Bosonic Mott Insulator with Meissner Currents

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We introduce a generic bosonic model exemplifying that (spin) Meissner currents can persist in insulating phases of matter. We consider two species of interacting bosons on a lattice. Our model exhibits separation of charge (total density) and spin (relative density): the charge sector is gapped in a bosonic Mott insulator phase with total density one, while the spin sector remains superfluid due to interspecies conversion. Coupling the spin sector to the gauge fields yields a spin Meissner effect reflecting the long-range spin superfluid coherence. We investigate the resulting phase diagram and describe other possible spin phases of matter in the Mott regime possessing chiral currents as well as a spin-density wave phase. The model presented here is realizable in Josephson junction arrays and in cold atom experiments.

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Interacting bosons in magnetic fields exhibit a range of interesting phenomena, from field expulsion in the Meissner-Ochsenfeld effect of superconductivity [1–3] to the realization of topologically ordered states [4]. The realization of ultracold atomic systems allows us to meticulously engineer such exotic phases of matter, in particular, through the realization of synthetic gauge fields [5–8]. The presence of multiple particle species has also been addressed [9–11]. Analogous phase transitions have been studied with Josephson-junction arrays in real magnetic fields [12–15]. With respect to systems with multiple species of particles, the phenomenon of interspecies coherence has been explored in Bose-Einstein condensates [16,17], bilayers of dipolar Fermi gases [18], quantum Hall bilayers [19], excitons in quantum wells [20], and bilayer graphene [21], polariton condensates [22]. Interspecies coherence and spin-charge separation have been studied for bosons [23–27], giving rise to a Meissner effect in the superfluid regime [23]. Similar physics has been studied with fermions [28]. Bosonic systems with time-reversal symmetry breaking and spin-charge separation yield rich phase diagrams [29–33].

In this Letter, we put such ingredients together and reexplore the phenomenon of spin and charge separation in a two-species bosonic system [26] incorporating the presence of (artificial) gauge fields.

In optical lattices, a transition between a bosonic superfluid to a Mott insulator has been observed experimentally [34], in agreement with theory [35,36], as well as disorder effects resulting in glassy phases [35,37,38]. Here, we restrict ourselves to a Mott insulating regime with total density one. The system under consideration constitutes an example of a time-reversal symmetry breaking Mott phase of bosons with chiral pseudospin currents. A prerequisite is the phase coherence between the two species which is realized by Josephson coupling, and explicitly breaks the $U(1)$ phase symmetry. Counter-flowing spin Meissner currents with zero net charge transfer can be induced by low-flux artificial magnetic fields. Our main result is a proof that the Meissner currents subsist as the system enters the total density Mott phase independently of the dimensionality of the system.

We consider two species of interacting lattice bosons where the conversion term mimics the Josephson-type coupling. In a generic gauge field, the Hamiltonian reads

$$H = -t\sum_{\alpha,\langle ij\rangle} e^{iaA^{\alpha}_{ij}} b^{\dagger}_{\alpha i} b_{\alpha j} - g\sum_{\alpha,i} e^{-ia'A_{\perp i}} b^{\dagger}_{2i} b_{1i} + \text{H.c.},$$
$$+\frac{U}{2}\sum_{\alpha,i} n_{\alpha i}(n_{\alpha i}-1) + V_{\perp}\sum_{i} n_{1i}n_{2i} - \mu\sum_{\alpha i} n_{\alpha i}. \qquad (1)$$

