Superconducting Vortex Lattices for Ultracold Atoms

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We propose and analyze a nanoengineered vortex array in a thin-film type-II superconductor as a magnetic lattice for ultracold atoms. This proposal addresses several of the key questions in the development of atomic quantum simulators. By trapping atoms close to the surface, tools of nanofabrication and structuring of lattices on the scale of few tens of nanometers become available with a corresponding benefit in energy scales and temperature requirements. This can be combined with the possibility of magnetic single site addressing and manipulation together with a favorable scaling of superconducting surface-induced decoherence.

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The ability to trap and manipulate ultracold atoms in lattice structures has led to remarkable experimental progress to build quantum simulators for Hubbard models, which are paradigmatic in condensed-matter physics. A prominent example is atoms in optical lattices (OLs) [1,2]. When loading an ultracold gas of neutral atoms into a lattice potential, atoms are positioned at the local minima of the lattice potential. In this situation, atoms can tunnel to neighboring lattice sites with tunnel coupling t and interact on site due to short-ranged collisional interactions with a strength U [2,3]. One of the significant interests for studying these type of models, in particular, for spin $= 1/2$ fermionic atoms [4,5], lies in the fact that in the strong coupling regime $U/t \gg 1$, superexchange processes (with coupling strengths $\sim t^2/U$) provide the basic mechanism for an antiferromagnetic coupling between spins on neighboring sites, which is closely related to studies of high-T_c superconductivity within the Hubbard model [6].

An important challenge to simulate Bose- and Fermi-Hubbard Hamiltonians in a regime not accessible to classical computers [7] is the development of better cooling schemes in order to reduce the entropy of the simulator [8]. In particular, one demands $k_B T$, $\hbar\Gamma \ll t^2/U \ll t < E_R$, where T is the temperature of the system (k_B is Boltzmann's constant), Γ the decoherence rate of the atoms, and $E_R = h^2/(8 m_a a^2)$ the recoil energy, where m_a is the mass of the trapped atoms, a the interlattice site distance, and $h = 2\pi\hbar$ Planck's constant. In OLs, a corresponds to half the optical wavelength, which leads to $E_R/k_B \sim 10^{-7}$ K (e.g., for rubidium and wavelength 852 nm). With present cooling techniques atoms can be prepared at few nanokelvins, which render the above set of inequalities very tight. Alternatively to designing better cooling schemes, one could loosen the above set of inequalities by reducing a and thereby boosting the energy scale of the physical parameters of the quantum simulator. Because of the diffraction limit, this requires to trap atoms near a surface without adding new sources of decoherence [9].

Here, we propose and analyze a new approach to trap and manipulate ultracold neutral atoms in arbitrary (periodic and nonperiodic) lattice potentials based on using a magnetic nanolattice generated by a controlled array of superconducting vortices in thin-film type-II superconductors [10]. With present technologies, superconducting vortices can be positioned in complex structures by artificially pinning them in nanoengineered arrays of, for instance, completely etched holes (antidots) of various sizes and shapes; see Ref. [11] and references therein. Our proposal hints at the possibility to exploit this technology to fabricate and structure arbitrary magnetic lattices for atomic physics at the fundamental length scales associated with superconducting vortices, the coherence length ξ, and London's penetration depth λ, which can be of a few tens of nanometers. Moreover, the combination of all-magnetic trapping and manipulation and superconducting surfaces leads, in principle, to very favorable scalings on surface-induced decoherence, as discussed below. These features make this proposal significantly distinctive from previous schemes for magnetic lattices [12–16], where denser lattices with low decoherence are challenging, as well as for magnetic traps where an atom is trapped by the field created by macroscopic currents flowing in type-II superconducting materials [17–22] and not by the field created by a few controlled superconducting vortices, as proposed here.

