}$ scheme, decoupling the top quark from the running. In the case of LO (NLO) contributions, we describe the running of the strong coupling constant with one-loop (two-loop) accuracy. The wave functions of the gluon and of the quarks are renormalized on-shell; i.e., the corresponding renormalization constants cancel the external self-energy corrections exactly.

The virtual amplitudes of $t\bar{t}Hj$ have been decomposed in terms of MIs using, for the first time, the integrand reduction via Laurent expansion [16], implemented in the C++ library NINJA. This new algorithm exploits the complete knowledge of the analytic expression of the integrand and of the residues at the multiple cut to ameliorate the determination of the coefficients of the MIs with respect to the canonical integrand reduction [26]. Elaborating on the techniques introduced in [41–43], the series expansion combined with the integrand decomposition lowers the computational load and improves the accuracy of the results. Within this new algorithm, the sampling of the numerators and the subtractions of the higher-point residues, characterizing the *triangular* system-solving approach of the original integrand-reduction procedure, are avoided. Instead, the series expansion allows for a *diagonal* system-solving strategy, where the polynomial subtractions of the residues, when needed, are replaced by universal correction terms which have to be added to

the coefficients of the Laurent series. These universal corrections, required only for the determination of the coefficients of two-point and one-point MIs, are obtained, once and for all, from the expansions of the generic polynomial forms of the residues at the triple and double cuts.

The NINJA library, which has been interfaced to GOSAM, implements the integrand reduction via the Laurent expansion using a semianalytic algorithm. The coefficients of the Laurent expansion of a generic integrand are efficiently computed by performing a *polynomial division* between the numerator and the set of uncut denominators [16].

The calculation of the NLO virtual corrections performed with NINJA has been checked using the independent reduction algorithm implemented in the library SAMURAI [31]. We verified the agreement of the virtual corrections obtained with the two reduction procedures in ten thousand phase-space points. The values of the double and the single poles, for each individual subprocess, conform to the universal singular behavior of dimensionally regulated one-loop amplitudes [44]. Our results fulfill gauge invariance, verified through the vanishing of the amplitudes when substituting the polarization vector of one or more gluons with the corresponding momentum.

The NINJA reduction algorithm proved to be numerically more efficient and stable. In fact, for the highly nontrivial process under consideration, only a small set of phase-space points, of the order of few per mill, were detected as unstable. All these points have been recovered using the tensorial reduction provided by GOLEM95 [45,46], thus avoiding the necessity of higher precision routines, which are extremely time consuming.

The time required for the computation of the full color- and helicity-summed amplitudes in one phase-space point is about 2.5 sec. The numerical values of the one-loop amplitudes for the two partonic processes listed in Eq. (1) in a nonexceptional phase-space point are collected in the Appendix.

In view of the later comparison between the processes $pp \rightarrow t\bar{t}H$ and $pp \rightarrow t\bar{t}Hj$ at NLO QCD accuracy, we also used the framework obtained combining GOSAM-NINJA with SHERPA to compute the cross section for $t\bar{t}H$ production. We found excellent agreement with the results presented in Refs. [7,47].

Numerical results.—In the following, we present results for the integrated cross section for a center-of-mass energy of 8 TeV. The mass of the Higgs boson is set to $m_H = 126$ GeV and the top quark mass is set to $m_t = 172.5$ GeV. The parameters of the electroweak sector are

TABLE I. Total cross section for $t\bar{t}Hj$ for different choices of the central scale at LO and NLO.

Central Scale	σ_{LO} (fb)	σ_{NLO} (fb)
$2 \times GA_T$	$80.03^{+35.64}_{-23.02}$	$100.6^{+0.00}_{-9.43}$
H_T	$88.93^{+41.41}_{-26.13}$	$102.3^{+0.00}_{-15.82}$

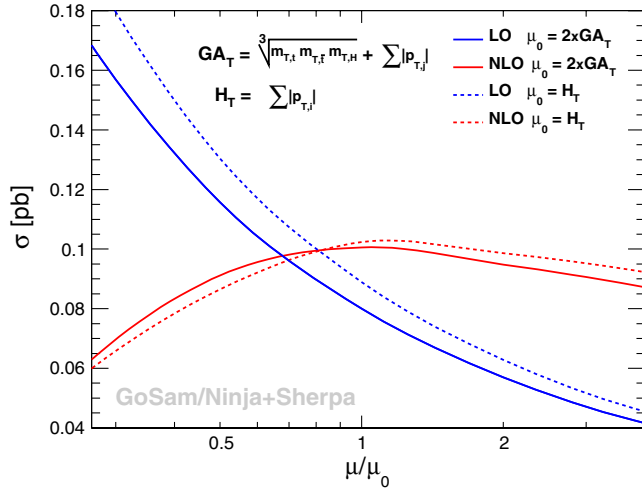


FIG. 2 (color online). Scale dependence of the total cross section at LO and NLO.

fixed by setting $M_W = 80.419$ GeV, $M_Z = 91.1876$ GeV, and $\alpha_{EW}^{-1} = 132.50698$.

