# Magnetic Field-Induced Giant Enhancement of Electron-Phonon Energy Transfer in Strongly Disordered Conductors

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Relaxation of soft modes (e.g., charge density in gated semiconductor heterostructures, spin density in the presence of magnetic field) slowed down by disorder may lead to giant enhancement of energy transfer (cooling power) between overheated electrons and phonons at low bath temperature. We show that in strongly disordered systems with time-reversal symmetry broken by external or intrinsic exchange magnetic field the cooling power can be greatly enhanced. The enhancement factor as large as $10^2$ at magnetic field $B \sim 10$ T in 2D InSb films is predicted. A similar enhancement is found for the ultrasound attenuation.

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*Introduction.*—A number of recent experiments show that energy transfer (the cooling power) $\mathcal{J}(T_e, T_{\rm ph}) = J(T_{\rm el}) - J(T_{\rm ph})$ between overheated electrons with temperature $T_{\rm el} > T_{\rm ph}$ and phonons at low bath temperature $T_{\rm ph}$ may vary by several orders of magnitude when measured per one electron per volume. The outflux $J(T) = WT^p$ may have different power-law temperature dependence with the exponent $p$ both smaller and larger than the classical result $p = 5$ valid for pure metals. In disordered metals with complete screening of Coulomb interaction and impurities that are fully involved in the lattice motion, one expects [1–3] a power law with $p = 6$ which corresponds to weaker energy transfer compared to the clean case. This is related to the "Pippard ineffectiveness condition" (denoted as PIC below) [4,5] formulated for the rate of inelastic electron-phonon scattering. A very accurate experiment in metal films of Hf and Ti [6] confirmed this theoretical expectation, including the value of the prefactor $W$ in front of $T^6$. At the same time, experiments on heavily doped Si [7], which also demonstrated the $T^6$ behavior, gave at low temperatures the value $W/n_e$ ($n_e$ is the carrier density) larger by a factor of $10^3$ than the theoretical prediction in Refs. [1–3]. Surprisingly, the $T^6$ behavior of the cooling rate with approximately the same values of $W/n_e$ as in Ref. [7] was extracted from recent experiments [8] on amorphous InO films showing *weakly insulating* behavior in the magnetic field of 11 T. In this case $W/n_e$ was larger by a factor of $5 \times 10^4$ than the theoretical prediction for a dirty metal approaching the Anderson transition. Clearly, neither of the above cases with anomalously large cooling rate correspond to the piezoelectric type of electron-phonon coupling where the PIC does not hold and the theory predicts $T^4$ temperature behavior of the cooling rate [9,10]. It is also dubious that the model of impurities, which is only partially involved in the lattice motion [2] that also leads to enhanced cooling rate with $T^4$ temperature behavior, is realistic for the cases in question. Thus, there was a quest from experiment for a different and more general mechanism of enhancement of cooling rate in strongly disordered conductors.

In the present Letter we demonstrate the existence of a general mechanism which is capable of enhancing by a factor $10^2$–$10^3$ both the cooling power $J(T)$ and the ultrasound attenuation $\tau_{\rm ph}^{-1}$ (for longitudinal phonons) at low temperatures. This mechanism is effective if the lattice motion is able to induce significant oscillations of *local* densities of certain globally conserved physical quantities. The deviations of these local densities from their equilibrium values are enhanced by the slow diffusive character of electron motion (characterized by both small frequency and small momentum) aimed to restore equilibrium. This leads to a significant retardation in the response and thus to the entropy production and dissipation. The proposed mechanism is reminiscent of the Mandelstam-Leontovich (ML) mechanism of phonon attenuation in liquids [11]. In contrast to PIC which suppresses the relaxation rate $\tau_{\rm ph}^{-1}$ at strong disorder and small carrier concentration, the ML mechanism is efficient at these conditions. The particular realizations of such a mechanism were studied previously in the literature. Specifically, the relatively weak Coulomb interaction between electrons in semiconductors, when the local electroneutrality condition is not strictly obeyed and the density fluctuations are not completely suppressed, was a cause of enhancement of cooling rate discussed in Ref. [12]. The asymmetric intervalley modes were shown [13] to lead to a significant enhancement of cooling rate in multiple-valley semiconductors such as Si. Below we reproduce some of these results from our general approach.