We consider a superconducting film of thickness $d \lesssim \lambda$. The film is a type-II superconductor, i.e., $\lambda/\xi > 1/\sqrt{2}$, where the value of ξ typically ranges between few to tens of nanometers [10,23]. In thin films, the effective penetration depth is given by $\Lambda = \lambda^2/d \gtrsim \lambda \gtrsim d > \xi$, which hence can potentially be as small as few tens of nanometers. The upper side of the film is situated at the x-y plane with $z = 0$ and contains a nanoengineered array of artificial pinning centers consisting of antidots of radius $R \gtrsim \xi$ [10,11] distributed in a Bravais lattice $\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2$, where $\mathbf{a}_{1(2)}$ are the lattice primitive vectors and $n_{1(2)}$ range through all integer values. The density of antidots is $1/a^2$, where $a^2 = |\mathbf{a}_1 \times \mathbf{a}_2|$ is the area of the primitive cell. By cooling the film in the presence of an external field whose flux density is commensurate to the density of antidots, a single vortex is pinned in each antidot (pinning multiple vortices in each antidot is also possible, see [24]). This assumes that the density of antidots does not exceed $1/a^2 \lesssim 1/\Lambda^2$, otherwise the vortex-vortex repulsion would prevent the vortices from sitting at the antidot lattice; see Supplemental Material (SM) [25] for further discussion. Once the film is cooled and the vortex lattice is prepared, the external magnetic field is switched off and the vortices remain, each of them with a magnetic flux given by $\Phi_0 = h/(2e)$, where e is the charge of the electron.

Our proposal consists in using the magnetic field created by the vortex lattice above the film $\mathbf{B}_V(\mathbf{r}, z > 0)$ to trap neutral atoms in a two-dimensional magnetic nanolattice with a geometry dictated by the nanoengineered antidot lattice. One can use well-known results in the field of superconductivity to approximate the field above the thin film as the one generated by an array of magnetic charges of strength $2\Phi_0$ situated at a distance $z = -\Lambda$ below the film; see Fig. 1(a) and SM [25]. This leads to

$$\begin{aligned} B_V^{x(y)}(\mathbf{r}, z) &\approx B_0 e^{-\Delta_z} g_{x(y)}(\mathbf{r}, z), \\ B_V^{z}(\mathbf{r}, z) &\approx B_0[1 + e^{-\Delta_z} g_z(\mathbf{r}, z)]. \end{aligned} \quad (1)$$

We have defined $B_0 \equiv \Phi_0/a^2$, $\Delta_z \equiv 2k(z + \Lambda) > 2k\Lambda \equiv \Delta_{\min}$, $k \equiv \pi/a$, and

$$\begin{aligned} g_{x(y)}(\mathbf{r}, z > 0) &\equiv \sum_{\mathbf{K}\neq\mathbf{0}} \frac{K_{x(y)}}{|\mathbf{K}|} \sin(\mathbf{K}\cdot\mathbf{r}) e^{\Delta_z(1-|\mathbf{K}|/k)}, \\ g_z(\mathbf{r}, z > 0) &\equiv \sum_{\mathbf{K}\neq\mathbf{0}} \cos(\mathbf{K}\cdot\mathbf{r}) e^{\Delta_z(1-|\mathbf{K}|/k)}, \end{aligned} \quad (2)$$

where the sum is over all reciprocal lattice vectors $\mathbf{K}$. The x and y components of $\mathbf{B}_V$ are equal to zero on top of the vortices, namely, at $\mathbf{r} = \mathbf{R}$. The z component is always positive and tends to an homogeneous field of strength B_0 at long distance from the surface due to the infinite extension of the plane. General nonperiodic structures obtained by nanoengineering can also be considered.

Alkali metal atoms in low fields of strength $\lesssim 30$ mT, where the Zeeman shift of hyperfine levels is linear, experience a potential of the form $V_{\rm lat}(\mathbf{r}, z) = \mu_{m_F}|\mathbf{B}(\mathbf{r}, z)|$ [26,27]. The local field interacting with the atoms is denoted by $\mathbf{B}$ and will be composed of the one generated by the vortices $\mathbf{B}_V$ plus additional bias fields, see below. The magnetic dipole moment is given by $\mu_{m_F} \equiv m_F g_F \mu_B$, where m_F is the magnetic quantum number, g_F is the Landé g factor, and the positive number μ_B is the Bohr magneton. Thus, low-field-seeking states $g_F m_F > 0$ can be trapped at the local minima of $|\mathbf{B}(\mathbf{r}, z)|$ [26,27].

