

Entanglement Detection from Conductance Measurements in Carbon Nanotube Cooper Pair Splitters

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Spin-orbit interaction provides a spin filtering effect in carbon nanotube based Cooper pair splitters that allows us to determine spin correlators directly from current measurements. The spin filtering axes are tunable by a global external magnetic field. By a bending of the nanotube, the filtering axes on both sides of the Cooper pair splitter become sufficiently different that a test of entanglement of the injected Cooper pairs through a Bell-like inequality can be implemented. This implementation does not require noise measurements, supports imperfect splitting efficiency and disorder, and does not demand a full knowledge of the spin-orbit strength. Using a microscopic calculation we demonstrate that entanglement detection by violation of the Bell-like inequality is within the reach of current experimental setups.

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The controlled generation and detection of entanglement is a necessary step toward the goal of using quantum states for applications. In a solid state nanostructure this control ideally allows us to manipulate and detect entanglement between selected pairs of electrons. A promising source of entangled electron pairs is the Cooper pair splitter (CPS). It consists of a superconductor that injects Cooper pairs through two quantum dots (QDs) into two outgoing normal leads, designed such that the Cooper pair electrons preferably split and leave the superconductor over different leads but preserve their spin entanglement [1,2]. Very recently several CPS experiments have been performed [3–7] and Cooper pair splitting efficiencies up to 90% have been reached [7]. So far, however, a proof that the electrons remain entangled is still lacking.

The present experiments do not allow us to resolve individual splitting events, and the results of the measurements are time averaged quantities, such as current or noise. These provide information on the average spin correlations of the injected Cooper pairs. In this Letter we demonstrate that this information can be extracted from the currents alone in a carbon nanotube (CNT) based CPS, if spin-orbit interaction (SOI) effects are taken into account [8]. This allows us to propose a general entanglement test, based on the Bell inequality [9,10], which does not require noise measurements [11].

Indeed, the SOI in CNTs leads to unique spin-energy filtering properties that directly modulate the Cooper pair splitting current flowing out of the CPS, and ideally suppress any noise. From conductance measurements it is then already possible to reconstruct all spin correlators contained in the Bell inequality, thus avoiding the need of ferromagnetic contacts as spin filters, which are challenging to implement. Without noise measurements we also avoid the associated problem of electron fluctuations in the

detectors [12]. The built-in energy filtering furthermore leads to an enhanced Cooper pair splitting efficiency [13].

The proposed CPS setup is shown in Fig. 1 and consists of a regular double-QD CPS built from a single-wall CNT, yet made with a (naturally) bent CNT such that there is an angle θ_{CNT} between the QD axes. Alternatively, the QDs can be built from separate CNTs with similar diameters and an angle θ_{CNT} between them. The SOI spin splits the QD levels. In combination with a global magnetic field \mathbf{B} , the fourfold spin-valley degeneracy of the QD levels is completely lifted. The split levels provide a unique spin filter for electron transport with two spin projection axes per QD, filtering directly the injected Cooper pair current.

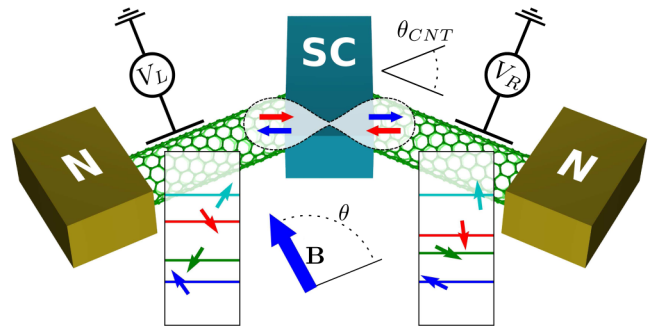


FIG. 1 (color online). Double quantum dot CPS based on a bent CNT in an external magnetic field \mathbf{B} . Because of \mathbf{B} , SOI, and the bending angle θ_{CNT} of the CNT, the spin-valley degeneracy of the QD levels is lifted, and the resulting 4 levels (boxes) are spin polarized as indicated by the arrows (see also Fig. 2). The superconductor SC injects Cooper pairs (hourglass shape) that split onto the QDs and provide a current to the normal leads N that is modulated by the spin projections of the QDs (tunable by the gates $V_{L,R}$) and can be used to determine the spin correlators for the Bell inequality.