To cluster the jets we use the anti- k_t algorithm implemented in FASTJET [48–50] with radius $R = 0.5$, a minimum transverse momentum of $p_{T,jet} > 15$ GeV and pseudorapidity $|\eta| < 4.0$. The LO cross sections are computed with the LO parton-distribution functions cteq6L1 [51], whereas at NLO we use CT10 [52].

In order to study the scale dependence of the total cross section, we employ two different choices of the renormalization and factorization scales $\mu_R = \mu_F = \mu_0$, namely, $\mu_0 = H_T$ and $\mu_0 = 2 \times GA_T$ with

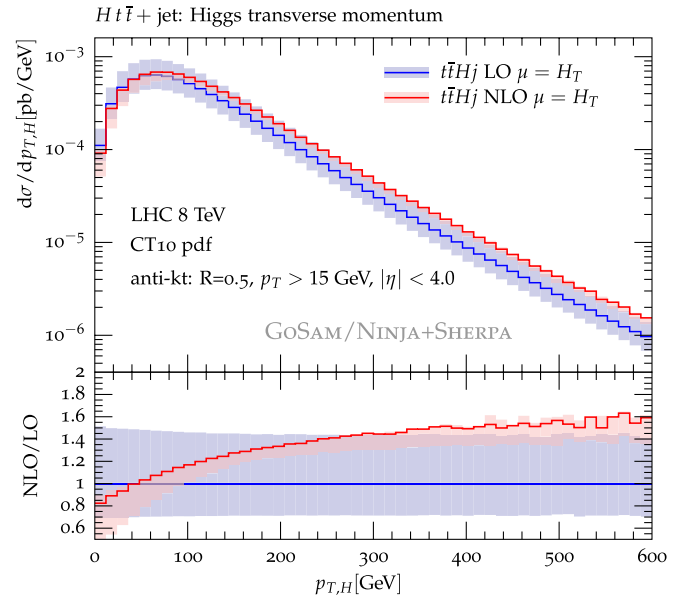


FIG. 4 (color online). Transverse momentum distribution of the Higgs boson at LO and NLO for $\mu = H_T$.

$$H_T = \sum_{\text{final states } f} |p_{T,f}|, \quad (2)$$

$$GA_T = \sqrt[3]{m_{T,H} m_{T,t} m_{T,\bar{t}}} + \sum_{\text{jets } j} |p_{T,j}|. \quad (3)$$

Within this setup, for the two scale choices, we obtain the total LO and NLO cross sections reported in Table I.

The scale dependence of the total cross section, depicted in Fig. 2, is strongly reduced by the inclusion of the NLO

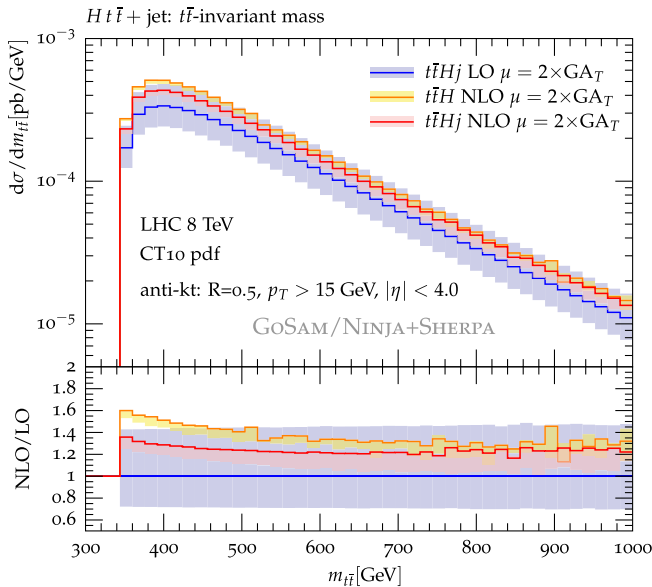


FIG. 3 (color online). Invariant mass distributions of the $t\bar{t}$ pairs for $t\bar{t}H$ and $t\bar{t}Hj$ at NLO relative to the $t\bar{t}Hj$ at LO for $\mu = 2 \times GA_T$.

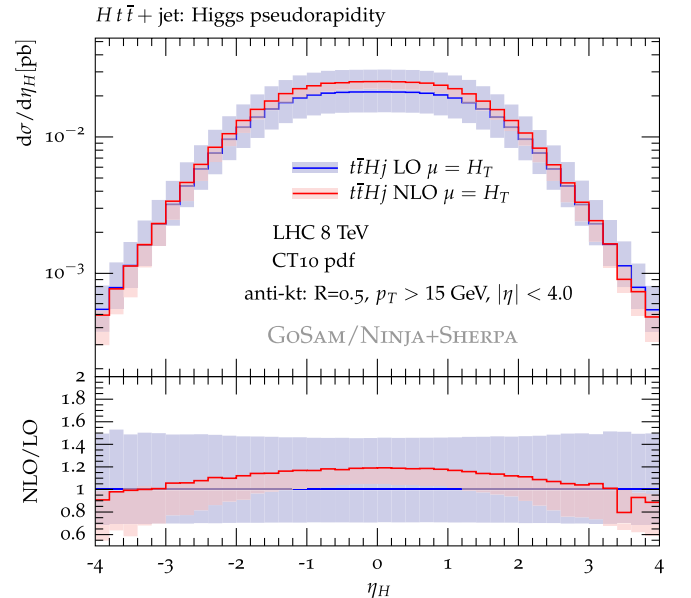


FIG. 5 (color online). Pseudorapidity η of the Higgs boson at LO and NLO accuracy for $\mu = H_T$.