The field generated by the vortices does not have local minima of $|\mathbf{B}_V|$ since the z component is always positive. For this reason, we propose to add a perpendicular bias field of the form $\mathbf{B}_1 = B_1(0, 0, -1)$ and define $\mathbf{B}_{\rm lat}(\mathbf{r}, z) = \mathbf{B}_V(\mathbf{r}, z) + \mathbf{B}_1$. The field $\mathbf{B}_1$ can be considered homogeneous even close to the surface provided that a thin film of $d \lesssim \lambda$ is used. We have validated this assumption by numerically calculating the field distribution and induced currents in a superconducting disk of finite radius and thickness using an energy minimization procedure [28,29]; see SM [25]. The strength of B_1 is limited by the fact that it should not induce extra vortices. This leads to the condition $B_1 < B^\star + \min_{\mathbf{r}} B_V^z(\mathbf{r}, 0)$, where $B^\star \approx \Phi_0/(4\pi\Lambda^2)$. The local minima of $|\mathbf{B}_{\rm lat}(\mathbf{r}, z)|$ are obtained on top of the vortices, $\mathbf{r} = \mathbf{R}$, when $B_1 > B_0$, at a position z_0 given by the solution of the equation $B_V^z(\mathbf{R}, z_0) = B_1$, see Fig. 1(b), where we have defined $\Delta \equiv \Delta_{z_0}$. In this case, the conditions $B^\star > B_1 > B_0$ can be fulfilled provided $\Lambda \gtrsim a$. In Fig. 2(a) we plot $|\mathbf{B}_{\rm lat}|^2$ [as the final form the potential depends on the modulus square, see Eq. (3)] in the x-z plane at $y = 0$. As a validation of the model, we also plot the same quantity in Fig. 2(b) for an array of 3×3 vortices in a finite plane numerically solving the London equation using the method presented in [30]. One can

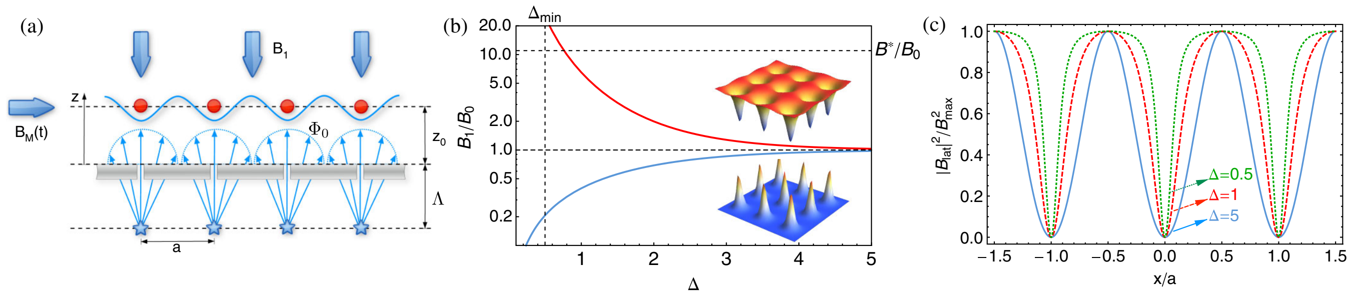

FIG. 1 (color online). (a) Schematic illustration of the superconducting vortex lattice for ultracold atoms. (b) Applied field B_1 as a function of Δ. Inset: $|\mathbf{B}_{\rm lat}(x, y, z_0)|^2$ for a square lattice, $\Delta = 2$, and $B_1 > B_0$ (deep wells) and $B_1 < B_0$ (sharp peaks). (c) $|\mathbf{B}_{\rm lat}(x, 0, z_0)|^2$ in units of $B_{\max}^2 \equiv |\mathbf{B}_{\rm lat}(x = a/2, 0, z_0)|^2$ as a function of x/a for a square lattice for different Δ.