Therefore, conductance measurements alone, at fixed \mathbf{B} , allow a reconstruction of all the spin correlators necessary for the Bell inequality. The spin projection axes are different in the two QDs due to the bending, and are tunable by \mathbf{B} . In the following we show that this tunability provides sufficient conditions for obtaining violations of the Bell inequality in an ideal CPS. We then proceed to a full microscopic calculation and demonstrate that the result remains robust under realistic conditions, as achievable by present experiments.

SOI in CNT quantum dots.—CNTs are graphene sheets rolled into a cylinder. They preserve the graphene band structure with two Dirac valleys but have enhanced SOI contributions due to the curvature. The corresponding model, including the effect of \mathbf{B} , is described by the sum of the Hamiltonians [14–17]

$$H_0 = \hbar v_F [k_t^0 \sigma_1 + k \tau_3 \sigma_2], \quad (1)$$

$$H_{cv} = \hbar v_F [\Delta k_t^{cv} \sigma_1 + \Delta k_z^{cv} \tau_3 \sigma_2], \quad (2)$$

$$H_{\text{SOI}} = \alpha \sigma_1 S_z + \beta \tau_3 S_z, \quad (3)$$

$$H_B = \mu_B g \mathbf{B} \cdot \mathbf{S} / 2 + |e| v_F R B_z \sigma_1 / 2, \quad (4)$$

which are matrices in the space spanned by the graphene sublattice indices $\sigma = A, B$ (with Pauli matrices $\sigma_{1,2,3}$), the valleys $\tau = K, K' = +, -$ (Pauli matrices $\tau_{1,2,3}$), and the spin projections $S = \uparrow, \downarrow$ (Pauli matrices $S_{x,y,z}$, with S_z oriented along the CNT axis). v_F is the Fermi velocity, k_t^0 the transverse quantized momentum (zero for metallic CNTs), k the longitudinal momentum, $\Delta k_{t,z}^{cv}$ are momentum corrections induced by the curvature, α, β determine the SOI, μ_B is the Bohr magneton, $g = 2$ the Landé g factor, e the electron charge, R the CNT radius, and B_z the component of \mathbf{B} along S_z . We have neglected terms leading to the formation of Landau levels since at the considered sub-Tesla fields they are of no consequence. For a QD, k is further quantized by the QD length [18–20].

Because of its momentum independence, the SOI takes the role of an internal valley (and QD orbital) dependent Zeeman field $\tau \mathbf{B}_{\text{SOI}}$ along S_z , which combines with \mathbf{B} to the effective field in each valley $\mathbf{B}_{\text{eff}}^\tau = \mathbf{B} + \tau \mathbf{B}_{\text{SOI}}$. These fields lift the spin degeneracy of the QD levels, while the orbital effect of Eq. (4) lifts the energy degeneracy between the two valleys for any $B_z \neq 0$. The QD levels turn into spin-valley-energy filters. The effective fields define the spin polarization axes $\mathbf{a}_\tau \propto \mathbf{B}_{\text{eff}}^\tau$, which are nonparallel if $\mathbf{B}_{\text{eff}}^K \neq \mathbf{B}_{\text{eff}}^{K'}$, tunable by \mathbf{B} , and such that the spin eigenstates in each valley $|\pm a_\tau\rangle$ fulfill $(\mathbf{S} \cdot \mathbf{a}_\tau)|\pm a_\tau\rangle = \pm |\pm a_\tau\rangle$ (full polarization). If $P_{\pm a_\tau} = |\pm a_\tau\rangle\langle\pm a_\tau|$, spin measurements can be reconstructed by electron transport over the different QD levels by $(\mathbf{S} \cdot \mathbf{a}_\tau) = P_{+a_\tau} - P_{-a_\tau}$.

Bell test in an ideal CNT-CPS.—In the double-QD system shown in Fig. 1, the CNT bending changes the

orientation of \mathbf{B}_{SOI} and so of $\mathbf{B}_{\text{eff}}^\tau$. The spin polarization axes \mathbf{a}_τ in the left QD become distinct from the axes in the right QD, which we call \mathbf{b}_τ . We consider an ideal CPS, characterized by a perfect Cooper pair splitting efficiency with valley-independent pair injection (see discussion below) and isolated sharp QD levels. Since any injected Cooper pair splits onto the different levels in each QD (the current consists only of split Cooper pairs), and the tunneling amplitude onto each dot is proportional to the spin projection, the current collected at the normal leads in resonant conditions for a given pair of levels is proportional to $\langle P_{\pm a_\tau} \otimes P_{\pm b'_\tau} \rangle$ and allows us to reconstruct the spin correlators $C_{\mathbf{a}_\tau, \mathbf{b}'_\tau} = \langle (\mathbf{S} \cdot \mathbf{a}_\tau) \otimes (\mathbf{S} \cdot \mathbf{b}'_\tau) \rangle$ [see Eq. (6)]. The availability of 2 spin projection axes per QD consequently allows us to test the Bell inequality [9]

