Light Composite Scalar in Twelve-Flavor QCD on the Lattice

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On the basis of lattice simulations using highly improved staggered quarks for twelve-flavor QCD with several bare fermion masses, we observe a flavor-singlet scalar state lighter than the pion in the correlators of fermionic interpolating operators. The same state is also investigated using correlators of gluonic interpolating operators. Combined with our previous study that showed twelve-flavor QCD to be consistent with being in the conformal window, we infer that the lightness of the scalar state is due to infrared conformality. This result shed some light on the possibility of a light composite Higgs boson ("technidilaton") in walking technicolor theories.

At the Large Hadron Collider, the existence of a new bosonic particle of mass $m_H \approx 125$ GeV [1,2], identified as the Higgs boson, has recently been confirmed. Although recent analyses show its consistency with the standard model Higgs boson, a possibility still allowed from current data is that this boson is a composite particle, coming from a new high-energy strongly interacting gauge theory. A typical example is the walking technicolor theory [3] (see also Refs. [4–6]), featuring approximate scale invariance and a large anomalous dimension $\gamma_m \approx 1$. Such a theory predicts a light composite Higgs "technidilaton" [3], a light scalar particle emerging as a (pseudo-)Nambu-Goldstone boson of the spontaneously broken approximate scale symmetry. In this context, we consider one family of theories, the SU(3) gauge theory with N_f massless Dirac fermions in the fundamental representation (dubbed large N_f QCD). Large N_f QCD has been studied by many groups using different lattice discretizations and techniques in the search for a candidate of the walking technicolor model (for reviews, see Refs. [7–9]).

We (LatKMI collaboration) put effort into studying $N_f = 4, 8, 12, 16$ QCD using lattice simulations with a common setup. Studies of the conformal and walking properties of the $N_f = 12$ and 8 theories have already been published [10,11]. Here, we would like to focus on the $N_f = 12$ theory that we found [10] to be consistent with the conformal theory and having unbroken chiral symmetry. Pressed and inspired by the impressive experimental discovery of the Higgs boson, we present our results for the mass of the lightest scalar bound state in the $N_f = 12$ SU(3) gauge theory using fully nonperturbative lattice Monte Carlo simulations. This measurement of

the nonperturbative scalar spectrum is important because it gives information on the possible conformal dynamics at low energy and it serves as a model towards study of the composite Higgs boson in the walking technicolor model, if not a candidate as it stands for the walking technicolor model. As we will explain in the following, the scalar spectrum is incredibly challenging, and those challenges are added to the usual problems faced by lattice calculations near the conformal window [7–9].

In this Letter, we report the results of our calculations, which show a scalar state lighter than π (N_f-flavor extension of the pion). A preliminary report on such a light scalar was given in Ref. [12]. In QCD, the lightest flavor-singlet scalar state is the $f_0(500)$ (σ) meson, whose mass has been reviewed in the latest Particle Data Group [13]. Several studies of the σ meson were carried out in lattice QCD [14–19]. Another example of a flavor-singlet scalar particle would be the 0^{++} glueball, whose existence as a resonance in QCD has yet to be proven (see Ref. [20] for a detailed review). Two-pions scattering states could also be relevant to this channel when combined in an s wave [21]. In QCD, we also expect a mixing between gluonic and fermionic degrees of freedom, and this could be the case also for larger N_f values. Note that the candidate for a composite Higgs or a technidilaton must be predominantly a fermionic bound state and not a glueball state, since the gluons do not carry $SU(2) \times U(1)$ charges. Our contribution is the first of this kind for a theory that could be relevant for the physics beyond the standard model.

To study flavor-singlet states using lattice simulations, the computation of disconnected diagrams is mandatory for a correct estimate of their mass. This requires

computationally expensive measurements and high statistics in order to give results with relatively small errors. Previous studies of the scalar spectrum using fermionic operators in $N_f = 12$ QCD either did not include the computation of disconnected diagrams [22] or were restricted to an unphysical region of the parameter space that is not related to the continuum limit physics of the asymptotically free theory [23].

We discretize the continuum SU(3) gauge theory with 12 degenerate fermions using 3 degenerate staggered fermion species of bare mass m_f (each coming in four tastes). In this Letter, all the dimensionful quantities are expressed in lattice units. At finite lattice spacing, where the simulations take place, the continuum flavor symmetry does not hold exactly. We use a tree-level Symanzik gauge action and the highly improved staggered quark [24] action without the tadpole improvement and the mass correction in the Naik term [25] for the fermions. The flavor symmetry breaking of this action is highly suppressed in QCD [25], and we observed that it is almost negligible in our $N_f = 12$ QCD simulations [10]. At fixed lattice spacing, defined by the bare coupling constant $\beta = 6/g^2 = 4.0$, we simulate three physical volumes L^3 with $L = 24, 30, 36$ and aspect ratio $T/L = 4/3$. We investigate the flavor-singlet scalar spectrum at four different bare quark masses $m_f = 0.05$, 0.06, 0.08, and 0.10. These parameters allow us to check for finite size systematics and to test hyperscaling [26,27].