However, a very new effect we are predicting is the giant enhancement of the cooling rate and ultrasound attenuation in the presence of external magnetic field or in ferromagnetic materials where the role of external magnetic field is

 

played by the intrinsic exchange field. In this case it is the spin-density mode, which can be excited by absorption of a phonon or terminated by creation of a phonon, that is responsible for the enhanced ultrasound attenuation or the enhanced cooling rate.

*Cooling power and ultrasonic attenuation.*—We start by considering the quantum kinetic equation [14,15] for the phonon distribution function $B_{\rm ph}(\omega, T)$ in the case of partial equilibrium in both electron and phonon subsystems with temperatures $T_{\rm el} > T_{\rm ph}$:

$$\partial_t B_{\rm ph}(\omega, T_{\rm ph}) = [B_{\rm ph}(\omega, T_{\rm el}) - B_{\rm ph}(\omega, T_{\rm ph})]\tau_{\rm ph}^{-1}(\omega, T_{\rm el}). \tag{1}$$

If the electron-phonon energy relaxation is much slower than the electron-electron one and the phonon system is well coupled to the thermostat (refrigerator), a quasiequilibrium situation with two temperatures is realized. In this approximation $B_{\rm ph}(\omega, T) \equiv B_{\rm ph}(\omega/T) = (1/2)[\coth(\omega/2T) - 1]$ is the equilibrium phonon distribution function. The phonon decay rate $\tau_{\rm ph}^{-1}$ is then given by the imaginary part of the phonon self-energy $\Sigma^R(\omega, q, T)$:

$$\tau_{\rm ph}^{-1}(\omega, T_{\rm el}) = \frac{1}{\rho_m \omega}\, {\rm Im}\Sigma^R(\omega, q, T_{\rm el})|_{\omega = v_s q}. \tag{2}$$

The phonon decay rate depends only on the electron temperature, since the (weak) electron-phonon interaction is considered in the leading approximation, and thus the phonon self-energy (which is second order in the $e$-ph coupling) is expressed in terms of electronic variables only. If in addition the effect of the electron-electron interaction is reduced to charge screening considered in the RPA approximation, the phonon relaxation rate $\tau_{\rm ph}^{-1}(\omega, T_{\rm el}) \equiv \tau_{\rm ph}^{-1}(\omega)$ does not depend explicitly on the electron temperature.

Now the energy flow $\mathcal{J} = dE_{\rm ph}/dt$ from hot electrons to cool phonons can be written as follows: $\mathcal{J} = J(T_{\rm el}) - J(T_{\rm ph})$, where

$$J(T) = \int_0^\infty d\omega\, \omega\, \nu_{\rm ph}(\omega) \frac{B_{\rm ph}(\omega/T)}{\tau_{\rm ph}(\omega)}, \tag{3}$$

and $\nu_{\rm ph}(\omega) = \omega^2/(2\pi^2 v_s^3)$ is the phonon density of states for 3D phonons with the sound velocity $v_s$. Equations (2) and (3) establish a relationship between the cooling rate $J(T)$ and the attenuation time $\tau_{\rm ph}(\omega)$ of ultrasound with the frequency $\omega$. In particular, it follows from Eq. (3) that, for the power-law dependence $\tau_{\rm ph}^{-1}(\omega) \propto \omega^\beta$, the cooling rate due to 3D phonons is proportional to $J(T) \propto T^{4+\beta}$.