$$Q = |C_{\mathbf{a}_K, \mathbf{b}_K} + C_{\mathbf{a}_K, \mathbf{b}_{K'}} + C_{\mathbf{a}_{K'}, \mathbf{b}_K} - C_{\mathbf{a}_{K'}, \mathbf{b}_{K'}}| \leq 2. \quad (5)$$

Any nonentangled state (including the steady state density matrix considered here) fulfills this inequality. A violation $Q > 2$ is sufficient to prove entanglement. For a spin singlet, a maximal $Q = 2\sqrt{2}$ is obtained by orthogonal $\mathbf{a}_K \perp \mathbf{a}_{K'}$, $\mathbf{b}_K \perp \mathbf{b}_{K'}$, and 45° between \mathbf{a}_K and \mathbf{b}_K . Such optimal axes cannot be generally obtained in the CNT-CPS, for which \mathbf{B}_{SOI} and θ_{CNT} are fixed by the sample fabrication, and only \mathbf{B} is tunable. Yet, as we show in Fig. 2, this tunability is sufficient to obtain $Q > 2$ as a function of the angle θ of a rotating in-plane field $\mathbf{B} = B(\sin\theta, 0, \cos\theta)$ (see Fig. 1), for $B \sim |\mathbf{B}_{\text{SOI}}|$. The shown result is generic and we find similar $Q > 2$ for most CNT chiralities, diameters, and QD lengths.

Realistic systems.—In a realistic setup, the two QDs remain coupled through the superconducting region, their levels are broadened by the contacts, the splitting

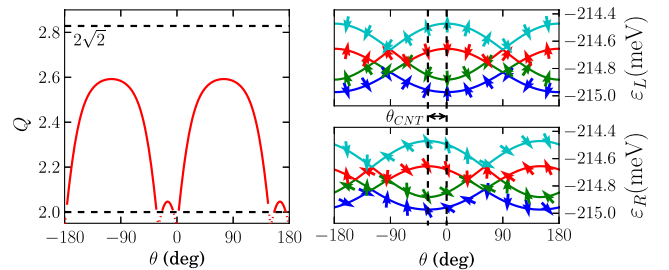


FIG. 2 (color online). Values Q of the Bell equation (5) (left panel) for an ideal bent CNT-CPS as a function of in-plane \mathbf{B} -field rotation angle θ , for the lowest valence band orbitals in a CNT of chirality (18, 10), $|\mathbf{B}| = 0.4$ T, $\theta_{\text{CNT}} = 30^\circ$, $\alpha = -0.08$ meV, $\beta = -0.15$ meV, and QD lengths of 200 nm. For this situation, $|\mathbf{B}|/|\mathbf{B}_{\text{SOI}}| \approx \mu_B g |\mathbf{B}| / 2 |\alpha - \beta| = 0.34$. The horizontal lines mark the threshold $Q = 2$ and the maximal possible $Q = 2\sqrt{2}$. The right panels show the θ dependence of the level energies of both QDs. The spectra are identical up to the shift by θ_{CNT} marked by the vertical dashed lines. The arrows indicate the spin polarizations in a global spin basis, as used for the determination of Q .

efficiency is imperfect and electron pairs can tunnel onto the same QD, the tunneling rates depend on the gate voltages, and electrons can interact. Any measurement probes the steady state density matrix ρ of the full CPS system and not an ideal singlet state. The projections $P_{\pm a_\tau}$, $P_{\pm b_{\tau'}}$ are obtained by narrowing the measurement to an energy window capturing the electron transport through the corresponding level of each QD, typically by differential conductance measurements tuned to the resonances corresponding to the levels. The modified ρ together with the measurement method leads to a distorted reconstruction of the spin correlators, and we need to distinguish between *local* and *nonlocal* distortion sources.

