Equivalence Principle and Bound Kinetic Energy

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We consider the role of the internal kinetic energy of bound systems of matter in tests of the Einstein equivalence principle. Using the gravitational sector of the standard model extension, we show that stringent limits on equivalence principle violations in antimatter can be indirectly obtained from tests using bound systems of normal matter. We estimate the bound kinetic energy of nucleons in a range of light atomic species using Green's function Monte Carlo calculations, and for heavier species using a Woods-Saxon model. We survey the sensitivities of existing and planned experimental tests of the equivalence principle, and report new constraints at the level of between a few parts in 10^6 and parts in 10^8 on violations of the equivalence principle for matter and antimatter.

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General relativity follows from the Einstein equivalence principle (EEP), which holds that in any local Lorentz frame about any point in spacetime, the laws of physics are described by the standard model of particle physics and special relativity [1]. Both general relativity and the standard model are believed to be the low energy limits of some as yet unknown complete theory of physics at high energy scales. Such a theory might generate violations of EEP at experimentally accessible energy scales [2–4], although its exact form is unknown. Thus, it is important to search for EEP violation in as many different places as possible. We use the standard model extension (SME) [4], a flexible and widely applied [5] framework for describing violations of EEP. The SME is an effective field theory that phenomenologically augments the standard model action with terms that break local Lorentz invariance and other tenets of EEP [6], while preserving energy conservation, gauge invariance, and general covariance. As in other models [2], EEP violation in the SME can manifest in multiple ways. In particular, it may be strongly suppressed in normal matter relative to antimatter [6,7]. Although the equivalence principle has been validated with extremely high precision for normal matter [8], the situation for antimatter is less clear.

In this Letter, we show that in the SME, EEP violation in antimatter can be constrained by tests using bound systems of normal matter. We clearly demonstrate how an anomaly that violates the weak equivalence principle for free particles generates anomalous gravitational redshifts in the energy of systems in which they are bound, in proportion to the systems' internal kinetic energy. Using a nuclear shell model, we estimate the sensitivity of a variety of atomic nuclei to EEP violation for matter and antimatter, and illustrate points of commonality between older representations of EEP violation based on neutron excess and baryon number, and that of the SME. We show that existing experimental [8–17] limits on spin-independent EEP violation in matter and antimatter [7] are up to ten times tighter than previously thought, and could be made tighter still, provided more precise estimates of the bound kinetic energy of particles in atomic systems. We focus on EEP violation in conventional matter (made up of protons, neutrons, and electrons), and as in prior work [5–7], assume that anomalies affecting force-carrying virtual particles are negligible. Using general covariance, we define our coordinates such that photons follow null geodesics, ensuring that electromagnetic fields do not violate EEP.

In the SME, spin-independent violations of EEP acting on a test particle of mass m^w are described in its action [6]

$$S=-\int\left(\frac{m^w c\sqrt{-[g_{\mu\nu}+2(\bar{c}^w)_{\mu\nu}]dx^\mu dx^\nu}}{1+(5/3)(\bar{c}^w)_{00}}+(a^w_{\rm eff})_\mu\right)dx^\mu, \tag{1}$$

where the superscript $w = p$, n, or e (for proton, neutron, or electron) indicates the type of particle in question, $g_{\mu\nu}$ is the metric tensor, dx^μ is the interval between two points in spacetime, and c is the speed of light. The $(\bar{c}^w)_{\mu\nu}$ tensor describes a fixed background field that modifies the effective metric that the particle experiences, and thus, its inertial mass relative to its gravitational mass. The four-vector $(a^w_{\rm eff})_\mu = \{(1-\frac{U\alpha}{c^2})(\bar{a}^w_{\rm eff})_0, (\bar{a}^w_{\rm eff})_j\}$, where U is the Newtonian potential, represents the particle's coupling to a field with a nonmetric interaction α with gravity. As $(a^w_{\rm eff})_\mu$ is *CPT* odd [4], this term enters with opposite sign in the action for an antiparticle $\bar{w}$. Both $(\bar{c}^w)_{\mu\nu}$ and $(a^w_{\rm eff})_\mu$ vanish if general relativity is valid. For convenience, Eq. (1) includes an unobservable scaling of the particle mass by $[1+\frac{5}{3}(\bar{c}^w)_{00}]$.