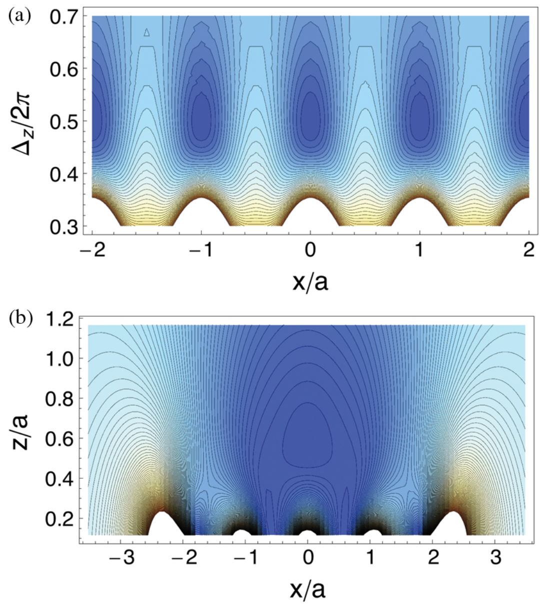

FIG. 2 (color online). (a) A contour plot of $|\mathbf{B}_{\mathrm{lat}}|^2$ at $y = 0$ is shown for an infinite square lattice as a function of x/a and Δ_z for $\Delta = \pi$ using Eq. (1). (b) The same plot as (a) is shown with a set of 3×3 localized vortices located on a squared lattice in a thin superconductor (the borders of the plane are at $x/a = \pm 2.45$) by numerically solving the London equation [30] and using $\Lambda \approx a$.

observe that there is also a minimum of $|\mathbf{B}_{\mathrm{lat}}(\mathbf{r}, z)|^2$ on top of the central vortex. In the case $B_1 < B_0$, one confines atoms between vortices with a much shallower trap; see Fig. 1(b).

Majorana losses [31], namely, spontaneous spin flips rendering the state of the atom into a high-field seeker, occur when $|\mathbf{B}| \sim 0$. Since the minima of $|\mathbf{B}_{\mathrm{lat}}|$ correspond to zero field, we suggest to use an effective time-averaged, orbiting potential [32] generated by adding to $\mathbf{B}_{\mathrm{lat}}$ an homogeneous time-dependent field parallel to the thin film $\mathbf{B}_M(t) = B_M(\sin\omega_M t, \cos\omega_M t, 0)$. By time averaging we have that $\langle \mathbf{B}_M(t) \rangle = \mathbf{0}$, but $\langle |\mathbf{B}_M(t)| \rangle = B_M$. Assuming $B_M \gg \max|\mathbf{B}_{\mathrm{lat}}(\mathbf{r}, z_0)|$ and considering that the total field experienced by the atoms is given by $\mathbf{B} = \mathbf{B}_{\mathrm{lat}} + \mathbf{B}_M$, we have $\langle |\mathbf{B}| \rangle \approx B_M + |\mathbf{B}_{\mathrm{lat}}|^2/(2B_M)$, which does not contain zero-field local minima. Using $\omega_t \ll \omega_M \ll \omega_L \equiv \mu_{m_F} B_M/\hbar$, the effective time-averaged magnetic potential for the atoms is given by [32]

$$V_{\mathrm{lat}}(\mathbf{r}, z) \approx \hbar\omega_L + \frac{\mu_{m_F}}{2B_M}|\mathbf{B}_{\mathrm{lat}}(\mathbf{r}, z)|^2. \tag{3}$$

This potential depends on $|\mathbf{B}_{\mathrm{lat}}|^2$, has nonzero field minima reducing Majorana losses to a rate given by $\Gamma_{ML}/2\pi \approx \omega_t \exp[-4\omega_L/\omega_t]$ [31], and confines atoms on a magnetic lattice whose geometry is dictated by the vortex lattice. Note that the bias field B_1 can be used to control the trapping height; see Fig. 1(b). This might be used to adiabatically load the ultracold atoms into the magnetic lattice from an external dimple trap [33].

The strength of the Hubbard parameters and the dependence with the physical parameters of the superconducting vortex lattice (SVL) proposed here can be obtained in analogy to OLs [1]. In particular, let us consider a square lattice with $B_1 > B_0$ and Δ sufficiently large, such that the potential Eq. (3) can be approximated to $V_{\mathrm{lat}}(\mathbf{r}, z) \approx V_0[\sin^2(kx) + \sin^2(ky)]$, where $V_0 \equiv 8B_0^2 \exp[-2\Delta]\mu_{m_F}/B_M$. This potential has the same form as the typical one obtained in OLs. The distance between the trapped atoms in the SVL is given by a; therefore, $2a$ plays the role of the optical wavelength in OLs. The role of the laser intensity in OLs, that can be used to modulate the trap depth, is taken in the SVL by the strength of the bias field B_1. Recall that Δ depends on B_1; see Fig. 1(b). In Fig. 3(a), we plot the tunneling rate $t \approx 4E_R(V_0/E_R)^{3/4} \times \exp[-2(V_0/E_R)^{1/2}]/\sqrt{\pi}$ [1] as a function of z_0/Λ for Li (using $a = 3\Lambda/2$ for $\Lambda = 100$ nm and $\Lambda = 20$ nm) and compare it with the decoherence rates that we discuss below. This is plotted in the range $50 > V_0/E_R > 1$ [see inset of Fig. 3(a)], which includes the tight-binding regime where $U \gg t$ [1]. The on-site repulsion $U < \hbar\omega_t \sim 2E_R(V_0/E_R)^{1/2}$ can be calculated solving the ultracold two-body collisional problem in a tight harmonic trap [9,34]. Comparing to OLs, the case $\Lambda = 20$ nm leads to