Local distortions in one QD are independent of the other QD and modify, e.g., $P_{\pm a_\tau}$ to $P_{+a'_\tau}$, $P_{-a''_\tau}$. We can write $P_{+a'_\tau} - P_{-a''_\tau} = \gamma(\mathbf{S} \cdot \tilde{\mathbf{a}}_\tau) + (1 - \gamma)P_{\tilde{\mathbf{a}}_\tau}$ for an intermediate axis $\tilde{\mathbf{a}}_\tau$, $0 \leq \gamma \leq 1$, and a remaining projection $P_{\tilde{\mathbf{a}}_\tau}$. The latter transforms any state into a product state, and local distortions therefore lower the ideal value of Q by an amount set by the various γ for the different QD levels. Assuming that the level broadening can be kept small so that there is only little overlap between nearby resonances (assisted also by a charging energy), the most important source of local distortions is disorder scattering within each QD. It mixes the wave functions in different valleys [21,22], and the $|\pm a_\tau\rangle$ are no longer the eigenstates. While of central importance in metallic CNTs, in semiconducting CNTs disorder scattering competes with the valley-preserving semiconducting gap of typically ~ 100 meV, which has opposite signs in opposite valleys. If the disorder scattering amplitude is smaller it has a negligible influence. Therefore, semiconducting CNTs are preferable for testing the Bell inequality.

Valley mixing at injection, however, is essential. Indeed, if valleys and spins are correlated, for instance, if the singlet splits always into opposite valleys, the transport through other valley combinations does not provide any information on the Cooper pairs and the construction of Q is no longer possible. For a valid spin correlator measurement the injection must mix valleys to produce a detectable signal through all resonances, yet the precise degree of mixing is unimportant.

Nonlocal distortions of the spin modify the spin projections as an effect of the entire CPS system, typically by hybridization between the two QDs, and the measured $P_{\pm a_\tau}$, $P_{\pm b_{\tau'}}$ become nonlocal operators. Such operators can generate additional entanglement through wave function mixing between the left and right QDs. In the CPS setup they are a source of error for detecting spin entanglement. Yet with the full microscopic calculation discussed next we can see that these nonlocal contributions can be kept under control in realistic conditions.

Microscopic model.—To quantitatively access a realistic system and to determine the optimal choice of measurements that allows us to gain insight in the effects of local

and nonlocal distortions, we have investigated a microscopic tight-binding model of the CNT-CPS. Our approach follows Ref. [23], which we have complemented to include magnetic fields by terms equivalent to Eq. (4) and valley mixing at injection. As a result, we obtain the partial conductances of the CPS due to Cooper pair splitting (crossed Andreev reflections, G_{CAR}), elastic cotunneling through the superconducting region (G_{EC}), and the local Andreev scattering contributions at each QD (G_{AL} , G_{AR}). From these quantities, transport from the superconductor to the normal leads is expressed by the conductances $G_j = 2(G_{\text{A}j} + G_{\text{CAR}})$ ($j = L, R$), and transport between the normal leads by the nonlocal conductance $G_{\text{nl}} = G_{\text{EC}} - G_{\text{CAR}}$.

In Fig. 3(a) we display a conductance map for a semiconducting CNT as a function of the QD gate voltages

