Transport Signatures of Floquet Majorana Fermions in Driven Topological Superconductors

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Floquet Majorana fermions are steady states of equal superposition of electrons and holes in a periodically driven superconductor. We study the experimental signatures of Floquet Majorana fermions in transport measurements and show, both analytically and numerically, that their presence is signaled by a quantized conductance sum rule over discrete values of lead bias differing by multiple absorption or emission energies at drive frequency. We also study the effects of static disorder and find that the quantized sum rule is robust against weak disorder. Thus, we offer a unique way to identify the topological signatures of Floquet Majorana fermions.

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Introduction.—The nonlocal quantum order characterizing the topological state of a gapped medium often necessitates the existence of topologically protected gapless states bound to bulk defects or the edge with a topologically trivial medium where the gap closes. The detection of these topological bound states is, therefore, a primary probe of the topological state. It has recently been understood that topological bound states may arise as steady states when a topologically trivial system is driven periodically $[1-3]$ $[1-3]$ $[1-3]$. In the superconducting state, these are equal superpositions of electrons and holes known as Floquet Majorana fermions [\[4](#page-4-0),[5](#page-4-1)] that exhibit non-Abelian statistics [[6](#page-4-2),[7\]](#page-4-3) and can be used for topological quantum computation [[8](#page-4-4)]. This possibility expands the systems and conditions that realize Majorana fermions as emergent quasiparticles [[9](#page-4-5)[–17\]](#page-4-6), but also poses fundamental questions as to how to detect and possibly manipulate such steady states. In particular, since the driven system is not in equilibrium, the experiments probing the equilibrium response of static Majorana fermions [\[18–](#page-4-7)[22](#page-4-8)] cannot be used directly for this purpose.

In this Letter, we address these questions by studying the nonequilibrium transport properties of Floquet Majorana fermions. We show, both analytically and numerically, that there is a quantized conductance sum rule, which we dub the ''Floquet sum rule,'' whenever Floquet Majorana fermions exist. The Floquet sum rule naturally generalizes the quantized zero-bias conductance of static Majorana fermions [[23](#page-4-9)[–28\]](#page-4-10). Moreover, we show that the Floquet sum rule is robust against moderate static disorder, owing to its topological character, while other peaks get suppressed. Remarkably, this suggests that disorder, usually detrimental to electronic properties, can be used as a ''sieve'' to find Floquet Majorana fermions. Transport studies in irradiated graphene, where the Floquet topological insulator was first proposed to exist [[1\]](#page-3-0), suggested quantized transport in the driven system is possible in certain geometries and for large drive frequencies [\[29,](#page-4-11)[30\]](#page-4-12). We use a systematic Green's function method that extends the previous studies to superconducting systems in any frequency range, and can, in principle, incorporate the effects of interactions.

Though our results are applicable to any realization of Floquet Majorana fermions, systems of cold atoms could prove specially useful in this regard due to a high degree of design control and newly developed experimental probes of their dynamics, such as single-atom imaging, tunneling, and transport [[31–](#page-4-13)[37](#page-4-14)]. Disorder can be introduced in cold atom systems controllably [\[38,](#page-4-15)[39](#page-4-16)] and could, therefore, play a key role in the detection and manipulation of Floquet Majorana fermions. In the solid state, such as in quantum wires [\[9](#page-4-5)[,14](#page-4-17),[15](#page-4-18)[,18](#page-4-7)[,19](#page-4-19)[,22](#page-4-8)], high-frequency irradiation of the order of the bandwidth is detrimental to the proximity-induced superconducting state. However, we find numerically that even at much lower frequencies, Floquet Majorana fermions can still be realized and have the same transport signatures, with or without disorder, as at higher frequencies.