We carry out the simulations by using the standard hybrid Monte Carlo algorithm using MILC code version 7 [28] with some modifications to suit our needs, such as the Hasenbusch mass preconditioning [29] to reduce the large computational cost at the smaller m_f value. Beside the excellent flavor (taste) symmetry, another important feature of our simulations is the large number of Monte Carlo trajectories from uninterrupted Markov chains obtained after more than 1000 trajectories for thermalization. For all sets of parameters explored, we collect between 8000 and 30000 trajectories, and we do measurements every two trajectories. This is a necessary step to contrast the rapid degradation of the signal in the flavor-singlet scalar correlators. The simulation parameters and number of trajectories for each parameter are tabulated in Table I. For the measurement of the ground-state mass of this channel we used interpolating operators including both the fermionic fields and the gauge fields, with the appropriate quantum numbers. The statistical errors for the fermionic and gluonic measurements are estimated by a jackknife method with bin sizes of 200 and 160 trajectories, respectively.

In our fermionic scalar calculation, we employ the local fermionic bilinear operator

$$\mathcal{O}_S(t) = \sum_{i=1}^{3} \sum_{\vec{x}} \bar{\chi}_i(\vec{x}, t)\chi_i(\vec{x}, t), \tag{1}$$

where the index i runs through different staggered fermion species. The explicit staggered spin-taste structure of the bilinear operator can be written as $\bar{\chi}_i(y + A) \times (\mathbf{1} \otimes \mathbf{1})_{AB}\chi_i(y + B)$ with y as an origin of the hypercube, and A, B as vectors in the hypercube. Note that this system has exact symmetry for exchanging the species. The taste symmetry breaking, which is to vanish in the continuum limit, is very small in our simulations. Therefore, a part of the full flavor symmetry is exact, and the rest is only broken by a small amount. From $\mathcal{O}_S(t)$ we calculate the correlator, which is constructed by both the connected $C(t)$ and vacuum-subtracted disconnected $D(t)$ correlators, $\langle \mathcal{O}_S(t)\mathcal{O}_S^\dagger(0)\rangle = 3D(t) - C(t)$, where the factor in front of $D(t)$ comes from the number of species. It is noted that the contribution of $D(t)$ with respect to $C(t)$ increases with $N_f = $ no. species $\times$ 4.

The operator $\mathcal{O}_S$ overlaps with the flavor-singlet scalar state (σ) but also with a flavor nonsinglet pseudoscalar

TABLE I. Parameters of lattice simulations for $N_f = 12$ QCD at fixed $\beta = 4.0$. N_{cfgs} is the number of saved gauge configurations. The second error of m_σ is a systematic error coming from the fit range. The values of m_π are from Ref. [10], but the ones with (*) have been updated. The error on m_σ/m_π comes only from m_σ.

$L^3 \times T$	m_f	N_{cfgs}	m_σ	m_π	m_σ/m_π
$24^3 \times 32$	0.05	11000	$0.237(13)\binom{02}{01}$	$0.3273(19)$*	$0.73(4)\binom{1}{0}$
$24^3 \times 32$	0.06	14000	$0.279(17)\binom{07}{01}$	$0.3646(16)$*	$0.77(5)\binom{2}{0}$
$24^3 \times 32$	0.08	15000	$0.359(21)\binom{01}{18}$	$0.4459(11)$	$0.81(5)\binom{0}{4}$
$24^3 \times 32$	0.10	9000	$0.453(42)\binom{37}{08}$	$0.5210(7)$	$0.87(8)\binom{7}{2}$
$30^3 \times 40$	0.05	10000	$0.275(13)\binom{21}{08}$	$0.3192(14)$*	$0.86(4)\binom{7}{3}$
$30^3 \times 40$	0.06	15000	$0.329(15)\binom{47}{12}$	$0.3648(9)$*	$0.90(4)\binom{13}{3}$
$30^3 \times 40$	0.08	15000	$0.382(21)\binom{03}{16}$	$0.4499(8)$	$0.85(5)\binom{1}{4}$
$30^3 \times 40$	0.10	4000	$0.431(51)\binom{06}{04}$	$0.5243(7)$	$0.82(10)\binom{1}{1}$
$36^3 \times 48$	0.05	5000	$0.283(23)\binom{01}{02}$	$0.3204(7)$*	$0.88(7)\binom{0}{1}$
$36^3 \times 48$	0.06	6000	$0.305(22)\binom{25}{06}$	$0.3636(9)$*	$0.84(6)\binom{7}{2}$