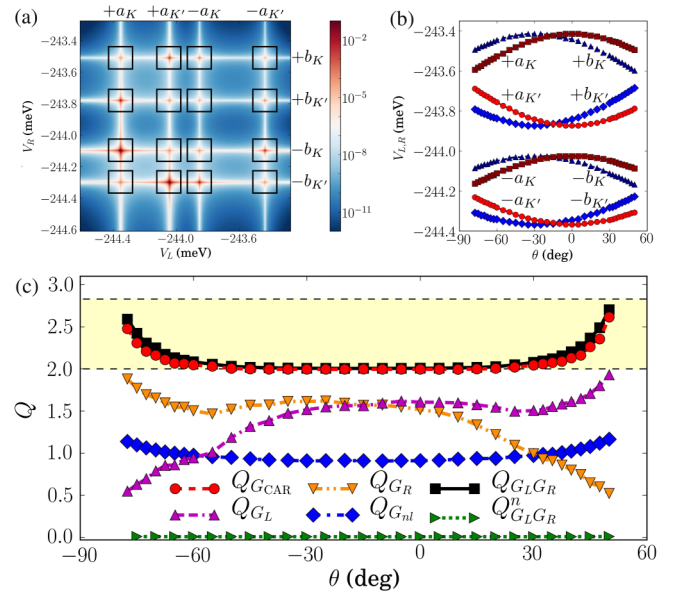


FIG. 3 (color online). Results from the microscopic calculation of a CPS, based on a zigzag CNT of chirality (20, 0) with a bending angle $\theta_{\text{CNT}} \approx 30^\circ$, in a field of $|\mathbf{B}| = 0.5$ T (see the Supplemental Material [24]). The SOI energies $\alpha = -0.10$ meV and $\beta = -0.40$ meV lead to $|\mathbf{B}|/|\mathbf{B}_{\text{SOI}}| \approx \mu_B g |\mathbf{B}| / 2|\alpha - \beta| = 0.10$. (a) Map of the conductance product $G_L G_R$ (units of e^4/h^2) as a function of QD gate voltages $V_{L,R}$ at $\theta = 25^\circ$. The 4 levels of each QD give rise to the 4 resonances labeled by $\pm a_\tau$, $\pm b_{\tau'}$. Inside the black squares, the CPS acts as a spin-valley filter for the projections $P_{\pm a_\tau} P_{\pm b_{\tau'}}$, and integrating the signal within each black square yields the corresponding observable. (b) $V_{L,R}$ values marking the positions of the resonances of the levels $\pm a_\tau$ and $\pm b_{\tau'}$ of the two QDs as a function of θ . The curves are identical up to the shift by θ_{CNT} . Levels in the same valley τ see the same field $\mathbf{B}_{\text{eff}}^\tau$ and are identified by having the same curvature as a function of θ . (c) Q as a function of θ for the conductances G given as subscripts of Q in the figure legend. The Q values are obtained by analyzing data as shown in panel (a) by the method described in the text. The yellow shaded region marks the allowed range of violation of the Bell inequality for the spin singlets in the steady state.

$V_{L,R}$ that tune the QD levels to resonance. Such a result is useful for a Bell test if all 4 resonances in each QD are well resolved and their 16 points of intersection, corresponding to the products $P_{\pm a_\tau} \otimes P_{\pm b_{\tau'}}$, form single peaks and not avoided crossings. To access this regime, we have chosen a coupling between the superconductor and the CNT on the order of the superconducting gap (≈ 1 meV), and tuned the coupling to the leads such that the resonances are well resolved (see the Supplemental Material [24]). Similar conditions have been obtained in experiments [21,22], and such a regime can be reached for a wide variety of samples and coupling strengths to the contacts.

To analyze the data we integrate the various conductances over regions centered at the crossings as shown by the black squares in Fig. 3(a). From the resulting 16 integrals $G_{\pm a_\tau, \pm b_{\tau'}}$ we construct the spin correlators

$$C_{\mathbf{a}_\tau, \mathbf{b}_{\tau'}} = \frac{\sum_{\nu, \nu' = \pm} \nu \nu' G_{\nu a_\tau, \nu' b_{\tau'}}}{\sum_{\nu, \nu' = \pm} G_{\nu a_\tau, \nu' b_{\tau'}}}, \quad (6)$$

which is a simple consequence from the fact that $P_{+a_\tau} - P_{-a_\tau} = (\mathbf{S} \cdot \mathbf{a}_\tau)$ and $P_{+a_\tau} + P_{-a_\tau}$ is the identity operator (see the Supplemental Material [24]). From these $C_{\mathbf{a}_\tau, \mathbf{b}_{\tau'}}$ we determine Q by Eq. (5), with the liberty of placing the $-$ sign in front of any term in Eq. (5) to obtain the maximum Q .

The Cooper pair splitting amplitude is directly described by G_{CAR} , and the corresponding curve $Q_{G_{\text{CAR}}}$ [Fig. 3(c)] captures indeed a similar behavior as the ideal case of Fig. 2, with $Q > 2$ in the θ regions where the levels of different valleys approach each other and the spin projections rotate [Fig. 3(b)]. The measurable conductances G_j , however, contain, with G_{A_j} , contributions that represent strong enough local distortions to suppress Q below 2. In the right QD the local distortions are enhanced by level overlaps close to $\theta = 60^\circ$ where the K and K' levels become degenerate [Fig. 3(b)], and indeed Q_{G_R} decreases in this region. In contrast, the left QD levels remain well separated, and Q_{G_L} mirrors the upturn of $Q_{G_{\text{CAR}}}$, with G_{CAR} overruling the G_{A_L} contribution. The same behavior with $G_L \leftrightarrow G_R$ is found near $\theta = -90^\circ$. On the other hand, G_{nl} corresponds to an experiment of electron injection through a normal lead and contains with G_{EC} a component describing the uncorrelated single-particle transport. Since we find that G_{EC} and G_{CAR} have a similar amplitude, we expect that $Q_{G_{nl}} \sim Q_{G_{\text{CAR}}}/2$. However, G_{EC} contains also the higher order tunneling processes that represent the nonlocal distortions, which may cause $Q_{G_{nl}}$ to increase again. Nonetheless, we find that $Q_{G_{nl}} \sim 1$ with a similar shape as $Q_{G_{\text{CAR}}}$, indicating that the nonlocal distortions have a negligible effect.