aA^{α}_{ij} is the Peierls phase acquired by a particle of species $\alpha = 1, 2$, and $a'A_{\perp i}$ the phase acquired upon species conversion. Within our notations, a and a' depict lattice spacing in the longitudinal and transverse directions, respectively (see Fig. 1). The model in Eq. (1) exhibits the Mott insulator to superfluid phase transition mentioned earlier. The phase boundaries can be calculated using variational mean-field theory and, for a one-dimensional lattice, exact density matrix renormalization methods [39] (these approaches are summarized in the Supplemental Material [40]). The Mott insulator is unambiguously characterized by vanishing total density fluctuations. In the limit of hard-core bosons ($U \to +\infty$), increasing either the interspecies coupling $V_{\perp}$ or the conversion g from zero is sufficient for the existence of the Mott phase with $\rho = 1$; the limits of Mott phase for vanishing kinetic terms are $\mu = -g$ and $\mu = V_{\perp} + g$. On the superfluid side, bosons condense (quasicondense in one dimension). Interspecies phase coherence can still remain in the Mott phase, $\langle b^{\dagger}_1 b_2\rangle \neq 0$, due to the Josephson coupling.

We define Meissner currents to satisfy the twofold condition: 1. vanishing between the species (there is no current proportional to g); 2. nonzero for the same species, and

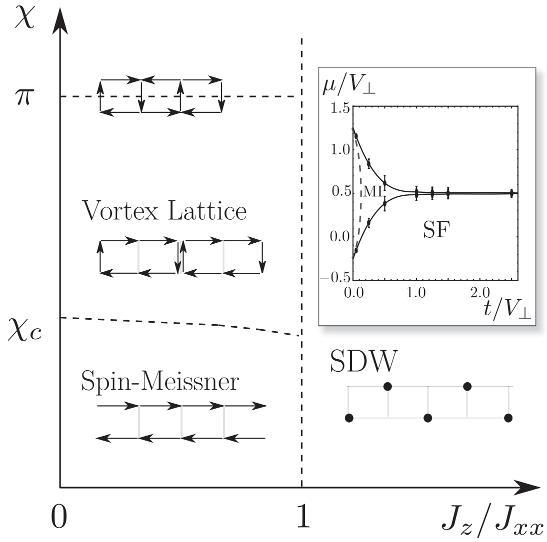

FIG. 1. Phase diagram for the effective gauged spin-$\frac{1}{2}$ model in Eq. (5) built for large repulsive terms U and $V_\perp$. In the XY limit, depending on flux, there is a spin Meissner phase or a vortex lattice phase (the direction of current patterns is shown in each phase). The inset shows the Mott lobe with total density $\rho = 1$ of interest, obtained using the density matrix renormalization group method for the one-dimensional model. The dashed line is the mean-field theory result.

proportional to minus the Peierls phase acquired by a particle. The current of the relative density operator $\dot{n}_{1i} - \dot{n}_{2i}$ separates into intraspecies and interspecies components $j_\sigma = j_\parallel(i \to j) + j_\perp(i)$; these are

$$\begin{aligned} j_\parallel &= it(-e^{iaA^1_{ij}} b^\dagger_{1i} b_{1j} + e^{iaA^2_{ij}} b^\dagger_{2i} b_{2j}) + \text{H.c.}, \\ j_\perp &= -2igb^\dagger_{1i} b_{2i} e^{ia' A_{\perp i}} + \text{H.c.} \end{aligned} \tag{2}$$

Outside the Mott lobe, the phase-angle representation is justified $b^\dagger_{1,2i} = \sqrt{n} e^{i\theta_{1,2i}}$ [in this reasoning, $n = \rho/2$ represents the mean (superfluid) density in each species]. The conversion takes the form of a Josephson coupling

$$- g \cos(a' A_{\perp i} + \theta_{1i} - \theta_{2i}). \tag{3}$$

For strong g, the superfluid phases will be pinned by this term such that $a' A_{\perp i} + \theta_{1i} - \theta_{2i} = 0$. Then $j_\perp$ vanishes and furthermore, in the small field limit, we may expand to obtain the Meissner form of the intraspecies current

$$\langle j_\parallel \rangle = -2tn\, \text{phase}_{ij}. \tag{4}$$

We have defined the phase around a plaquette, $\text{phase}_{ij} = (A^2_{ij} - A^1_{ij})a + (A_{\perp i} - A_{\perp j})a'$, which is invariant under a lattice gauge transform with scalars φ^α_i, $A^\alpha_{ij} \to A^\alpha_{ij} + (\varphi^\alpha_j - \varphi^\alpha_i)/a$ and $A_{\perp i} \to A_{\perp i} + (\varphi^2_i - \varphi^1_i)/a'$. As expected, there is a Meissner effect in the superfluid sector in the low field limit, as checked in Ref. [23], for example, in the specific case of one-dimensional systems.