*Local and diffusion contribution to cooling rate.*—In impure conductors there are two distinctly different contributions to the phonon relaxation time. One is local and determined by the small distances $|\mathbf{r} - \mathbf{r}'| \sim l$ between the points $\mathbf{r}$ and $\mathbf{r}'$ of phonon absorption and reemission. The other one allows many scattering events of electrons off impurities between the points $\mathbf{r}$ and $\mathbf{r}'$. This is the diffusive contribution. With increasing disorder and decreasing the mean free path $l$, the local contribution diminishes. This leads to the so-called PIC when the relaxation rate $\tau_{\rm ph}^{-1}(\omega)$ of phonons with momentum $q$ is proportional to $lq^2 \ll q$ instead of $\tau_{\rm ph}^{-1} \propto q$ for longitudinal phonons in the clean case [4,5]:

$$\frac{1}{\tau_{\rm ph}^{\rm (PIC)}(\omega)} = c_\alpha \frac{2\nu p_F^2}{\rho_m} Dq^2 \sim \frac{mn_e}{\rho_m} Dq^2, \tag{4}$$

where $\nu$ is an electron density of states per spin [16], $p_F = mv_F$ is Fermi momentum, $n_e$ and $\rho_m$ are the density of electrons and the mass density of material, $D = v_F l/d_e$ is the diffusion coefficient, $q = \omega/v_s \ll 1/l$ is the phonon momentum, and $d_e$ is the dimensionality of electron motion. The subscript $\alpha$ corresponds to the choice of either transverse (tr) or longitudinal ($l$) phonons; correspondingly, numerical coefficients $c_\alpha$ are defined as $c_{\rm tr} = 1/(2 + d_e)$ and $c_l = 2(1 - d_e^{-1})/(2 + d_e)$.

The diffusion contribution has an opposite trend and increases with increasing disorder. The goal of our Letter is to analyze this very contribution in different physical situations.

We will use the comoving frame of reference (CFR) bound to the lattice and impurities rigidly imbedded in it and moving in the laboratory frame of reference (LFR). Then, for a single branch of electrons, one finds [1,17] for electron-phonon interaction in the CFR

$$H_{e\text{-ph}} = -\sum_{\mathbf{p},\mathbf{q}} p_\alpha (v_\beta \nabla_\beta u_\alpha)_q \cdot \bar{\psi}_p \psi_{p+q}, \tag{5}$$

where $\mathbf{p}$ and $\mathbf{v}$ denote electron momentum and velocity, respectively, and $\mathbf{u}$ is the lattice displacement. Note that this term appears due to the inhomogeneous Galilean shift $\mathbf{p}d\mathbf{u}(\mathbf{r}(t), t)/dt$ of the energy of a quasiparticle at a point $\mathbf{r} = \mathbf{r}(t)$, while the usual deformation potential in LFR is canceled by the modification of $e$-$e$ interaction due to inhomogeneous coordinate transformation [18] $\hat{H} \to \hat{U}^{-1}\hat{H}\hat{U}$, with $\hat{U} = 1 + (1/2)\{\mathbf{u}, \boldsymbol{\nabla}\}$. The tensor structure of Eq. (5) is crucial for local processes only, while for diffusion processes, it is sufficient to average the $e$-ph vertex over the Fermi surface. For a metal with isotropic electron dispersion, one finds $\Gamma = \overline{p_\alpha (v_\beta \nabla_\beta u_\alpha)_q} = pv/d_e {\rm div}\mathbf{u}$. In general, $\Gamma$ may contain other contributions. In particular, for semiconductors $\Gamma$ is known [19] to be much larger than $E_F$ due to contribution $\Gamma_{bs}$ originating from the shift of conduction band edge $E_b$.