While G_{CAR} produces the purest indicator of spin entanglement, it is only indirectly accessible by experiments. On the other hand, the directly measurable G_j are obscured by the local contributions of the G_{A_j} . A method of

circumventing this problem is to consider products of the G_j , such as $G_L G_R$. Since the projections $P \equiv P_{\pm a_\tau} \otimes P_{\pm b_{\tau'}}$ eliminate all QD degrees of freedom, the product $G_L G_R$ is equivalent to a nonlocal current measurement with a density matrix ρ' whose nonlocal contribution is encoded in $P \rho' P \propto P \rho^2 P$. By the higher power of ρ and the projections, the relative weight of the local contributions can be reduced, while a spin singlet in $P \rho P$ remains a spin singlet in $P \rho^2 P$. In Fig. 3(c) we see that the corresponding curve $Q_{G_L G_R}$ follows almost perfectly $Q_{G_{\text{CAR}}}$, showing that the multiplication $G_L G_R$ is powerful enough to suppress the local distortions in the G_j . Therefore, a high splitting efficiency of a CPS is not a primary requirement for the proposed Bell test.

To demonstrate that the large Q value is indeed an effect of superconductivity, we show with $Q_{G_L G_R}^n$ the corresponding curve for $G_L G_R$ obtained for the normal state. The fact that $Q_{G_L G_R}^n \approx 0$ is the strongest indicator that $Q_{G_L G_R}$ demonstrates indeed the spin entanglement.

Finally, we have truncated the curves in Fig. 3 close to $\theta = 60^\circ$ and -90° where QD levels strongly overlap [Fig. 3(b)] and spin correlators can no longer be reconstructed. It is indeed important to maintain well separated QD levels. Hence, the charging energy of the QDs, which has been neglected in the microscopic calculation, plays here an important role as it increases the level separation but has much reduced exchange coupling due to the SOI induced spin projections of the QD levels.

Conclusions.—We have demonstrated that due to SOI effects bent CNT-CPS (or two CNTs under an angle) can be used for entanglement detection in the steady state by a violation of the Bell inequality. Notable for the Bell inequality is that the set of axes \mathbf{a}_τ , $\mathbf{b}_{\tau'}$ along which the spin correlators must be measured can be arbitrary and the precise axis orientations, i.e., the precise SOI strengths, do not need to be known. This is an advantage over entanglement witnesses [25] or quantum state tomography. Although discussed for CNTs, the introduced concept of entanglement detection is general and can be implemented in any system allowing tunable spin-energy filtering. For an ideal CNT-CPS, a violation of the Bell inequality can be achieved for most CNTs over a large range of orientations of an external field \mathbf{B} with strength $B \sim |\mathbf{B}_{\text{SOI}}|$, which for usual CNTs are < 1 T. The robustness of this behavior was confirmed by a microscopic calculation that incorporates the local and nonlocal imperfections of a realistic system. From the results we propose the use of the product of conductances $G_L G_R$ as the optimal observable for testing the Bell inequality. We have furthermore argued that the spin reconstruction in semiconducting CNTs is robust against disorder.

To conclude, it should be noted that a bending of the CNT is not an absolute requisite. An equivalent effect can be obtained by applying individual \mathbf{B} fields on the QDs or by providing a constant field offset on one QD by placing a

ferromagnet in its vicinity, if sufficient control of the typical field strengths $|\mathbf{B}| \sim |\mathbf{B}_{\text{SOI}}| < 1$ T can be granted. If two separate CNTs are connected to the superconductor, they should have similar diameters such that their \mathbf{B}_{SOI} are comparable.

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