In fact, as we argue below, the same remains true inside the Mott phase with total density $\rho = 1$. To show this, we place ourselves in the limit of large Mott gap favored by the interplay between the prominent Hubbard term U and the interspecies repulsion $V_\perp$. In this Mott phase at $\rho = 1$, the density $\rho = (n_1 + n_2)$ is not fluctuating. The limit of strong interactions has been achieved in ultracold atoms [41]. A gauged spin-$\frac{1}{2}$ model is easily obtained in the limit of strong interactions, as summarized in the Supplemental Material [40]. The two species are the Schwinger bosons in the representation of spin $\rho/2$ operators. The relative density corresponds to $\sigma_z = b^\dagger_1 b_1 - b^\dagger_2 b_2$. As demonstrated in boson language, σ_z fluctuates in the Mott phase. This is due to a transverse magnetic field in the $x - y$ plane, $-g\cos(a' A_{\perp i})\sigma^x_i + g \sin(a' A_{\perp i})\sigma^y_i$. (We have used $\sigma^x = b^\dagger_1 b_2 + \text{H.c.}$ and $\sigma_y = -ib^\dagger_1 b_2 + \text{H.c.}$).

The generic Hamiltonian for pseudospin that we obtain is

$$\begin{aligned} H_\sigma = &-\sum_{\langle ij \rangle} [2J_{xx}(\sigma^+_i \sigma^-_j e^{iaA^\sigma_{ij}} + \text{H.c.}) - J_z \sigma^i_z \sigma^j_z] \\ &- g\sum_i [\sigma^x_i \cos(a' A_{\perp i}) - \sigma^y_i \sin(a' A_{\perp i})], \end{aligned} \tag{5}$$

with $J_{xx} = (t^2/V_\perp)$ and $J_z = t^2[-(2/U) + (1/V_\perp)]$, and $A^\sigma = A^1 - A^2$. Setting $V_\perp = U/2$ or $J_z = 0$ yields the gapless XY phase of Eq. (5) and the Heisenberg antiferromagnetic chain is reached for $U \to +\infty$. In the absence of gauge fields, the XY term is ferromagnetic. For experimentally feasible values, the Ising term is antiferromagnetic ($J_z > 0$). These types of spin models have been addressed in various contexts [28,42–44].

At weak Ising interactions, the ferromagnetic XY order corresponds to superpositions $\alpha b^\dagger_1 + \beta b^\dagger_2$. These are just two distinct regimes for the unit density Mott phase of Fig. 1. The pseudospin current associated with σ^z is

$$\begin{aligned} j_\parallel &= 2J_{xx}[\cos(A^\sigma_{ij})(\sigma^y_i \sigma^x_j - \sigma^x_i \sigma^y_j) \\ &\quad + \sin(A^\sigma_{ij})(\sigma^x_i \sigma^x_j + \sigma^y_i \sigma^y_j)], \\ j_\perp &= -2g[\cos(a' A_{\perp i})\sigma^y_i + \sin(a' A_{\perp i})\sigma^x_i]. \end{aligned} \tag{6}$$