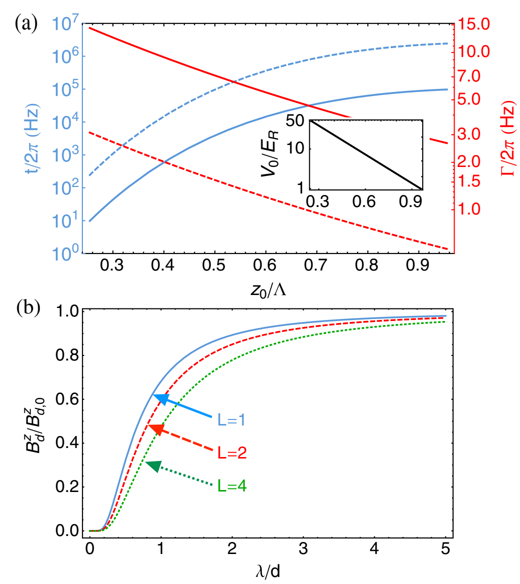

FIG. 3 (color online). (a) Tunneling rate t (left axis), decoherence rate (right axis) $\Gamma = \Gamma_{ML} + \Gamma_{\mathrm{sf}} + \Gamma_{0\to 1}$, and V_0/E_R (inset) as a function of the trapping position z_0 in units of Λ for a square lattice at sufficiently large Δ. The range of the plot is limited to the values of z_0/Λ for which $50 > V_0/E_R > 1$. We used the atomic parameters of lithium, $a = 3\Lambda/2$, $\Lambda = 100$ nm (solid lines) and $\Lambda = 20$ nm (dahsed lines), B_M such that $\omega_{ML} = 5\omega_T$, and $\eta/\Lambda = 10^{-7}$ Kg/(sm) [44]. (b) The ratio $B_d^z/B_{d,0}^z$ as a function of λ/d for $L = 1, 2, 4$.

more than 2 orders of magnitude larger values of E_R, and, therefore, significantly less stringent low-entropy requirements for the simulation of quantum magnetism in Hubbard Hamiltonians [5], as discussed in the introduction. In contrast to OLs, the potential Eq. (3) also permits us to design dense lattices with higher Fourier components. Here, by reducing Δ, more reciprocal vectors of different frequencies enter into play; see Fig. 1(c). For $B_1 < B_0$ this can lead to interesting potentials with sharp repulsive structures; see the inset of Fig. 1(b).

Atoms in magnetic traps are subjected to decoherence (spin flips and motional heating) due to magnetic field fluctuations at the Larmor frequency ω_L [35]. In metal surfaces, these fluctuations are generated by thermally excited motion of electrons (Johnson noise) [36]. Superconducting vortex-free surfaces have been predicted to dramatically reduce Johnson noise by 6–12 orders of magnitude [37–39]. However, experiments in superconducting atom chips have only shown a moderate improvement since they operate in a regime where uncontrolled superconducting vortices are present [40–43]. In the SVL proposed here, it is clear that a source of magnetic field fluctuations will be given by the thermal jiggling of the pinned vortices. In the SM [25] we use a standard phenomenological model of vortex dynamics (see [44] and references therein) in order to estimate the spin-flip rate $\Gamma_{\rm sf}$ and the motional heating rate $\Gamma_{0\to1}$ [35] induced by the thermal motion of vortices. They are given (up to some constant factors) by

$$\begin{aligned} \Gamma_{\rm sf} &\sim 3\pi^3 \frac{\mu_{m_F}^2}{\hbar^2}\frac{k_B T}{k_p}\frac{\omega_d}{\omega_L^2+\omega_d^2}\frac{B_0^2}{a^2\Delta^4}, \\ \Gamma_{0\to1} &\sim (2\pi)^5 \frac{\mu_{m_F}^2}{\hbar^2}\frac{k_B T}{k_p}\frac{\omega_d}{\omega_t^2+\omega_d^2}\frac{x_0^2 B_0^2}{a^4\Delta^6}. \end{aligned} \tag{4}$$