We focus on the isotropic subset of this model [6], in which $(\bar{c}^w)_{\mu\nu}$ is diagonal and traceless, and the spatial terms in the vector $(a^w_{\rm eff})_\mu$ vanish. In this limit, EEP violation is described by the comparatively poorly constrained $(\bar{c}^w)_{00}$ coefficients [5], and the $(\bar{a}^w_{\rm eff})_0$ terms, which cannot be measured in nongravitational experiments [4]. In the nonrelativistic, Newtonian limit, less the rest mass energy and assuming that any violations of EEP must be small, the single particle Hamiltonian produced by the action (1) is given by

$$H = \frac{1}{2} m^w v^2 - m^w_g U, \tag{2}$$

where the effective gravitational mass m^w_g is given by

$$m^w_g = m^w \left(1 - \frac{2}{3}(\bar{c}^w)_{00} + \frac{2\alpha}{m^w}(\bar{a}^w_{\rm eff})_0\right).$$

Experimentally observable EEP violations are proportional to the particle's gravitational to inertial mass ratio

$$\frac{m^w_g}{m^w} = 1 - \frac{2}{3}(\bar{c}^w)_{00} + \frac{2\alpha}{m^w}(\bar{a}^w_{\rm eff})_0 \equiv 1 + \beta^w, \tag{3}$$

and are described here, as elsewhere [7,9,18], by the parameter β^w. From Eq. (3), we see that both $(\bar{c}^w)_{00}$ and $(\bar{a}^w_{\rm eff})_0$ are responsible for violations of the weak equivalence principle, an aspect of EEP [19], since they produce particle-dependent rescalings of the effective gravitational potential. In addition, EEP violation is not apparent in the nonrelativistic motion of a free particle if $\alpha(\bar{a}^w_{\rm eff})_0 = (m^w/3)(\bar{c}^w)_{00}$, although it remains manifest in the motion of the antiparticle $\bar{w}$, for which $\beta^{\bar{w}} = -2\alpha/m^w(\bar{a}^w_{\rm eff})_0 - 2/3(\bar{c}^w)_{00}$, a limit known as the isotropic parachute model [6]. As we now demonstrate, however, the antimatter anomaly $\beta^{\bar{w}}$ does contribute to tests involving nongravitationally bound systems of matter, thanks to the anomalous gravitational redshift produced by $(\bar{c}^w)_{00}$ in the energies of bound systems.

For a bound system of particles, the total Hamiltonian is a sum of single-particle Hamiltonians, plus an interaction energy $V_{\rm int}$ that is assumed to be free of EEP-violating terms. As implicit in Eq. (2), we take the system's squared center of mass velocity $\bar{v}^2$ to be small, and of similar order as the relevant change ΔU it explores in the gravitational potential. Since the system is nongravitationally bound, however, we cannot assume that the same is true of its constituent particles. Thus, we must include terms proportional to $v^2_{w,j}U/c^2$ in our Hamiltonian, where $v_{w,j}$ is the instantaneous velocity of the jth bound particle of species w. In the limit that $\bar{v} \ll v_{w,j} \ll c$, we may approximate $v^2_{w,j} = (\bar{v} + \delta v_{w,j})^2 \approx \bar{v}^2 + (\delta v_{w,j})^2$, [dropping the mixed $\bar{v}(\delta v_{w,j})$ terms which make little contribution to the bound kinetic energy] and obtain

$$H = V_{\rm int} + \sum_w \left[\frac{1}{2} m^w N^w \bar{v}^2 - m^w N^w U(1 + \beta^w) + \frac{1}{2}\sum_{j=1}^{N^w} (\delta v_{w,j})^2 \left(1 + \frac{3U}{c^2} + \frac{2U}{3c^2}(\bar{c}^w)_{00}\right)\right]. \tag{4}$$

The second line in Eq. (4) represents the system's internally bound kinetic energy $T_{\rm int}$, and includes a term that contributes to the system's conventional gravitational redshift, as well as a term proportional to $(\bar{c}^w)_{00}$ and the gravitational potential U. This last term corresponds to an anomalous gravitational redshift of the bound state energies. To evaluate this term for bound quantum states, we recast it in terms of the momenta $\delta\vec{p}_{w,j}$ conjugate to the particle displacements $\delta x_{w,j} = x_{w,j} - \bar{x}$ from the system's center of mass $\bar{x}$. The momenta satisfy $\delta\vec{p}_{w,j} = \partial H/\partial(\delta\vec{v}_{w,j})$, and so

$$(\delta\vec{p}_{w,j}) = m^w(\delta\vec{v}_{w,j})\left(1 + \frac{3U}{c^2} + \frac{2U}{3c^2}(\bar{c}^w)_{00}\right). \tag{5}$$