Model.—We study the model Hamiltonian $H(t) =$ $H_w(t) + H_c + H_l$, where the last term describes the leads,

$$
H_{\rm w}(t) = \frac{i}{2} \gamma^{\rm T} A(t) \gamma,
$$
 (1)

is the Hamiltonian of the system (wire) in the Majorana basis $\gamma^T = (\gamma_1, ..., \gamma_{2N})$ with a real, skew-symmetric matrix $A(t)$, and the contact Hamiltonian $H_c =$ matrix $A(t)$, and the contact Hamiltonian $H_c = \sum_{\lambda} a^{\lambda \dagger} K^{\lambda} \gamma + \text{H.c.}$ with $a^{\lambda \dagger}$ is the row of electronic creation operators in lead λ and K^{λ} a contact matrix tion operators in lead λ , and K^{λ} a contact matrix.

Our analytical results are presented for a general realization of Majorana fermions. For numerical calculations, we choose the simple model of a single-band quantum wire with superconducting pairing in a spin-polarized electronic band [[9](#page-4-5),[40](#page-4-20)]. This model can be effectively realized in the solid state [\[14,](#page-4-17)[15](#page-4-18)[,18](#page-4-7)[,19](#page-4-19)[,22\]](#page-4-8) and potentially in cold atom systems [[4](#page-4-0)[,41–](#page-4-21)[44\]](#page-4-22). There are two Majorana operators $(\gamma_{r1}, \gamma_{r2}) \equiv \gamma_r^{\dagger}$ at sites $r = 1, ..., L$. The contact matrix
elements $K^{\lambda} \propto (1, i)$ in the Majorana basis at each site. The elements $K^{\lambda} \propto (1, i)$ in the Majorana basis at each site. The nonzero elements of A are nonzero elements of A are

$$
A_{r,r} = -i\mu_r \sigma_y, \qquad A_{r,r+1} = \Delta_r \sigma_x + i w_r \sigma_y, \quad (2)
$$

and $A_{r+1,r} = -A_{r,r+1}$, where the real parameters μ_r , Δ_r , and w_r are, respectively, the chemical potential at site r, the superconducting pairing, and the hopping integral on $(r, r + 1)$ bond, and $(\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices. The static, uniform wire with bandwidth $W = 4|\text{Re } w|$ has a topological phase transition at $2|\mu| = W$ where the gap closes. There is an unpaired Majorana fermion at each end of the wire with zero energy (the energy is referenced to chemical potential of the wire) in the topological phase $2|\mu| < W$ and none in the trivial phase $2|\mu| > W$ [[9\]](#page-4-5).

When the system is driven with period $T = 2\pi/\Omega$,
repeated valution of the time-periodic Schrödinger equathe general solution of the time-periodic Schrödinger equation (we set $\hbar = 1$) $H(t)|\psi(t)\rangle = i\partial_t |\psi(t)\rangle$ is found in terms of the Floquet functions $|\psi_{\alpha}(t)\rangle = e^{-i\epsilon_{\alpha}t} |\phi_{\alpha}(t)\rangle$,
where $|\phi_{\alpha}(t+T)\rangle = |\phi_{\alpha}(t)\rangle$ is an eigenket of the effecwhere $|\phi_{\alpha}(t+T)\rangle = |\phi_{\alpha}(t)\rangle$ is an eigenket of the effective Hamiltonian $H_{\text{eff}}(t) = H(t) - i\partial_t$, $H_{\text{eff}}(t)|\phi_\alpha(t)\rangle =$ $\epsilon_{\alpha}|\phi_{\alpha}(t)\rangle$. The quasienergies ϵ_{α} are restricted to $(-\Omega/2, \Omega/2]$ by the map $\epsilon_{\alpha} \mapsto \epsilon_{\alpha} + k\Omega$, $|\phi_{\alpha}(t)\rangle \mapsto$
 $e^{ik\Omega t}|\phi_{\alpha}(t)\rangle$ We shall compute the Floquet spectrum $\{\epsilon\}$ $e^{ik\Omega t}|\phi_{\alpha}(t)\rangle$. We shall compute the Floquet spectrum $\{\epsilon\}$
using the evolution operator $I(t)|\psi(0)\rangle = |\psi(t)\rangle$ and conusing the evolution operator $U(t)|\psi(0)\rangle = |\psi(t)\rangle$ and constructing the Floquet Hamiltonian $H_F = (i/T) \log[U(T)]$. Then, $H_F|\phi_\alpha(0)\rangle = \epsilon_\alpha |\phi_\alpha(0)\rangle$. The periodic eigenkets can be resolved in a Fourier series $|\phi(t)\rangle = \sum_{k} e^{-ik\Omega t} |\phi^{(k)}\rangle$.
We shall use a shorthand $|\phi\rangle$ tor vectors in the extended We shall use a shorthand $|\phi_{\alpha}\rangle$ for vectors in the extended Hilbert space spanned by $|\phi_{\alpha}^{(k)}\rangle$, with the inner product $\langle A'|A\rangle = \sum_{\alpha} |\phi_{\alpha}^{(k)}(t)|A(t)\rangle = f[A(t)]A(t)\rangle dt/T$ [45] $\langle \langle \phi' | \phi \rangle \rangle \equiv \sum_{k} \langle \phi'^{(k)} | \phi^{(k)} \rangle = \int_{0}^{T} \langle \phi'(t) | \phi(t) \rangle dt / T$ [[45\]](#page-4-23).
Floquet Majorana fermions — Floquet Majorana fermions