Considering the XY ordered phase, we define the expectation values of spin operators in this state as $\langle \sigma^x_i \rangle = \cos(\Theta_{\sigma i})$ and $\langle \sigma^y_i \rangle = \sin(\Theta_{\sigma i})$ (we define $\Theta_{\sigma i} = \theta_{1i} - \theta_{2i}$). A minimization of the resulting variational energy for *strong* g shows that these phases are pinned $\Theta_{\sigma i} + a' A_{\perp i} = 0$. Then, interspecies current $\langle j_\perp \rangle = 2g \sin(\Theta_{\sigma i} + a' A_{\perp i})$ vanishes, and a similar Meissner current to that of Eq. (4) is obtained, $\langle j_\parallel \rangle = -2J_{xx}\, \text{phase}_{ij}$. This strong coupling form is analogous to the form of Eq. (4) computed in the superfluid phase, where J_{xx} has replaced the kinetic term t. The condition of strong g coupling is in fact naturally achieved via renormalization group arguments. Associated with the spin-charge separation, there are two relevant energy scales, the Mott scale and the scale associated with phase coherence or the

Meissner effect, on which two-point correlations of $b_1^\dagger b_2$ are observable. In the strongly interacting regime $(U, V_\perp) \gg (t, g)$, the Mott energy scale is formally "infinite" compared to the scale of the Meissner phase.

Considering first the one-dimensional limit, we use the technique of bosonization and a renormalization-group treatment to draw the phase diagram of the model in Eq. (5). The standard treatment is to express the spin $\frac{1}{2}$ operators in terms of fermion field operators via the Jordan-Wigner transformation [37,45,46]. The resulting free part of the Hamiltonian has dispersion $\epsilon_k = -4J_{xx}\cos(ka - \chi)$ and Fermi velocity $v_F = |4aJ_{xx}|$. Within our notations, the flux χ reads "phase$_{ii+1}$", and the Fermi surface is delimited by $k_F = \pm(\pi/2a) + (\chi/a)$ for the half filled band with an additional flux. The Ising term produces next neighbor interactions.

The low-energy spectrum is then mapped to a continuum bosonic theory [37,45,46]. Introducing fields ϕ_σ, θ_σ with commutator $[\nabla\theta_\sigma(x), \phi_\sigma(x')] = -i\pi\delta(x - x')$, the continuum Hamiltonian has the form

$$H_\sigma = \frac{1}{2\pi}\int dx\left[u_\sigma K_\sigma(\nabla\theta_\sigma - A^\sigma)^2 + \frac{u_\sigma}{K_\sigma}(\nabla\phi_\sigma)^2\right] - \frac{2J_z}{(\pi^2 a)}\int dx\cos(4\phi_\sigma) - \frac{2g}{\sqrt{2}\pi a}\int dx\cos[(\theta_\sigma(x) + a' A_\perp)][1 + (-1)^{x/a}\cos 2\phi_\sigma]. \tag{7}$$

The sine-Gordon term in Eq. (7) has been approximated by keeping only $q \sim 0$ terms in the density operators. The speed of sound is $u_\sigma = v_F[1 + 16aJ_z/\pi v_F]^{1/2}$; the Luttinger parameter $K_\sigma = [1 + 16aJ_z/\pi v_F]^{-(1/2)}$ is a measure of interaction strength. $K_\sigma = 1$ for the xy limit and decreases as antiferromagnetic $J_z > 0$ is turned on. Gauge invariance can be checked simply by shifts of $\theta_\sigma \to \theta_\sigma + \varphi$.

We now turn to the phase diagram in Fig. 1 for our effective model. Whenever $J_z > J_{xx}$, dominant Ising interactions induce an antiferromagnetic spin density wave and there is no (Meissner) current. The corresponding inset shows a charge density wave of the bosons $b_{1,2}$, depicted as localized in two layers. The ϕ_σ-dependent sine-Gordon term is irrelevant if $K_\sigma > \frac{1}{2}$, or $J_z < J_{xx}$. The remaining sine-Gordon term is $\propto g\cos(\theta_\sigma + \chi(x/a))$, where we have chosen the Landau gauge with all flux on the conversion term. For infinitesimal flux, we may neglect the influence of χ. For $K_\sigma > \frac{1}{8}$, this term flows to strong coupling, and it is associated with the following energy gap [40] (we define $g_\sigma = ga/u_\sigma$)