The bound kinetic energy $T_{\rm int}$ in Eq. (4) is, thus,

$$T_{\rm int} = \sum_w \sum_{j=1}^{N^w} \frac{(\delta p_{w,j})^2}{2m^w}\left(1 - \frac{3U}{c^2} - \frac{2U}{3c^2}(\bar{c}^w)_{00}\right). \tag{6}$$

Note that, in general, to ensure that the system's mass defect is subject to a conventional gravitational redshift in the absence of EEP violation, $V_{\rm int}$ must depend upon U. If EEP is satisfied, the variation of the mass defect $m'_A = (V_{\rm int} + T_{\rm int})/c^2$ for a system A in a gravitational potential U is such that the ratio $m'_A(U_1)/m'_A(U_2) = 1 + (U_1 - U_2)/c^2$. Because of our initial scaling of the particle mass in Eq. (1), the factor in parentheses in Eq. (6) contains terms proportional to 1, U, $U(\bar{c}^w)_{00}$, but not $(\bar{c}^w)_{00}$ alone. This, along with our assumption that $V_{\rm int}$ is independent of $(\bar{c}^w)_{00}$ and $(\bar{a}^w_{\rm eff})_0$, implies that the ratio $m'_A(U_1)/m'_A(U_2)$ does not generate additional cross terms in $U(\bar{c}^w)_{00}$, and we can, therefore, write the total Hamiltonian for a bound system A as

$$H = \frac{1}{2} M_A \bar{v}^2 - M_A U\left(1 + \beta^A + \frac{2}{3}\sum_w \frac{T^w_{\rm int}}{M_A c^2}(\bar{c}^w)_{00}\right), \tag{7}$$

where $M_A = (\sum_w N^w m^w) - m'_A$ incorporates the conventional components of $V_{\rm int} + T_{\rm int}$, the total kinetic energy of all w particles in the system is $T^w_{\rm int} = \sum_{j=1}^{N^w}\langle(\delta p_{w,j})^2/2m^w\rangle$, and

$$\beta^A \equiv \frac{1}{M_A}\sum_w N^w m^w \left(\frac{2\alpha}{m^w}(\bar{a}^w_{\rm eff})_0 - \frac{2}{3}(\bar{c}^w)_{00}\right). \tag{8}$$

Since $(\bar{c}^w)_{00} = -(3/4)(\beta^w + \beta^{\bar{w}})$, this demonstrates that EEP tests using nongravitationally bound systems of normal matter can constrain phenomena that would otherwise only be apparent for free antimatter particles.

We now apply Eq. (7) to evaluate the phenomenological reach of existing experiments using conventional matter. Violation of EEP is described by six independent parameters. Three for matter: β^p, β^n, and β^e; and three for antimatter: $\beta^{\bar{p}}$, $\beta^{\bar{n}}$, and $\beta^{\bar{e}}$. For any particular EEP test comparing the effects of gravity acting on systems A and B, the observable anomaly is given by $\beta^A - \beta^B$, where β^A and β^B are defined in Eqs. (7) and (8). Since all high-precision tests of EEP are performed on charge-neutral systems, and since normal matter has a substantially similar ratio of proton to neutron content, the expression for $\beta^A - \beta^B$ can be usefully expressed in terms of an effective neutron excess $\tilde{\Delta}_j$, effective mass defect $\tilde{m}'_j$, and kinetic energy components $T^w_{j,\mathrm{int}}$ of the two systems, where

$$\tilde{\Delta}_j \equiv \frac{m^n}{m^p}\frac{m^e + m^p}{m^n} N^n_j - N^p_j, \tag{9}$$

$$\tilde{m}'_j \equiv m'_j - \frac{(m^n - m^p)(m^e + m^p)}{m^n} N^p_j, \tag{10}$$

and $j \in \{A, B\}$. The EEP-violating observable can then be written in terms of linear combinations of the free particle (β^w) and antiparticle ($\beta^{\bar{w}}$) anomalies as