Under the condition of strict electroneutrality, the scalar vertex $\Gamma$ is screened out completely and the classical result Eq. (4) is valid. This is not the case, however, when $N$ different types of quasiparticles are present [13]. Then the interaction can be written as

$$H_{e\text{-ph}} = \sum_{j=1}^{N} \Gamma^{(j)} \mathrm{div}\mathbf{u} (\bar{\psi}\psi)_j, \tag{6}$$

where ($\nu_j$ is the partial DOS at the Fermi level):

$$\Gamma^{(j)} = -p^{(j)} v^{(j)}/d_e + \Gamma_{bs}. \tag{7}$$

Note that Eq. (6) is principally different from the $e$-ph interaction in the LFR, even when $\Gamma_{bs} = 0$. The latter contains the deformation potential $\Gamma_{\mathrm{def}} = \sum_j \nu_j (p_F^{(j)} v_F^{(j)}/d_e)/\sum_j \nu_j$, which is *symmetric* in the electron branch indices $j$, as well as in the moving-impurity part [1–3]. The latter part leads to the mode asymmetry of the $e$-ph interaction in LFR which in CFR is provided by the Galilean shift term.

The Coulomb interaction is able to screen out only the single mode corresponding to the total density $n = \sum_j (\bar{\psi}\psi)_j$, whereas $N-1$ asymmetric modes survive screening [13,17]. Their slow, diffusive character in strongly disordered conductors may lead to a considerable enhancement of the cooling rate and ultrasound attenuation. The particular case of the effect of such unscreened diffusion modes was studied in Ref. [13] for the case of $N$ species of electrons corresponding to $N$ inequivalent valleys in semiconductors.

Below we present a simple derivation of the diffusion-enhanced contribution to the phonon relaxation rate $\tau_{\mathrm{ph}}^{-1}$ in terms of macroscopic equations for the current and density of electrons; an alternative diagrammatic derivation is presented in [20], Sec. III. In the CFR the continuity and diffusion equations for each $i$th species of quasiparticles read

$$\partial_t n^{(i)} + \mathrm{div}\mathbf{j}^{(i)} = 0, \qquad \mathbf{j}^{(i)} = -D\nabla n^{(i)} - \kappa_i \mathbf{F}^{(i)}, \tag{8}$$

where ($i$) stands for the quasiparticle branch number, $n^{(i)}$ is the electron density, $\mathbf{j}^{(i)}$ is the particle number current, $\kappa_i = \nu_i D_i$ is the mobility, $D_i$ is the diffusion coefficient for the $i$th branch, $\mathbf{F}^{(i)} = -\nabla U^{(i)}$, and $U^{(i)}$ is the potential energy. In the simplest derivation we assume no interbranch mixing and thus the continuity equations in Eq. (8) imply that each of the partial electron densities $n^{(i)}$ are conserved separately. Generalization to the case where there is mixing between the branches will be done at the end of the Letter. The potential energy $U^{(i)} = U_C + \Phi^{(i)}$ in Eq. (8) consists of the Coulomb part $U_C$ and the phonon part $\Phi^{(i)} = \Gamma^{(i)}\mathrm{div}\mathbf{u}$:

$$U^{(i)} = \int \mathcal{V}_0(\mathbf{r}-\mathbf{r}') \sum_j \delta n^{(j)}(\mathbf{r}') + \Gamma^{(i)}\mathrm{div}\mathbf{u}, \tag{9}$$

where $\mathcal{V}_0(\mathbf{r})$ is the bare Coulomb potential acting between conduction electrons [below we use its Fourier transform $V_0(q)$]. Note that $V_0(q)$ does not include screening by conduction electrons in the sample.

Equations (8) and (9) are a full set of equations describing the diffusion and screening of partial densities $n_i$. Let us first study their solution in the case of perfect screening and multiple electron branches ($N > 1$). It formally corresponds to $\Pi(\omega, q)V_0(q) \gg 1$, where $\Pi(\omega, q)$ is the total polarization function. For the density modulation $n^{(i)}(\omega, q)$ induced by the phonon with frequency $\omega$ and momentum $q$, one finds from Eqs. (8) and (9),

$$n^{(i)}(\omega, q) = \Pi_i(\omega, q)[\Phi_i(\omega, q) - \Phi_C(\omega, q)], \tag{10}$$

where $\Phi_i(\omega, q) = \Gamma^{(i)}\mathrm{div}\mathbf{u}$, $\Phi_C = \sum_j \Phi_j \Pi_j / \sum_j \Pi_j$ represents dynamical screening of the Coulomb interaction, and $\Pi_i = \nu_i D_i q^2/(-i\omega + D_i q^2)$ is the partial polarization function. The solution Eq. (10) obeys charge neutrality: $n_{\mathrm{tot}} = \sum n^{(i)} = 0$.