Floquet Majorana fermions.—Floquet Majorana fermions are bound states with quasienergy $\epsilon_0 = 0$ or $\epsilon_{\pi} =$ $\Omega/2$ [[4\]](#page-4-0). The particle-hole symmetry, $H_{\rm w}^{\rm T} = -H_{\rm w}$,
requires $H_{\rm w}^{\rm T} = -H_{\rm w}$ so the quasienergies come in pairs requires $H_F^{\dagger} = -H_F$, so the quasienergies come in pairs $(\epsilon - \epsilon)$. In the Nambu basis $(\epsilon_{\alpha}, -\epsilon_{\alpha})$. In the Nambu basis,

$$
\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \phi_{\alpha} = \begin{pmatrix} u_{\alpha} \\ v_{\alpha} \end{pmatrix}
$$

the ϵ_0 and ϵ_{π} Floquet Majorana fermions satisfy $v_0(t)$ = $u_0^*(t)$ and $v_\pi(t) = e^{i\Omega t} u_\pi^*(t)$.
In Fig. 1, we show the Flo

In Fig. [1,](#page-1-0) we show the Floquet spectrum of a wire with a square-wave periodic chemical potential $\mu(t)$ alternating with frequency Ω between μ_1 and μ_2 , respectively, over time intervals t_1 and t_2 in each period. Note that $\mu(t)$ is not in the topological range at any time. As Ω/W decreases, the Floquet band spreads and ϵ crosses ϵ_{π} , giving rise to a pair of ϵ_{π} Floquet Majorana fermion bound states. Additional gap-closing level crossings lead to the appearance or disappearance of ϵ_0 and ϵ_{π} Floquet Majorana fermions.

A high-frequency approximation for $\Omega \gg W$ can be
adequation the Baker-Campbell-Hausdorff formula made using the Baker-Campbell-Hausdorff formula, $TH_F = t_1H_1 + t_2H_2 + t_1t_2[H_2, H_1] + (1/12)t_1t_2[t_2H_2$ t_1H_1 , $[H_2, H_1]] + \cdots$. The first two terms yield a static quantum wire with an averaged chemical potential $\langle \langle \mu \rangle \rangle$ = $(t_1\mu_1 + t_2\mu_2)/T$. The only terms contributing to the commutator $[H_2, H_1]$ are $\Delta \sigma_x$ in $A_{r,r+1}$ and $-i\mu \sigma_y$ in $A_{r,r}$, yielding a σ_z term in $A_{r,r+1}$ that contributes to Im w. Physically, Im w introduces a supercurrent in the chain,