$$\begin{aligned}\beta^A - \beta^B = {} & \frac{(m^n)^2}{(m^n)^2 + (m^e + m^p)^2}\Bigg[\left(\frac{\tilde{\Delta}_A}{M_A} - \frac{\tilde{\Delta}_B}{M_B}\right) m^p \beta^{e+p-n} \\ & - \left(\frac{\tilde{m}'_A}{M_A} - \frac{\tilde{m}'_B}{M_B}\right)\beta^{e+p+n}\Bigg] \\ & - \frac{1}{2}\sum_w \left(\frac{T^w_{A,\mathrm{int}}}{M^A c^2} - \frac{T^w_{B,\mathrm{int}}}{M^B c^2}\right)(\beta^w + \beta^{\bar{w}}),\end{aligned} \tag{11}$$

where M_A and M_B are the masses of the two test bodies, and

$$\beta^{e+p-n} \equiv \beta^{e+p} - \frac{m^e + m^p}{m^n}\beta^n, \tag{12}$$

$$\beta^{e+p+n} \equiv \frac{m^e + m^p}{m^n}\beta^{e+p} + \beta^n, \tag{13}$$

in which

$$\beta^{e+p} \equiv \frac{m^e}{m^p}\beta^e + \beta^p, \tag{14}$$

after the notation of [6]. We can define a similar set of terms $\beta^{\bar{e}+\bar{p}}$, $\beta^{\bar{e}+\bar{p}-\bar{n}}$, and $\beta^{\bar{e}+\bar{p}+\bar{n}}$ for antimatter. Note that Eq. (11) has a close parallel with older studies of EEP violation [2], since

$$\left(\frac{\tilde{m}'_B}{M_B} - \frac{\tilde{m}'_A}{M_A}\right) = \left(\frac{\tilde{A}_B}{M_B} - \frac{\tilde{A}_A}{M_A}\right) m^n, \tag{15}$$

where the effective baryon number $\tilde{A}_j$ is given by

$$\tilde{A}_j \equiv N^n_j + \frac{m^p}{m^n}\frac{m^e + m^p}{m^n} N^p_j. \tag{16}$$

Thus, the quantities $m^p\beta^{e+p-n}$ and $m^n\beta^{e+p+n}$ in the SME may be understood as parametrizing an anomalous gravitational coupling to a given particle's neutron excess and total baryon number "charges" [2].

In our prior analysis [7], the kinetic energy of protons and neutrons bound within a given nucleus was estimated by treating the nucleons as Fermi gases confined within a square potential well. This model did not account for the nucleons' angular momentum, treated the Coulomb potential in a heuristic way by shifting the depth of the proton potential, and did not account for the nucleons' spin-orbit interaction. The latter is of particular significance, because it can affect the occupation number of states with a given kinetic energy. Here, we improve upon that work by modeling the nucleons as single particles bound within fixed, spherically symmetric rounded square well potentials. These Woods-Saxon potentials [20] are taken to be of the form developed by Schwierz *et al.* [21]. Nuclide data are taken from Audi *et al.* [22], and isotopic abundances (for deriving the EEP-violating signal in bulk materials) from Laeter *et al.* [23]. A complete summary of our calculated kinetic energies can be found in the Supplemental Material [24]. Better estimates of the nucleons' bound kinetic energies are available for light nuclei using Green's function Monte Carlo (GFMC) calculations of the many-nucleon wave functions for nuclides with $A \leq 12$ [25]. The GFMC estimates of the bound kinetic energy of the constituent protons and neutrons in ^{6}Li, ^{7}Li, ^{9}Be, ^{10}B, and ^{12}C are summarized in Table I, and are compared with the corresponding predictions of our Woods-Saxon potential. Using these estimates, we can determine the contribution of the matter-sector $\beta^{e+p\pm n}$ and antimatter-sector $\beta^{\bar{e}+\bar{p}\pm\bar{n}}$ parameters to any observed violation of EEP in the motion of two (normal matter) test masses. These contributions are summarized in Fig. 1. Species with particular relevance to existing or planned tests of EEP [26–32] are explicitly labeled.

In most experiments, β^{e+p-n} is dominant, as it scales with the neutron excess. The next most accessible are the β^{e+p+n} term, which scales with the mass defect, and the antimatter term $\beta^{\bar{e}+\bar{p}-\bar{n}}$, which scales with the excess of the neutrons' kinetic energy over that of the protons, followed by $\beta^{\bar{e}+\bar{p}+\bar{n}}$. In some cases, (e.g., tests comparing lead and aluminium [28]) the signal from the antimatter $\beta^{\bar{e}+\bar{p}-\bar{n}}$ may actually be stronger than that from β^{e+p+n}. These terms represent four of the 6 degrees of freedom describing isotropic EEP violation, primarily for protons, neutrons, and their antiparticles. Electronic EEP violation is described by $\beta^{e-p} + \beta^{\bar{e}-\bar{p}} \equiv -\frac{4}{3}[(\bar{c}^e)_{00} - \frac{m^e}{m^p}(\bar{c}^p)_{00}]$, and has thus far been constrained largely by gravitational redshift

TABLE I. Comparison between calculated bound kinetic energies (in MeV) of protons and neutrons in light nuclei, obtained from many-body GFMC calculations [25], and a single-particle calculation using a modified Woods-Saxon potential.