The diffusion contribution to the phonon decay rate may be expressed as $\tau_{\mathrm{ph,ML}}^{-1} = |Q_t|/E_w$, where $Q_t$ and $E_w$ are the dissipation power and the acoustic wave energy in a unit volume, respectively:

$$Q_t = \frac{1}{2}\mathrm{Re}(\mathbf{j}^* \cdot \mathbf{F}), \qquad E_w = \frac{\rho_m}{2}\omega^2 u_m^2. \tag{11}$$

Here, $u_m$ is an amplitude of ionic displacement and $\mathbf{u} = (\mathbf{q}/q)u_m \exp[-i\omega t + i\mathbf{q}\cdot\mathbf{r}]$. Below we apply Eq. (11) to compute $\tau_{\mathrm{ph}}^{-1}(\omega)$.

*Giant enhancement by magnetic field.*—The case of $N = 2$ quasiparticle branches has a very important application. It corresponds to the two spin projections. However, they should be inequivalent with respect to the coupling to phonons. This is a consequence of the general statement that the spin density can be excited by phonons only if time-reversal invariance is broken. First, we discuss the case when time-reversal invariance is broken by external magnetic field.

For $N = 2$ a simple calculation based on Eqs. (10) and (11) leads to the following expression for the diffusion contribution to the decay rate of acoustic phonon:

$$\tau_{\mathrm{ph}}^{-1}(q) = \frac{(\Gamma_1 - \Gamma_2)^2}{\rho_m} \frac{\nu_* D_* q^2}{v_s^2 + D_*^2 q^2}, \tag{12}$$

where $v_s = \omega/q$ is the sound velocity, while $\nu_* = (\nu_1^{-1} + \nu_2^{-1})^{-1}$ and $D_* = \nu_*^{-1}[(\nu_1 D_1)^{-1} + (\nu_2 D_2)^{-1}]^{-1}$ are the effective density of states and diffusion coefficient, respectively.

When a parallel magnetic field $H$ is applied to a two-dimensional electron gas, the bottom of the spin-down and spin-up conduction bands gets shifted by $\pm(1/2)\mu H$ with respect to their position at $H = 0$. This leads to a change of $\delta(p_F v_F) = \pm\mu H$, where $(1/2)\mu = (g/2)\mu_B$ is the electron magnetic moment. Thus, from Eq. (7) we conclude that an asymmetry $\Gamma_\uparrow - \Gamma_\downarrow = (2/d_e)\mu H$ arises due to the Galilean shift of the quasiparticle energy. Then, according to Eq. (12), the phonon relaxation rate acquires an $H$-dependent contribution that may become dominant at sufficiently strong field and low phonon frequencies. Adding the local contribution (4) and

the magnetic-field-controlled diffusion contribution, Eq. (12), one finds for the full phonon decay rate $\tau_{\rm ph}^{-1} = [\tau_{\rm ph}^{(\rm PIC)}]^{-1}\mathcal{F}_H(q,h)$, where for $\mathbf{q}$ parallel to 2D gas