$$\Delta_\sigma \sim \frac{u_\sigma}{a} g_\sigma^{1/2-(1/4K_\sigma)}. \tag{8}$$

This expression assumes that the bare value of $g \ll J_{xx}$. For nonzero fluxes χ, the energy scale in Eq. (8) defines the critical flux χ_c at which the system undergoes a transition to a vortex lattice phase of the commensurate-incommensurate type [37]. Below this critical field, the phase is the spin-Meissner low-field Mott phase, characterized by zero interspecies (or bulk) currents and counterflowing intraspecies currents. The following correlation function $\langle\sigma^+(x)\sigma^-(0)\rangle \sim \langle e^{-i\theta_\sigma(x)}e^{+i\theta_\sigma(0)}\rangle \sim \langle e^{-i\theta_\sigma(x)}\rangle\langle e^{i\theta_\sigma(0)}\rangle$ is asymptotically constant at large distances. This situation corresponds to XY order polarized (definite $\langle\theta_\sigma\rangle$) due to the in-plane field g. To return to the original boson operators, $\theta_\sigma = 0$ corresponds to a "bonding" state produced by the operator $(b_1^\dagger + b_2^\dagger)/\sqrt{2}$.

Above the critical field χ_c, currents organize in a vortex lattice, corresponding to commensurate values of the flux [23]. A flux of $(p/q)2\pi$ corresponds to p vortices in q unit cells as found from the expectation value of the current operator $\langle j_\perp\rangle \propto g\sin((\pi/q) + (2\pi p/q)(x/a))$. When the flux is further increased to half the elementary flux per plaquette, $\chi = \pi$, the sine-Gordon term oscillates $(-1)^{x/a} g\cos(\theta_\sigma)$ and is naively irrelevant, but at second order in perturbation theory [23], the oscillatory part disappears and the contribution is proportional to $(g^2/u_\sigma)\cos(2\theta_\sigma)$. This pins the field θ_σ to a new minimum which gives a staggered current configuration $\langle j_\perp\rangle \propto (-1)^{x/a}$ as shown in Fig. 1 (horizontal line at $\chi = \pi$). This phase corresponds to the "chiral Mott insulator" phase of boson ladders discussed in Ref. [31], and which exists in fermion ladders at weak field [28]. For completeness, we have checked the precise Meissner current pattern by exact diagonalization of small systems. Each species is localized in one of two chains composing a ladder. We have considered ladders of up to ten rungs. We confirmed numerically the Meissner current of Eq. (4), at small flux, as well as the vortex lattice and staggered current configurations depicted as insets in the phase diagram of Fig. 1.

The derivation of the effective XY model of Eq. (7) can be extended to $d + 1$ dimensions via a variational approach (See Supplemental Material at Ref. [40]; to substantiate our analysis of two dimensional systems, we also consider an array of coupled ladders). Starting from Eq. (5), we introduce the following pseudospin coherent state $|\psi\rangle = \prod_i(\cos\phi_{\sigma i}|\uparrow\rangle_i + e^{i\theta_{\sigma i}}\sin\phi_{\sigma i}|\downarrow\rangle_i)$. The azimuthal and polar angles are $2\phi_\sigma$ and θ_σ, respectively. Expanding about a saddle point corresponding to XY order, taking the continuum limit and expanding in gradients, we arrive at the following continuum Hamiltonian

$$H_\sigma[\theta_\sigma, \phi_\sigma] = \frac{1}{2}\int\frac{d^d x}{a^{d-2}} J_{xx}(\nabla\theta_\sigma - A^\sigma)^2 - \int\frac{d^d x}{a^d} g\cos(\theta_\sigma + a' A_\perp). \tag{9}$$

Firstly, if we restore the quantum character of θ_σ, ϕ_σ, this form is identical to the one of Eq. (7) in the one-dimensional limit with J_z taken to zero. In addition, the argument proving the existence of the Meissner current was independent of dimension.