	GFMC		Woods-Saxon	
Species	$T^{\mathrm{p}}_{\mathrm{int}}$	$T^{\mathrm{n}}_{\mathrm{int}}$	$T^{\mathrm{p}}_{\mathrm{int}}$	$T^{\mathrm{n}}_{\mathrm{int}}$
^{6}Li	77	78	64	65
^{7}Li	88	108	67	84
^{9}Be	124	135	95	112
^{10}B	162	164	116	122
^{12}C	219	219	145	153

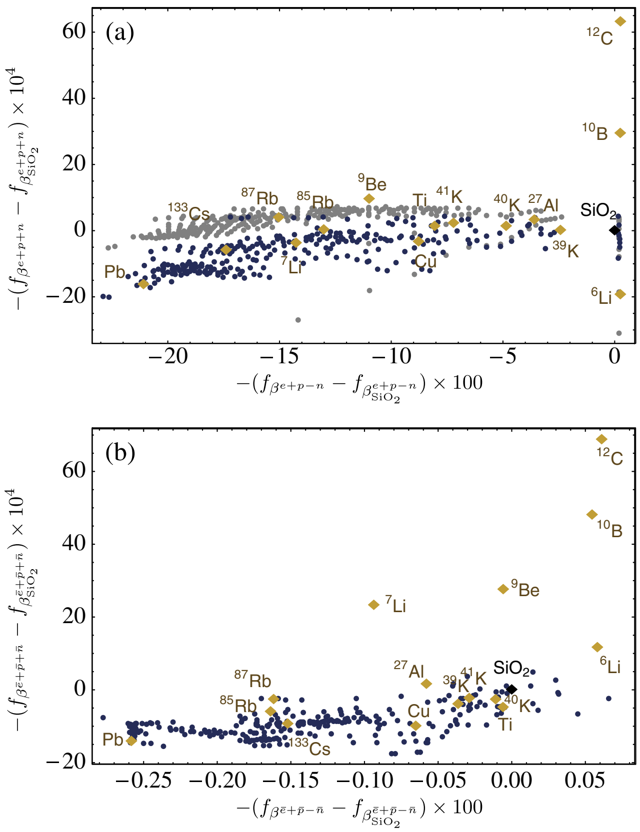

FIG. 1 (color online). Scatterplot of the contribution of $\beta^{e+p\pm n}$ and $\beta^{\bar{e}+\bar{p}\pm\bar{n}}$ parameters to observable EEP violation in normal nuclides with lifetimes in excess of 1 Gyr, when compared to SiO_2, from Eqs. (7) and (8). The anomalous fractional acceleration $\beta = f_{\beta^{e+p-n}}\beta^{e+p-n} + f_{\beta^{e+p+n}}\beta^{e+p+n} + f_{\beta^{\bar{e}+\bar{p}-\bar{n}}}\beta^{\bar{e}+\bar{p}-\bar{n}} + f_{\beta^{\bar{e}+\bar{p}+\bar{n}}}\beta^{\bar{e}+\bar{p}+\bar{n}}$. Tests that compare two or more widely separated species are more sensitive than tests involving neighboring isotopes. Plot (a) shows each species' relative sensitivity to matter-sector EEP violation, and (b) depicts their sensitivities to antimatter-sector anomalies. Gray points in (a) indicate the range of sensitivities obtained without accounting for nucleons' kinetic energies. Sensitivities of ^{6}Li, ^{7}Li, ^{9}Be, ^{10}B, and ^{12}C are taken from GFMC calculations, all others from a Woods-Saxon model (see Supplemental Material [24]).

tests [11–16], and tests of local Lorentz invariance [33,34]. The sixth degree of freedom, $\beta^{e-p} - \beta^{\bar{e}-\bar{p}} \propto \alpha(\bar{a}^p_{\mathrm{eff}})_0 - \alpha(\bar{a}^e_{\mathrm{eff}})_0$, is only observable in tests on charged bodies [6,18].