$$\mathcal{F}_H(q,h) = 1 + \frac{v_F^2 h^2}{v_s^2 + (Dq)^2}. \tag{13}$$

Here, $\tau_{\rm ph}^{(\rm PIC)}$ is given by Eq. (4) for longitudinal phonons and $h = (|\mu|H/2\varepsilon_F)$ (we assume here $h \ll 1$). The enhancement factor $\mathcal{F}_H$ can become very large for strong spin polarization, $h \sim 1$. In particular, for low phonon momentum, $ql \leq v_s/v_F$, the factor $\mathcal{F}_H$ is of the order of inverse adiabatic parameter $(v_F/v_s)^2 \sim 10^5$. The strong spin-orbit interaction which leads to mixing of spin-up and spin-down branches sets a limitation on the enhancement factor. Its maximum value becomes $\mathcal{F}_{\max} \sim \tau_{\rm SO}/\tau$ ($\tau_{\rm SO}$ is the spin relaxation time and $\tau \ll \tau_{\rm SO}$ is the momentum relaxation time) instead of $\mathcal{F}_m \sim (v_F/v_s)^2$. This makes the optimization of parameters to maximize the enhancement factor a hard problem, since materials with a large $g$ factor (to maximize $\mu H$) usually have large spin-orbit coupling. Nevertheless, the example of InSb films shows that $\mathcal{F} \sim 10^2$ is experimentally achievable (see Fig. 1).

*Enhancement of cooling rate in ferromagnetic metals.*—Another relevant example is provided by ferromagnetic metals with strong intrinsic band splitting due to the exchange field. In the case of Fe, $\mu H^* \approx 1.8$ eV, $\varepsilon_F = 11.1$ eV, $v_F = 1.98\times10^8$ cm/s, $v_s \approx 6\times10^5$ cm/s. The spin relaxation rate may be estimated as $\tau/\tau_{\rm SO} \sim (\alpha Z)^4 \sim 10^{-3}$, $\alpha = 1/137$ and $Z = 26$ being the fine structure constant and the atomic number, respectively, resulting in the maximum enhancement of the phonon relaxation time as large as $\mathcal{F}_H \sim (\mu H^*/\varepsilon_F)^2(\tau_{\rm SO}/\tau) \sim 10$.

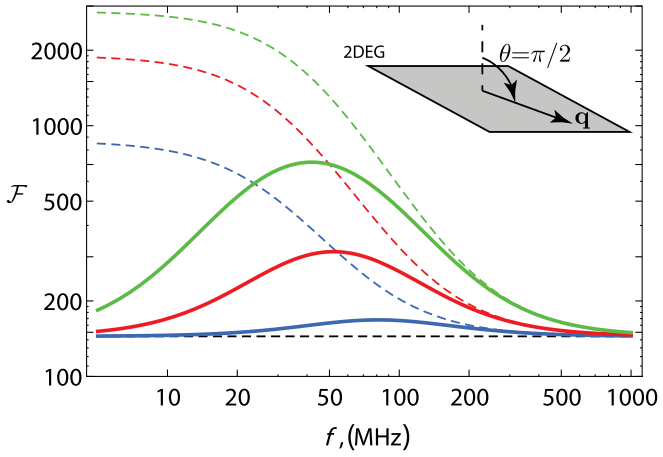


FIG. 1 (color online). The total enhancement factor $\mathcal{F} = \tau_{\rm ph}^{(0)}/\tau_{\rm ph}$ at $\theta = \pi/2$ of ultrasound attenuation in the 2D semiconductor InSb. The parameters taken are $n = 10^{11}$ cm$^{-2}$, $p_F l = 50$, $\Delta_{\rm SO} = 0.1$ meV, and magnetic fields are 3 T (blue curves), 5 T (red curves), and 7 T (green curves). Dashed curves represent the result in the absence of SO relaxation, $\Delta_{\rm SO} = 0$. The black dashed curve gives the enhancement $\mathcal{F}_C$ by incomplete screening.