Here, $x_0 = \sqrt{\hbar/(2m_a\omega_t)}$, $k_p = \Phi_0^2/(2\mu_0\Lambda a^2)$ (provided $2\pi\Lambda \gg a$) is the spring constant given by the repulsive force with the lattice, see [45], and $\omega_d = k_p/\eta$, with η being the vortex viscosity coefficient, is the so-called depinning frequency, which marks the crossover between elastic motion, dominant at lower frequencies, and purely dissipative motion, arising at higher frequencies [44]. Using typical numbers for the vortex viscosity, these rates are remarkably small compared to the tunneling rate in Hubbard Hamiltonians; see Fig. 3(a).

Other sources of decoherence and practical considerations might be relevant in the eventual experiment. For instance, as analyzed in the SM [25], the position of the vortices has to be very accurate, with an error less than 1%–2% in the distance between them. Otherwise, the trap depths and thereby the tunneling rates will fluctuate throughout the lattice. This can constitute a serious challenge in the nanofabrication of regular antidot lattices. In this respect, using triangular lattices spontaneously formed in a film without artificial pinning might be advantageous. The randomness in size and shapes of the antidots might also lead to imperfections; nevertheless, the flux in each vortex is given by the constant of nature Φ_0. Time-dependent fields might induce dissipation in the vortices. As discussed before, we apply a time-dependent field $\mathbf{B}_M(t)$ to avoid Majorana losses. This field is parallel to the film, and in the ideal case, would not interact with the vortices. In any case, to reduce dissipation it will be convenient to use $\omega_M \ll \omega_d$.

Let us discuss the possibility to perform magnetic local addressing of the atoms in the lattice. We propose to place a magnetic tip close to the bottom side of the film, at position $z = -d - a_d$, to locally interact with the atoms trapped above. In the SM [25], we obtain the analytical expression of the magnetic field above the film for a given thickness d and London penetration depth λ. The ratio between the magnetic field B_d^z at $z = z_0$ in the presence of a superconducting film with London penetration depth λ, with the corresponding one in the case of not having the film $B_{d,0}^z$, depends on the dimensionless parameters λ/d and $L \equiv (z_0 + a_d)/d$. In Fig. 3(b), this ratio is plotted as a function of λ/d for different L. Note that even for $\lambda = d$, $B_d^z \sim B_{d,0}^z/2$ for $L = 2$. As discussed in [46], the minimum distance for which the dipole will create a vortex is given by $a_1 = \Lambda\sqrt{\mu_0 m_d/[\Phi_0\Lambda\ln(\Lambda/\xi)]}$; this equation is valid for $a_1 > \Lambda$. With typical numbers ($\Lambda = 100$ nm and $\xi = 10$ nm), $a_1 \sim 2\Lambda$ for a dipole of magnetic moment $m_d \sim 10^8\mu_B$. Using the maximum magnetic moment and $\lambda = d = \Lambda = z_0 = a_d/2 = 100$ nm (note that $L = 3$), this leads to a coupling to the atom of $g_d \sim \mu_B B_d^z/\hbar \sim 0.4\mu_B B_{d,0}^z/\hbar \sim 2\pi \times 10^8$ Hz. Since the neighbor atoms are farther away from the dipole, the coupling is reduced at least by a factor of 2, which permits us to perform local addressing by adding different phases into the internal state of the atoms. In order to measure the collective state of the atoms, one can release them from the trap and perform time-of-flight measurements [1].

To conclude, we have shown that the control and manipulation of superconducting vortices in thin films can be used to trap neutral atoms in a dense lattice near a surface in a superconducting state, whose macroscopic coherence leads to promising scalings regarding surface decoherence. The interplay between superconductivity and atomic physics, as proposed here, might pave the way towards all-magnetic schemes for quantum computation and simulation with neutral atoms. Moreover, this hints at the possibility of using ultracold atoms to probe important properties of high-T_c superconductivity.

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