state ($\pi_{\overline{SC}}$), which is the staggered parity partner of σ; therefore, in the large-time limit, the correlator above behaves as

$$3D(t) - C(t) = A_\sigma(t) + (-1)^t A_{\pi_{\overline{SC}}}(t), \quad (2)$$

where $A_H(t) = A_H(e^{-m_H t} + e^{-m_H(T-t)})$, and the pseudo-scalar state has a ($\gamma_5\gamma_4 \otimes \xi_5\xi_4$) spin-taste structure but is species singlet.

Because $C(t)$ can be regarded as a flavor nonsinglet scalar correlator, it should have a contribution from the lightest nonsinglet scalar state (a_0) (e.g., $a_0(980)$ in QCD [13]) and its staggered parity partner (π_{SC}). When t is large, we can therefore write

$$-C(t) = A_{a_0}(t) + (-1)^t A_{\pi_{SC}}(t), \quad (3)$$

where both a_0 and π_{SC} are species nonsinglet and have the same taste structure as σ and $\pi_{\overline{SC}}$, respectively. The π_{SC} state is degenerate with ($\gamma_5 \otimes \xi_5$) π and also with $\pi_{\overline{SC}}$ ($m_{\pi_{SC}} = m_\pi = m_{\pi_{\overline{SC}}}$) when the taste symmetry and, thus, the full flavor symmetry is recovered.

The disconnected correlator $D(t)$, which is essential to obtain the σ mass, can be calculated by inverting the staggered Dirac operator at each space-time point ($\vec{x}, t$). The computational cost of this inversion is mitigated by using a stochastic noise method. Moreover, its large fluctuations from the random noise in the method are dealt with by using a variance reduction method already employed for the flavor-singlet pseudoscalar [30,31] and chiral condensate [32] in usual QCD and for the flavor-singlet scalar meson in $N_f = 12$ QCD [23]. We employ 64 spacetime random sources for this reduction method. From Eqs. (2) and (3), the large-time asymptotic form of $3D(t)$ can be written as

$$3D(t) = A_\sigma(t) - A_{a_0}(t) + (-1)^t[A_{\pi_{SC}}(t) - A_{\pi_{\overline{SC}}}(t)]. \quad (4)$$

A typical result for $-C(t)$ and $3D(t)$ is shown in Fig. 1. In the large-time region, $3D(t)$ behaves as a smooth function of t in contrast to $-C(t)$, which has a clear oscillating
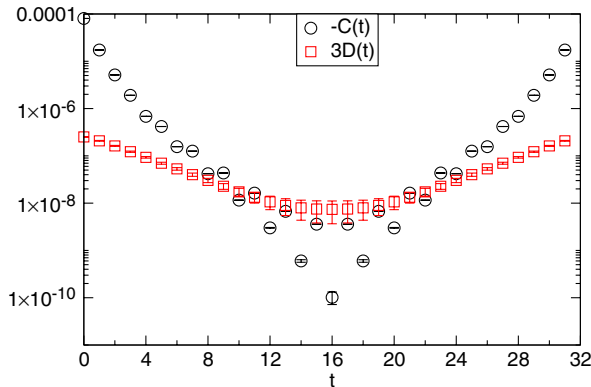
behavior. This means that the taste-symmetry breaking between $A_{\pi_{SC}}(t)$ and $A_{\pi_{\overline{SC}}}(t)$ in Eq. (4) is small, as expected from our previous work [10].

In order to minimize $A_{\pi_{\overline{SC}}}(t)$ in $3D(t) - C(t)$, we adopt a projection $C_+(t) = 2C(t) + C(t + 1) + C(t - 1)$, at even t. Figure 2 shows that the effective mass of $3D_+(t) - C_+(t)$ at large t is smaller than m_π, whereas the error is large. As an alternative method, we also employ $D(t)$ to extract m_σ, and its effective mass is also shown in the figure. The effective mass plateau of $D(t)$ is consistent with the one of $3D_+(t) - C_+(t)$ in the large-time region. Furthermore, the plot clarifies the importance of using $D(t)$ to extract m_σ because it performs better in identifying the lightest scalar state, even at small temporal separations. This might be caused by a reasonable cancellation among contributions from excited scalar states and the a_0 state in $D(t)$. It should be noted that because of the small m_σ value the exponential damping of $D(t)$ is slow, which helps in preventing the rapid degradation of the signal-to-noise ratio.