Secondly, viewing Eq. (9) as the energy of a classical two-dimensional system, the first term in Eq. (9) can be rewritten as $\frac{1}{2}\int d^2x\rho_\sigma[\nabla\theta_\sigma(x) - A^\sigma]^2$,where $\rho_\sigma \equiv J_{xx}$ is the pseudospin rigidity, which is accessible experimentally. This gauged XY model undergoes a Berezinskii-Kosterlitz-Thouless transition: below $T^{\mathrm{BKT}} \sim \rho_\sigma = J_{xx}$, there is a phase of bound vortex-antivortex pairs.

Thirdly, we could consider alternate gauge field configurations in this two-dimensional system [47,48]. If the magnetic field is normal to the plane, interspecies currents vanish, while intraspecies currents follow the curl of the gauge field. If the field is uniform, intralayer currents cancel in the bulk but not on the sample boundary. The edge state currents in the two layers are parallel flowing, giving nonzero density current and zero pseudospin current. Consequently, such edge currents would be observable in the superfluid phase, but not in the Mott phase, unlike the spin-Meissner currents discussed so far. More details on gauge field configurations are offered in the Supplemental Material [40].

Finally, let us note that compared to the energy gap of Eq. (8), the Mott energy scale dominates, $\Delta_\rho \gg \Delta_\sigma$, consistent with our assumptions of strong coupling. There is a distinct regime in which the Mott and phase coherence energy scales are inverted. Previous work on two-leg bosonic ladders has shown that it is possible to achieve the Mott transition at significantly lower energy scales than the phase coherence: $\Delta_\rho \ll \Delta_\sigma$ [23–25]. This occurs in the regime of weakly coupled chains where g is perturbative compared to all other energy scales. Defining $\theta_{\rho,\sigma} = (\theta_1 \pm \theta_2)/\sqrt{2}$ together with the canonically conjugate $\phi_{\rho,\sigma} = (\phi_1 \pm \phi_2)/\sqrt{2}$, the one-dimensional limit of the system in Eq. (1) reduces to a sum of Luttinger liquid Hamiltonians represented by parameters $K_{\rho,\sigma} \sim \sqrt{t/U}(1 \pm V_\perp/U)^{-1/2}$ and $u_{\rho,\sigma} = a\sqrt{tU}(1 \pm V_\perp/U)^{1/2}$ (plus corresponds to ρ) for relatively weak interactions. Additionally, there is a sine-Gordon term of the form $g\cos(\sqrt{2}\theta_\sigma + a'A_\perp)[1 + 2\cos(\sqrt{8}\phi_\rho)]$ [24].

Renormalization-group equations show that the θ_σ field becomes gapped first, leading to asymptotically constant correlation functions as in the strongly interacting case. Apart from the difference in parameters, the energy gap below which the correlations have this property is given by Eq. (8) with $1/4K_\sigma$ being replaced by $1/2K_\sigma$ [23]. On energy scales below Δ_σ, the term in $\cos(\sqrt{8}\phi_\rho)$ remains. The Mott gap takes the form ($g_\rho = ga/u_\rho$)

$$\Delta_\rho \sim \Delta_\sigma g_\rho^{1/(2-2K_\rho)}. \tag{10}$$

Since, as compared to the strongly interacting regime, the two energy scales are inverted, $\Delta_\rho \ll \Delta_\sigma$, observation of the Mott phase along with the Meissner phase requires probing correlators at very low energy scales Δ_ρ. This can be improved by increasing $V_\perp$, which lifts both energy scales Δ_ρ and Δ_σ. This is consistent with our conclusion that $V_\perp$ favors the $\rho = 1$ Mott phase, according to the phase diagram of Fig. 1. The introduction of anisotropies drives down the energy scale Δ_σ. Such anisotropies can be between hopping terms $t_1 \neq t_2$ or intraspecies interactions $U_1 \neq U_2$. In this sense, the isotropic case introduced in Eq. (1) is optimal.