FIG. 1 (color online). The evolution of the Floquet spectrum of a quantum wire with L sites vs Ω for a square-wave chemical potential. The arrows show the frequencies used for the transport calculations in Fig. [3.](#page-2-0)

which renormalizes the spectral gap but, when small, leaves the topological phase boundary unchanged [[40\]](#page-4-20). The same is true for the next term shown. Therefore, when $2\langle\langle|\mu|\rangle\rangle > W$, there are no Floquet Majorana fermions in the high-frequency limit. In the low-frequency limit $\Omega \ll W$ multiple exchange processes with energy Ω
become important and result in qualitative differences become important and result in qualitative differences between the static energy and quasienergy spectra. We note here that, as can be seen in Fig. [1](#page-1-0), Floquet Majorana fermions are found numerically for a much wider range of parameters, including $\Omega \ll W$ and $2\langle\langle|\mu|\rangle\rangle > W$ [[7](#page-4-3)[,46\]](#page-4-24).
Transport—Electrons in a driven system do not follo

Transport.—Electrons in a driven system do not follow the usual statistics in a closed system. In the transport problem we can address this issue by assuming that the leads are static and follow the usual Fermi-Dirac statistics at the distant past. The scattering problem between the leads can then be formulated by integrating out the leads using their Green's function [[47](#page-4-25)[,48](#page-4-26)]. This procedure adds to H_w an imaginary self-energy $i\Gamma(t) = i\sum_{\lambda} \Gamma^{\lambda}(t)$, where $\Gamma^{\lambda}(t) = 2\text{Im}[\mathbf{K}^{\lambda \dagger} \sigma^{\lambda}(t) \mathbf{K}^{\lambda}]$ and σ^{λ} is the Green's function $\Gamma^{\lambda}(t) = 2\text{Im}[K^{\lambda \dagger} g^{\lambda}(t) K^{\lambda}],$ and g^{λ} is the Green's function
of lead λ . The wire's Green's function is periodic of lead λ . The wire's Green's function is periodic, $G(t + T, t' + T) = G(t, t')$, and satisfies

$$
[\partial_t - A(t)]G(t, t') - (\Gamma * G)(t, t') = -i\delta(t - t'), \quad (3)
$$

where $\Gamma * G$ is the convolution $\int_{-\infty}^{t} \Gamma(t-s)G(s, t')ds$.
Then the steady state (time-averaged) current in lead λ Then the steady state (time-averaged) current in lead λ , $J^{\lambda} = ie \langle \langle [H(t), a^{\lambda \dagger} a^{\lambda}] \rangle \rangle$, can be computed with the Green's function. Assuming the leads' density of states Green's function. Assuming the leads' density of states ρ^{λ} is constant over the scattering energy range, we find an energy-independent $\Gamma^{\lambda} = -(\xi^{\lambda} + \xi^{\lambda}T)/2$, ξ^{λ}
 $2\pi K^{\lambda}T \partial^{\lambda} K^{\lambda}$ and the differential conductance σ^{λ} $2\pi K^{\lambda \dagger} \rho^{\lambda} K^{\lambda}$ and the differential conductance $\sigma^{\lambda} =$ $dJ^{\lambda}/dV_{\lambda}$, with bias V_{λ} , reads

$$
\sigma^{\lambda} = -\frac{e^2}{2\pi} \int d\omega \Bigg[\sum_n \{ \text{Tr}[\xi^{\lambda \dagger} G^{(n)}(\omega) \xi^{\lambda} G^{(n) \dagger}(\omega)] f'_{\lambda}(\omega) + \text{Tr}[\xi^{\lambda} G^{(n)}(\omega) \xi^{\lambda \dagger} G^{(n) \dagger}(\omega)] f'_{\lambda}(-\omega) \} + C_{\lambda}(\omega) f'_{\lambda}(\omega) \Bigg], \tag{4}
$$

FIG. 2 (color online). Differential conductance $\sigma = dI/dV$ vs bias V in a single-terminal setup for a static system with $L = 80$, $\Delta/w = 0.6$, $\mu/w = 0.25$, and $\nu/w = \pi/25$. The (blue) dots are calculated from the analytical expression of peak heights.