We fit $D(t)$ between $t = 4$ and $t = 8$, assuming a single light state propagating in this region, to obtain m_σ for all the parameters. A systematic error coming from the fitting range choice is estimated by the difference of central values obtained with several fit ranges. The results of m_σ and m_π are reported in Table I. We find that $m_\sigma < m_\pi < m_{a_0}$ for all the investigated fermion masses. The difference of m_σ and m_π is more than 1 standard deviation when the statistic and systematic errors are combined in quadrature, except for $m_f = 0.06$ on $L = 30$, where there is a sizable systematic error.

As mentioned before, bound states in the 0^{++} channel for a non-Abelian strongly interacting gauge theory can contain gluonic degrees of freedom as well as fermionic ones. This gluonic content has been studied already in the SU(2) gauge theory with two adjoint fermions [33]. The method to measure glueball masses on the lattice employs
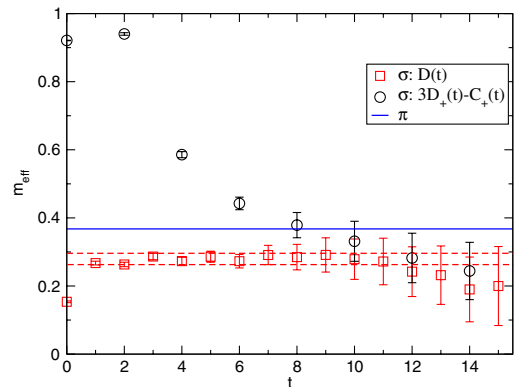

FIG. 2 (color online). Effective scalar mass m_σ from correlators in Eq. (2), with the projection explained in the text, and in Eq. (4) for $L = 24$ and $m_f = 0.06$. The dashed and solid lines highlight the fit result for m_σ with statistical error band and m_π, respectively.

FIG. 1 (color online). Connected $-C(t)$ and disconnected $3D(t)$ correlators for $L = 24$ and $m_f = 0.06$.

a large number of different interpolating operators built from gauge-invariant combinations of gauge links in such a way that a robust basis for a variational ansatz can be created [see for example Refs. [34–37]]. We build gauge-invariant and zero-momentum interpolating operators $\mathcal{O}_G^\alpha(t)$ with scalar rotational quantum numbers. By using differently shaped spatial Wilson loops, we construct 32 different basis operators for the scalar glueball. Each of these operators is smeared at several levels (five or six), and we obtain a large variational basis $\mathcal{O}_G^\alpha(t)$, $\alpha = 1, \ldots, 160(192)$.

The variational ansatz proves to be successful in extracting a signal for the ground state from vacuum subtracted cross-correlation matrices of the form

$$C^{\alpha\beta}(t) = \langle \mathcal{O}_G^\alpha(t) \mathcal{O}_G^\beta(0) \rangle - \langle \mathcal{O}_G^\alpha \rangle \langle \mathcal{O}_G^\beta \rangle. \qquad (5)$$

We analyze the correlator after a projection on the eigenstate corresponding to the smallest mass. Figure 3 shows the effective mass of such a state for $m_f = 0.06$ on the $L = 24$ volume, in comparison with the one obtained from the fermion bilinear (which has already been shown in Fig. 2). Remarkably, the asymptotic plateaus from both operators agree, though the statistical noise is larger in the gluonic case. The agreement indicates that the gluonic operator has an overlap with the light scalar state that couples to the fermion bilinear. On the $L = 24$ volume, we estimate the scalar mass m_G by fitting the large-time behavior ($t = 6$–8) of the correlator, and we obtain $m_G = 0.242(68)$ at $m_f = 0.05$, $m_G = 0.246(79)$ at $m_f = 0.06$, and $m_G = 0.28(12)$ at $m_f = 0.08$. These m_G are all lighter than m_π by more than 1 standard deviation, whereas the statistical errors are large.