*Enhancement by incomplete screening.*—In the case of a single quasiparticle branch, the general approach, Eqs. (8), (9), and (11), describes the diffusion-enhanced dissipation due to violation of the charge neutrality condition at a large screening length. In this case, we obtain $n = 2\nu Dq^2[-i\omega + Dq^2 + 2\kappa q^2 V_0(q)]^{-1}\Phi(\omega,q)$ and the enhancement factor

$$\mathcal{F}_C = 1 + \frac{c_l^{-1}(\Gamma/p_F v_F)^2}{(v_s/v_F)^2 + d_e^{-2}(q^2l^2)[1 + 2\nu V_0(q)]^2}. \tag{14}$$

For the 2D gas with Coulomb interaction and the constant dielectric permittivity $\varepsilon$ of surrounding media, we have $V_0(q) = V_{\rm 2D}(q) = 2\pi e^2/\varepsilon q$. In the relevant range of $\mathbf{q}$ parallel to the 2D gas, the $\mathcal{F}_C(q)$ factor reduces to a constant. This corresponds to the cooling rate $J(T) \propto T^6$ [12] but with the enhanced prefactor proportional to $\varepsilon^2/g_\square^2$ (where $g$ is dimensionless conductance per square in $e^2/h$ units) at strong disorder and large dielectric constant $\varepsilon$. In 3D conductors, when $V_0(q) \propto q^{-2}$, Eq. (14) has a regime were $\mathcal{F}_C \propto q^2$. Correspondingly, the cooling rate appears to be $J(T) \propto T^8$ [12].

An interesting situation arises in 2D electron gas in the presence of a gate that additionally screens Coulomb interaction and allows the density to fluctuate stronger. For this geometry and $\mathbf{q}$ parallel to 2D gas, $V_0(q) \to V_g(q) = V_{\rm 2D}(q)(1 - e^{-2qb})$, $b$ being the distance between the 2D electron gas and the gate. For phonons with wavelengths $1/q \geq b$, the effective potential $V_g(q) \approx V(q)2qb \approx$ const, and the presence of an adiabatic parameter in the denominator of (14) does become important at low enough temperatures:

$$\mathcal{F}_{C_{\rm gate}} = 1 + \frac{4(\Gamma/p_F v_F)^2}{(v_s/v_F)^2 + (q^2l^2)(4\pi\nu e^2 b/\varepsilon)^2}, \tag{15}$$

where Coulomb interaction is still assumed to be relatively strong: $2\pi\nu e^2 b/\varepsilon \gg 1$. In this case there is a regime where the enhancement factor is proportional to $q^{-2}$, and the cooling rate $J(T) \propto T^4$.

*Mode mixing and a realistic example.*—Finally, we collect results of the ML enhancement due to both the charge density and the spin density fluctuations, also taking into account mixing of spin projections by the spin-orbit interaction characterized by the parameter $\tau_{\rm SO}^{-1}$. We also consider the dependence of relaxation rate on the direction of phonon propagation relative to the 2D gas [20]. Both effects lead to the replacement $Dq^2/(-i\omega + Dq^2) \Rightarrow D\mathbf{q}_\parallel^2/(-i\omega + D\mathbf{q}_\parallel^2 + 1/2\tau_{\rm SO})$. It results in the total enhancement factor of the form

$$\mathcal{F} = \mathcal{F}_{C_{\rm 2D}} + \frac{\mathbf{q}_\parallel^2 v_F^2 h^2}{(qv_s)^2 + (D\mathbf{q}_\parallel^2 + 1/2\tau_{\rm SO})^2}. \tag{16}$$

For 2D electrons and 3D phonons, $|\mathbf{q}_\parallel| = q\sin\theta$ is the phonon momentum component parallel to the 2D system which appears in all the terms originating from electron diffusion. In this case, $\mathcal{F}_{C_{2D}}$ is independent of $q$, and $\mathcal{F}$ has a maximum as a function of $\omega$. The spin fluctuation effect given by the second term vanishes at small $\omega$ because of the mixing of branches caused by spin-orbit interaction. It also decreases at large $\omega$ because the dissipation power increases slower with $\omega$ than does the acoustic wave energy. At large enough Zeeman splitting $h$ when the effect of spin fluctuations in its maximum is large, there is a wide frequency region (the falling part of the curve $\mathcal{F} \propto \omega^{-2}$ in Fig. 1), where $\tau_{\rm ph}^{-1}$ is almost frequency independent. In this region the cooling or heating rate $J(T) \propto T^4 \ln T$ for the quasi-2D case. This temperature dependence is almost the same as in the case of impurities which are not fully involved in the lattice motion [21]. The extra logarithmic factor arises because of the angular averaging of $1/\tau_{\rm ph}(\theta)$ dominated by the small values of $\theta$. To illustrate this behavior we consider a thin film of semiconductor InSb ($g$ factor $|g| \approx 50$). At strong (and parallel to the 2D plane) magnetic fields $|g|\mu_B H \gg \Delta_{\rm SO}$ classification in terms of the spin subbands is still valid approximately, in spite of the Rashba spin-orbit coupling $\Delta_{\rm SO}$. The analysis presented in [20], Secs. IV and VI, leads to Eq. (16) and is summarized in Fig. 1.