where $f_{\lambda}(\omega) = [1 + e^{(\omega - \epsilon V_{\lambda})/\tau_{\lambda}}]^{-1}$ is the Fermi-Dirac dis-
tribution $G^{(n)}(\omega) = (1/T) \int_T^T \int e^{in\Omega t} e^{i\omega s} G(t, t - s) ds dt$ tribution, $G^{(n)}(\omega) = (1/T) \int_0^T \int e^{in\Omega t} e^{i\omega s} G(t, t - s) ds dt$, and $C_{\lambda}(\omega) = \sum_{\kappa \neq \lambda, n} \text{Tr}[(\xi^{\kappa} + \xi^{\kappa}) G^{(n)}(\omega) \xi^{\lambda} G^{(n) \dagger}(\omega)]$
[48] The static case is found by setting [\[48\]](#page-4-26). The static case is found by setting $G^{(n)}(\omega) = \delta_{n,0}G(\omega).$
For a single lead

For a single lead with a point contact, C_{λ} vanishes identically and the contact matrix is zero except at the contact site where $\xi = \nu(1 - \sigma_y)/2$ with $\nu = 2\pi \rho |w|^2$.
Then Then,

$$
\sigma = -\frac{e^2 \nu^2}{2\pi} \sum_n \int [|\mathcal{G}_{\text{he}}^{(n)}(\omega)|^2 + |\mathcal{G}_{\text{eh}}^{(n)}(-\omega)|^2] f'(\omega) d\omega,
$$
\n(5)

where the G_{eh} and G_{he} are the off-diagonal elements of the Nambu-Gorkov Green's function at the contact site,

$$
G^{(n)}(\omega) = \sum_{\alpha k} \frac{|\varphi_{\alpha}^{(n+k)}\rangle\langle\bar{\varphi}_{\alpha}^{(k)}|}{\omega - \epsilon_{\alpha} - n\Omega + i\delta_{\alpha}},\tag{6}
$$

with $-i\delta_{\alpha}$ the self-energy correction to quasienergy ϵ_{α} , and $|\varphi_{\alpha}\rangle$ and $\langle\bar{\varphi}_{\alpha}|$, respectively, the right and left Floquet eigenvectors of the effective (non-Hermitian) Hamiltonian $H_w + i\Gamma$ at level α .

In the weak-contact limit $\nu/w \ll 1$, we can employ perturbation theory in Γ . To the leading order, we find $|\varphi_{\alpha}\rangle = |\phi_{\alpha}\rangle$, $\langle \bar{\varphi}_{\alpha}| = \langle \phi_{\alpha}|$ (i.e., the same as eigenvectors of H_w), and $\delta_\alpha = - \langle \langle \phi_\alpha | \Gamma | \phi_\alpha \rangle \rangle$ [[45](#page-4-23)]. Let us first work out the static case. Then, $\delta_{\alpha} = (1/2)\nu (|u_{\alpha}^c|^2 + |v_{\alpha}^c|^2)$ with u^c and v^c evaluated at the contact site. At zero temperabut the static case. Then, $\sigma_{\alpha} = (1/2)\nu(|u_{\alpha}| + |v_{\alpha}|)$ with u_{α}^c and v_{α}^c evaluated at the contact site. At zero temperature, $\lim_{V \to E_\alpha} \sigma(V) = \sigma_\alpha L(V - E_\alpha/\delta_\alpha)$ where E_α is an energy level of the static system, $L(z) = (1 + z^2)^{-1}$ is the I original and the peak value Lorentzian, and the peak value,

$$
\sigma_{\alpha} = \frac{2e^2}{2\pi} \left| \frac{2u_{\alpha}^{\rm c}v_{\alpha}^{\rm c}}{|u_{\alpha}^{\rm c}|^2 + |v_{\alpha}^{\rm c}|^2} \right|^2. \tag{7}
$$

For the zero-energy Majorana fermion, $u_0 = v_0^*$, so $\sigma_0 = 2e^2/h$ in restored units as is well known [25.49] $\sigma_0 = 2e^2/h$ in restored units, as is well known [\[25,](#page-4-27)[49\]](#page-4-28). In Fig. [2](#page-2-1), we compare this analytical expression with a full numerical solution.