Using multivariate normal analysis of the results of an ensemble of EEP tests, including matter-wave [7,9,10], clock comparison [11–17], and torsion pendulum experiments [8], we obtain new limits on the five isotropic EEP-violating degrees of freedom that are observable in neutral systems, summarized in Table II. These bounds improve upon prior [7] gravitational constraints on these SME coefficients by factors of 2 to 10, and are also stated in terms of the five matter and antimatter $\beta^{e+p\pm n}$, $\beta^{\bar{e}+\bar{p}\pm\bar{n}}$, and $\beta^{e-p} + \beta^{\bar{e}-\bar{p}}$ coefficients. Though the limits reported in Table II are necessarily model dependent, they are stable

TABLE II. Global limits ($\times 10^6$) on isotropic EEP violation, obtained via multivariate normal analysis on the results of an ensemble of precision tests of EEP. Limits are stated in the Sun-centered, celestial equatorial frame [5], and are expressed in terms of the β^w parameters as well as the individual $(\bar{c}^w)_{TT}$ and $\alpha(\bar{a}^w_{\mathrm{eff}})_T$, with $(\bar{a}^{e+p}_{\mathrm{eff}})_T \equiv (\bar{a}^e_{\mathrm{eff}})_T + (\bar{a}^p_{\mathrm{eff}})_T$. The limits on the $\alpha(\bar{a}^w_{\mathrm{eff}})_T$ coefficients are stated in units of GeV/c^2.

$(\beta^{e-p} + \beta^{\bar{e}-\bar{p}})$	0.019 ± 0.037	$(\bar{c}^e)_{TT}$	-0.014 ± 0.028
β^{e+p-n}	-0.013 ± 0.021	$(\bar{c}^n)_{TT}$	1.1 ± 1.4
β^{e+p+n}	2.4 ± 3.9	$(\bar{c}^p)_{TT}$	0.24 ± 0.30
$\beta^{\bar{e}+\bar{p}-\bar{n}}$	1.1 ± 1.8	$\alpha(\bar{a}^n_{\mathrm{eff}})_T$	0.51 ± 0.64
$\beta^{\bar{e}+\bar{p}+\bar{n}}$	-4.1 ± 6.7	$\alpha(\bar{a}^{e+p}_{\mathrm{eff}})_T$	0.22 ± 0.28

against small variations in the estimated value of T^w/Mc^2 for the relevant nuclides, and are consistent with the limits obtained using substantially different nuclear models [35].

Despite the fact that torsion pendulum tests [8] set limits on specific combinations of β parameters at the level of 10^{-12} (having constrained $\Delta a/a$ to the level of 10^{-13}), the best bounds reported in Table II are at the level of 10^{-8}. This apparent discrepancy is due to the fact that such tests do not span the full parameter space considered here. Thus, the limits on the individual β's summarized in Table II are strongly correlated with one another. Analysis of these correlations reveals that some combinations of the β's are indeed constrained at the level of 10^{-9}, 10^{-11}, and 10^{-12}, thanks to matter-wave interferometer and torsion pendulum results. Unfortunately, the specific combinations of β's subject to these constraints are sensitive to small errors in our estimates of the nuclides' bound kinetic energy, due to disparities between the precision of torsion pendulums and of other EEP tests. Formal limits on EEP violation at the level of an effective field theory like the SME must, therefore, await the development of more reliable nuclear models [35] or the results of additional high precision EEP tests presently in development, using matter waves [30–32], clocks [27], or macroscopic masses [28,29].

We have demonstrated that EEP tests on nongravitationally bound systems of normal particles can set indirect constraints on EEP violation in antimatter, thanks to the interaction between the EEP-violating terms and the system's bound kinetic energy. We have explicitly derived the link between anomalous gravitational redshifts and violations of the weak equivalence principle. This occurs whenever EEP is violated by introducing a particle-specific metric. In the context of the SME, accounting for these interactions results in significantly improved constraints on EEP violation in the standard model Lagrangian, for both matter and antimatter. The precision of these bounds is limited by that of existing nuclear models, and uneven experimental coverage of EEP-violating parameter space. New EEP tests with precision comparable to that of existing torsion pendulum experiments [27–32] may substantially eliminate this model-dependent limitation. Better nuclear modeling could also improve limits on EEP violation in the

SME by up to 8 orders of magnitude, the pursuit of which will be the subject of future work.

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