In conclusion, we demonstrated the existence of a general relaxation mechanism that leads to enhancement of both the $e$-ph cooling power and the phonon decay rate. In particular, it may lead to a strong enhancement of the cooling power in disordered conductors in the external magnetic field or in disordered ferromagnetic metals.

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[1] A. Schmid, Z. Phys. **259**, 421 (1973).
[2] M. Reyzer and A. V. Sergeev, Zh. Exp. Theor. Fiz. **92**, 2291 (1987) [Sov. Phys. JETP **65**, 1291 (1987)].
[3] V. I. Yudson and V. E. Kravtsov, Phys. Rev. B **67**, 155310 (2003).
[4] A. B. Pippard, Philos. Mag. **46**, 1004 (1955).
[5] A. I. Akhiezer, M. I. Kaganov, and G. Ya. Lyubarskyi, Zh. Eksp. Teor. Fiz. **32**, 837 (1957).
[6] M. E. Gershenson, D. Gong, T. Sato, B. S. Karasik, and A. V. Sergeev, Appl. Phys. Lett. **79**, 2049 (2001).
[7] A. Savin, J. Pekola, M. Prunnila, J. Ahopelto, and P. Kivinen, Phys. Scr. **T114**, 57 (2004).
[8] M. Ovadia, B. Sacepe, and D. Shahar, Phys. Rev. Lett. **102**, 176802 (2009).
[9] D. V. Khveshchenko and M. Reizer, Phys. Rev. B **56**, 15 822 (1997).
[10] M. E. Gershenson, Yu. B. Khavin, D. Reuter, P. Schafmeister, and A. D. Wieck, Phys. Rev. Lett. **85**, 1718 (2000).
[11] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, England, 1977).
[12] A. Sergeev, M. Yu. Reizer, and V. Mitin, Phys. Rev. Lett. **94**, 136602 (2005).
[13] M. Prunnila, P. Kivinen, A. Savin, P. Torma, and J. Ahopelto, Phys. Rev. Lett. **95**, 206602 (2005); M. Prunnila, Phys. Rev. B **75**, 165322 (2007).
[14] A. Sergeev and V. Mitin, Europhys. Lett. **51**, 641 (2000).
[15] For application of kinetic approach, see, e.g., A. D. Semenov, G. N. Goltsman, and R. Sobolewski, Supercond. Sci. Technol. **15**, R1 (2002).
[16] In the case of 2D electron gas we consider an array of 2D electron planes with the distance $L$ between them. Then in Eq. (1), $\nu \to \nu_{2D}/L$.
[17] E. I. Blount, Phys. Rev. **114**, 418 (1959).
[18] T. Tsuneto, Phys. Rev. **121**, 402 (1961).
[19] M. Cardona and N. E. Christensen, Phys. Rev. B **35**, 6182 (1987).
[20] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.111.166603 for seven chapters, where we present some details that were omitted in the text. In particular, in the first chapter we present the Hamiltonian of the electron-phonon interaction in both laboratory (LFR) and comoving frames of reference (CFR), while in the third chapter we rederive our results via the diagrammatic approach. The expressions for the heat flow are given in the sixth chapter.
[21] A. Sergeev and V. Mitin, Phys. Rev. B **61**, 6041 (2000).