Floquet sum rule.—In the driven system, the peaks at $V = \epsilon_0$ and $V = \epsilon_{\pi}$ are not quantized even when Floquet Majorana fermions are present. This is because energies ϵ_{α} + n Ω are all connected via the drive force by emission
and absorption processes. Instead, we find a Floquet sum and absorption processes. Instead, we find a Floquet sum rule for the sum of differential conductance at these energies [\[48\]](#page-4-26),

FIG. 3. Differential conductances σ (top row) and $\tilde{\sigma}$ (bottom row) of the driven system as a function of bias V/Ω in a two-terminal setup. The parameters Δ , μ_1 μ_1 , μ_2 , and t_1/t_2 are as in Fig. 1, $\nu/w = 2\pi/25$, and the other parameters are (a),(d) $L = 40$, $\Omega/2w = 0.37$,
(b) (e) $L = 70$, $\Omega/2w = 0.49$ (c) (f) $L = 40$, $\Omega/2w = 0.75$. The fre (b),(e) $L = 70$, $\Omega/2w = 0.49$, (c),(f) $L = 40$, $\Omega/2w = 0.75$. The frequencies are marked by arrows in Fig. [1](#page-1-0).

$$
\tilde{\sigma}(V) = \sum_{n} \sigma(V + n\Omega). \tag{8}
$$

At zero temperature, $\lim_{V \to \epsilon_{\alpha}} \tilde{\sigma}(V) = \tilde{\sigma}_{\alpha} L(V - \epsilon_{\alpha}/\delta_{\alpha})$ is, again, a Lorentzian with the peak value,

$$
\tilde{\sigma}_{\alpha} = \frac{2e^2}{2\pi} \left| \frac{2\|u_{\alpha}^{\rm c}\| \|v_{\alpha}^{\rm c}\|}{\|u_{\alpha}^{\rm c}\|^2 + \|v_{\alpha}^{\rm c}\|^2} \right|^2, \tag{9}
$$

where $||z||^2 = \sum_k |z^{(k)}|^2$. By particle-hole symmetry $||u_0|| = ||u_0||$ and $||u|| = ||u_1||$ Thus if there is a Floquet $||v_0||$ and $||u_\pi|| = ||v_\pi||$. Thus, if there is a Floquet
Maiorana fermion at ϵ_0 and/or ϵ Majorana fermion at ϵ_0 and/or ϵ_{π} ,

$$
\tilde{\sigma}_0 \equiv \tilde{\sigma}(\epsilon_0) = \frac{2e^2}{h}
$$
 and/or $\tilde{\sigma}_{\pi} \equiv \tilde{\sigma}(\epsilon_{\pi}) = \frac{2e^2}{h}$, (10)

respectively. These relations can be generalized for nonpoint contact terms as well. This is our central result.

The two Floquet Majorana fermions overlap and split away from ϵ_0 or ϵ_{π} by an amount λ that is exponentially small in their separation. When $\lambda > \nu$, $\tilde{\sigma}$ also splits with the peak values $\tilde{\sigma}_0$ and $\tilde{\sigma}_{\pi}$ shifting to $V = \epsilon_0 \pm \lambda$ and $V = \epsilon_0 \pm \lambda$ respectively each with half the widths of the $\epsilon_{\pi} \pm \lambda$, respectively, each with half the widths of the central peak. Therefore, the total weight stays the same central peak. Therefore, the total weight stays the same. This is a general feature: the total weight $\int_0^{\Omega} \tilde{\sigma}(V) dV = \int_{-\infty}^{\infty} \sigma(V) dV \propto \sum_n \int |G_{eh}^{(n)}(\omega)|^2 d\omega$ is constant at all temperatures temperatures.