TABLE II. Benchmark phase-space point for $t\bar{t}Hj$ production.

Particle	E	p_x	p_y	p_z
p_1	250.000 000 000 000 00	0.000 000 000 000 000 0	0.000 000 000 000 000 0	250.000 000 000 000 00
p_2	250.000 000 000 000 00	0.000 000 000 000 000 0	0.000 000 000 000 000 0	-250.000 000 000 000 00
p_3	177.223 423 328 684 67	-31.917 865 771 774 753	-19.543 909 461 587 205	-15.848 571 666 570 733
p_4	174.899 512 849 077 35	13.440 699 620 020 803	24.174 898 117 950 033	-8.277 166 758 962 957 6
p_5	126.374 789 176 344 35	6.835 563 367 274 222 2	-3.265 280 159 088 275 2	6.099 209 645 529 803 0
p_6	21.502 274 645 893 632	11.641 602 784 479 652	-1.365 708 497 274 517 5	18.026 528 780 003 872

contributions. It is worthwhile to notice that both choices for the central value of the scale provide an adequate description, being close to the physical scale of the process.

In Fig. 3, we compare the distributions for the invariant mass of the top quark pair in $pp \rightarrow t\bar{t}Hj$ at LO and NLO with the NLO curve for $pp \rightarrow t\bar{t}H$. For $t\bar{t}Hj$, going from LO to NLO accuracy, we observe an increase in the distribution by 20%–35% over the full kinematical range. On the other hand, when comparing the NLO $t\bar{t}H$ prediction with the NLO $t\bar{t}Hj$ curve, the cross section decreases due to the presence of the additional jet which takes away energy from the $t\bar{t}$ system. This is particularly evident near the $t\bar{t}$ production threshold, while for high values of the $t\bar{t}$ invariant mass the two NLO curves get closer. The scale for this comparison is set to $\mu = 2 \times GA_T$.

In Fig. 4 and 5, we display the distributions of the transverse momentum p_T and the pseudorapidity η of the Higgs boson, respectively. Each plot contains the distributions at LO and NLO accuracy, for a value of the scale set to $\mu = H_T$. The NLO corrections are particularly important for high values of the p_T , which are the kinematical regions involved in the boosted analyses [15,53].

These distributions show the potential of the framework obtained combining GOSAM-NINJA with SHERPA, which can be successfully used to compute NLO predictions for multileg processes involving massive particles. Moreover, they shed some light on the impact of further jet activity in $pp \rightarrow t\bar{t}H$, one of the most important processes for the direct determination of the coupling of the Higgs boson to fermions. The NLO QCD corrections reduce the scale uncertainty and their numerical impact can be sizable. Therefore they could be helpful for an accurate simulation of the signal in the experimental searches looking for Higgs production in association with a top quark pair at the LHC.

TABLE III. Numerical results for the two subprocesses listed in Eq. (1) evaluated at the phase-space point of Table II for a scale $\mu = 2p_1p_2 = 500$ GeV. The accuracy of the result is indicated by the underlined digits.

	$q\bar{q} \rightarrow t\bar{t}Hg$	$gg \rightarrow t\bar{t}Hg$
a_0	-71.888 645 827 431 1	-37.184 246 033 542 95
a_{-1}	-32.690 291 018 053 5	-35.921 749 743 763 58
a_{-2}	-5.666 666 666 638 01	-9.000 000 000 056 112

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Appendix: Benchmark phase-space point.—In this appendix we collect numerical results for the renormalized virtual contributions to the processes, Eq. (1), in correspondence to the phase-space point in Table II. The results are collected in Table III and are computed using dimensional reduction. The coefficients a_i are defined as follows:

$$\frac{a_{-2}}{\epsilon^2} + \frac{a_{-1}}{\epsilon} + a_0 \equiv \frac{2\text{Re}\{\mathcal{M}^{\text{tree-level}*} \mathcal{M}^{\text{one-loop}}\}}{(\alpha_s/2\pi)|\mathcal{M}^{\text{tree-level}}|^2}.$$

The reconstruction of the renormalized pole can be checked against the value of a_{-1} and a_{-2} obtained by the universal singular behavior of the dimensionally regularized one-loop amplitudes [44], while the precision of the finite parts is estimated by reevaluating the amplitudes for a set of momenta rotated by an arbitrary angle about the axis of collision.

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