Firstly, the setup presented here has long been possible with Josephson junction arrays [12,13]. We present such a realization with realistic experimental estimates in the Supplemental Material [40]. An early study of the vortex lattice in Josephson-junction arrays, but without considering the Mott transition, has been performed in Ref. [49]. In the simplest realization, each species corresponds to a Josephson junction chain. The chains are coupled through a Josephson coupling as well as a visible capacitive interaction and there is a real magnetic field threading the interchain plaquettes. The prerequisite of one Cooper pair per rung necessary to access the Mott phase can be achieved through current technology [50]. Another realization of the Hamiltonian of Eq. (5) can be obtained as proposed in Ref. [44], by placing an array of Josephson junctions in the vicinity of a bulk superconductor. The spin degree of freedom then describes total density fluctuations on the superconducting islands.

Secondly, with cold atoms a one-dimensional setup is possible [51]. Recently, staggered artificial gauge fields have been realized [6,7]. Very recently, uniform artificial magnetic fields have been realized [8]: ^{87}Rb atoms have been loaded into tilted optical square lattices; the tilt in one direction suppressed the hopping due to a detuning between neighboring sites. An additional pair of lasers whose detuning was matched to that of the tilt reinstated a complex hopping term, which mimics the Peierls phases acquired by charged particles in a magnetic field. Implementation of a two-leg ladder based on this system requires merely confining the condensate to two columns by use of a parabolic potential.

In general the on site interactions dominate $V_\perp \ll U$ [36]. Interspecies interaction can be enhanced by the introduction of an additional fermion species that interacts with the bosons $H_f = -\sum_{\langle ij\rangle} t_f f_i^\dagger f_j + \mathrm{H.c.}$, $H_{bf} = V\sum_{\alpha i} n_{\alpha i} n_{fi}$. Integration of the fermions leaves bosons with repulsive interaction interspecies. This reads $V^2 a/(4\pi|v_F|)\sum_{\alpha\alpha' i} n_{\alpha i} n_{\alpha' i}$, where α denotes species. Longer-range interaction with the fermions induces longer-range interaction between the bosons. Alternatively, the $V_\perp$ is currently realizable with dipolar interactions [52]. In cold atom experiments, the Mott insulating phase can be probed by measuring local density fluctuations [53] $\langle\rho_i^2\rangle - \langle\rho_i\rangle^2$. The Meissner phase is characterized by nonvanishing relative density fluctations $\langle\sigma_i^2\rangle - \langle\sigma_i\rangle^2$. Both can be accessed with *in situ* measurements [54] whereas the total density is locked. Currents can be probed by studying density modulations following anisotropic quenching of the kinetic energy [55]. Additionally, the vortex lattice can be imaged in an ultracold ^{87}Rb gas [56]. The current response in the spin sector to a magnetic field is given by Eq. (4), which is the Meissner response. More details on the cold atom realization are discussed in the Supplemental Material [40]

To summarize, it is possible to realize a bosonic insulating phase with a spontaneous and persistent response which directly opposes the magnetic field in a case of a two-component bosonic Hubbard model with total density one. The associated fluxon quantization in a loop type geometry encodes topological aspects of the spin superfluid. The phase coherence in the Mott insulating regime can be analyzed with current technology in ultracold atoms [57]. In the strong-field limit where the spin Meissner effect is impossible, we recover the chiral Mott phase with a staggered current pattern found in Ref. [31]. Our analysis could be extended to high-Tc superconductors in the underdoped regime [58–62] and to low-dimensional symmetry protected topological phases [63,64].

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