We have numerically investigated the Floquet sum-rule quantization in the quantum wire. The plots in Fig. [3](#page-2-0) show the steady differential conductance calculated for a twolead setup with symmetric biases $\pm V$. It is clear that, within our numerical precision, $\tilde{\sigma}_0$ and/or $\tilde{\sigma}_{\pi}$ are quantized at $2e^2/h$ exactly when ϵ_0 and/or ϵ_π Floquet Majorana fermions appear. Note that the individual peaks of σ at $V = n\Omega$ (for ϵ_0 Floquet Majorana fermion) or $V = (2n + 1)\Omega/2$ (for ϵ Floquet Majorana fermion) are not $(2n+1)\Omega/2$ (for ϵ_{π} Floquet Majorana fermion) are not quantized. Indeed, the main contribution is not even from quantized. Indeed, the main contribution is not even from $n = 0$ [[48](#page-4-26)]. The Floquet spectrum is naturally reflected in $\tilde{\sigma}$: The quantized peaks at ϵ_0 (or ϵ_{π}) are separated from the other peaks by a value of $V \sim \epsilon_{g,0}$ (or $\epsilon_{g,\pi}$), i.e., the gap in the quasienergy gap separating the respective Floquet Majorana fermions from the other states. The quasienergy gaps in Fig. [3](#page-2-0) are $\sim 0.1 \Omega$ except for $\epsilon_{g,0} \sim 0.05 \Omega$ in
Fig. 3(a) Fig. $3(e)$.

Effects of disorder.—The natural question to answer at this point is whether and how $\tilde{\sigma}$ could be measured in an actual experiment. It is especially important to be able to tell apart a quantized peak from the other features, which is complicated if $\epsilon_{g,0}$ and $\epsilon_{g,\pi}$ are small. A possible way around is to exploit the topological character of the quantization of $\tilde{\sigma}_0$ and $\tilde{\sigma}_{\pi}$. Specifically, they must be protected against disorder while the other features are not. We have studied the effects of disorder numerically by adding a static, uncorrelated, random $\delta \mu_r$ to the wire's chemical potential at site r, i.e., $\langle \delta \mu_r \rangle_{\text{dis}} = 0$ and $\langle \delta \mu_r \delta \mu_{r'} \rangle_{\text{dis}} =$ $\mu_d^2 \delta_{rr'}$ where μ_d is the disorder strength. A typical result

FIG. 4. Differential conductance in a two-terminal setup averaged over 50 disorder configurations. The parameters are as in Fig. [3\(a\)](#page-2-2) and [3\(d\)](#page-2-2) and disorder strength $\mu_d = 0.28w = 0.38 \Omega$.

for the disorder-averaged $\langle \tilde{\sigma} \rangle_{dis}$ at moderate disorder is shown in Fig. [4.](#page-3-2) The central quantized peak remains nearly unchanged, while the other peaks are suppressed significantly. Note that here $\mu_d > \epsilon_{g,0}$. For stronger disorder the quantized peak is suppressed as well [[48](#page-4-26)].

Concluding remarks.—In sum, we find that the differential conductance summed over periodic drive harmonics, $\tilde{\sigma}$, signals Floquet Majorana fermions with a topologically protected quantized value $2e^2/h$ at the Floquet Majorana quasienergy. The quantization is robust and most prominent in the presence of weak disorder. This suggests disorder can be used as a knob to probe Floquet Majorana fermions. At lower frequencies where rotating-wave and similar approximations $[2,3]$ $[2,3]$ fail, we have numerically found steady state Floquet Majorana fermions, with similar transport signatures with or without disorder [\[48\]](#page-4-26). This is important for possible realization schemes in solid-state systems. The finite temperature behavior is discussed in the Supplemental Material [[48](#page-4-26)].

Other transport signatures of Floquet Majorana fermions, such as noise and heat transport, are interesting, open problems. A thorough study of the low-frequency regime is also quite important. Our inclusion of static disorder is appropriate if disorder is intrinsic to the wire itself and not the drive. Other disorder configurations, e.g., in the contacts or the external drive itself, would be interesting to study in the future. Finally, the effects of disorder at finite temperature as well as interactions are left to future studies.

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