Figure 4 presents the flavor-singlet scalar spectrum as a function of m_f. All the m_G values are consistent with m_σ at each parameter. For m_σ on the largest two volumes at each

m_f value, finite size effects are negligible in our statistics. For a check of consistency with the hyperscaling of m_π, we fit m_σ on the largest volume data at each m_f using the hyperscaling form $m_\sigma = C(m_f)^{1/1+\gamma}$ with a fixed $\gamma = 0.414$ estimated from m_π [10], which gives a reasonable value of $\chi^2/\text{DOF} = 0.12$. The fit is shown in Fig. 4. We remind here that the fitted data points have $m_\pi L > 11.5$, as can be checked from Table. I. We also estimate the ratio m_σ/m_π at each parameter and report it in Table I. All the ratios are smaller than unity by more than 1 standard deviation including the systematic error, except the one at $m_f = 0.06$ on $L = 30$, as previously explained. A constant fit with the largest volume data at each m_f gives 0.86(3). These results are consistent with the theory being infrared conformal. Moreover, they do not show an abnormal m_f dependence of m_σ similar to the one observed in Ref. [23], by which an effect of an unphysical phase boundary would have been suspected.

To summarize, we performed the first study of the scalar flavor-singlet state in $N_f = 12$ QCD using fermionic and gluonic interpolating operators. The most striking feature of the measured scalar spectrum is the appearance of a state lighter than the π state, as it is shown in Fig. 4. Such a state appears in both gluonic and fermionic correlators at small bare fermion mass. Clear signals in our simulations were possible thanks to the following salient features: (1) Small taste-symmetry breaking, (2) efficient noise-reduction methods, (3) large configuration ensembles, and (4) slow damping of $D(t)$ because of a small m_σ value.

We regard the light scalar state observed for $N_f = 12$ in this study as a reflection of the dilatonic nature of the conformal dynamics, since otherwise the p-wave bound state (scalar) is expected to be heavier than the s-wave one (pseudoscalar). Thus, it is a promising signal for a walking theory, where a similar conformal dynamics in a wide
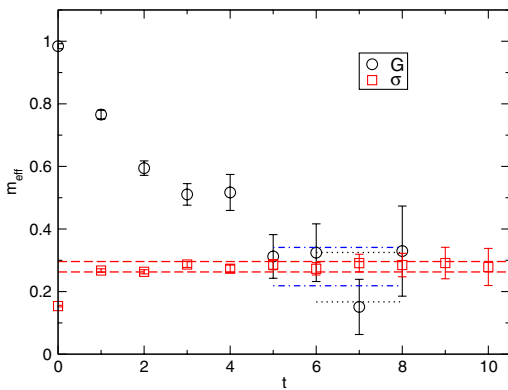

FIG. 3 (color online).   Fermionic m_σ and gluonic m_G effective masses [respectively from correlators in Eqs. (4) and (5)] for $L = 24$ and $m_f = 0.06$. The fitted masses are highlighted by dashed and dotted-dashed lines for the gluonic correlators and dotted lines for the fermionic one. The systematics effects on the gluonic mass are not relevant given the larger statistical error.
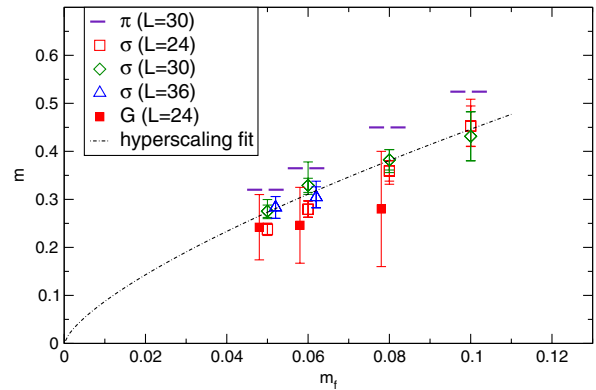

FIG. 4 (color online).   The mass of the flavor-singlet scalar meson σ (see Table I) compared to the mass of the pseudoscalar π state and the mass m_G from gluonic operators. Errors are statistical, and systematics are added in quadrature. The hyperscaling curve is described in the text. The triangle and filled square symbols are slightly shifted for clarity.

infrared region should be operative in the chiral limit to form a dilatonic state with mass of $\mathcal{O}(F_\pi)$, in such a way that the tiny spontaneous-breaking-scale F_π plays the role of m_f (cf. Ref. [11]).

While further investigation of the scalar state in $N_f = 12$ QCD, such as a possible lattice spacing dependence, is important, the most pressing future direction is to look at more viable candidates for walking technicolor models. For example, it will be interesting to investigate the scalar spectrum of the $N_f = 8$ SU(3) theory, which was shown to be a good candidate for the walking technicolor model [11], where the scalar state could be identified with the technidilaton, a pseudo-Nambu-Goldstone boson coming from the dynamical breaking of conformal symmetry. There actually exists an indication of such a light scalar in $N_f = 8$ QCD [38].

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