Finite-Temperature Behavior of Glueballs in Lattice Gauge Theories

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We propose a new method to compute glueball masses in finite temperature lattice gauge theory which at low temperature is fully compatible with the known zero temperature results and as the temperature increases leads to a glueball spectrum which vanishes at the deconfinement transition. We show that this definition is consistent with the Isgur-Paton model and with the expected contribution of the glueball spectrum to various thermodynamic quantities at finite temperature. We test our proposal with a set of high precision numerical simulations in the 3D gauge Ising model and find a good agreement with our predictions.

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While the physics of glueballs in pure lattice gauge theory (LGT) at zero temperature is by now rather well understood [1–3] a similar level of understanding for the finite temperature behavior of the glueball spectrum is still lacking. The standard method used to compute finite T glueball masses [4,5] is to measure the correlator of the glueball operator (for instance a simple plaquette, if one is interested in the 0^{++} glueball). along the compactified time direction of length $1/T$. The glueball spectrum obtained in this way turns out to be almost constant as the temperature increases and seems not to be affected by the deconfinement transition: glueball masses were measured even deeply in the deconfined phase showing values similar to the zero temperature ones (or slightly smaller, depending on the procedure adopted in the calculation) [4,5]. However, this picture is unsatisfactory for at least two reasons.

First, one of the most successful phenomenological descriptions of glueballs is the well known Isgur-Paton model [6]. This model, and its recent generalizations [7], is able not only to predict the general structure of the spectrum (like, for instance, the fact that the mass of the 2^{++} state is lower than the mass of the 1^{++}) but also its fine details and shows a remarkable agreement with the lattice estimates. In this model, glueballs are considered as "rings of glue," kept together by the same string tension which appears in the interquark potential and, thus, should vanish at the deconfinement point when the string tension vanishes. If we trust this picture, then it could also be used to predict the T dependence of the glueball spectrum for low temperatures. In fact, the Isgur Paton model predicts values of the zero temperature glueball masses $m_i(T=0)$ as adimensional ratios $m_i(0)/\sqrt{\sigma(0)}$ [where $\sigma(0)$ is the zero temperature string tension]. In a finite temperature setting, we expect the same ratios, but with $\sigma(0)$ substituted by the finite temperature string tension $\sigma(T)$, i.e.,

$$m_i(T) = \frac{m_i(0)}{\sqrt{\sigma(0)}}\sqrt{\sigma(T)}, \tag{1}$$

which is a decreasing function of T. This expectation is in complete disagreement with the almost constant T dependence proposed in [4,5] for this range of temperatures.

Second, recently, very precise estimates of various thermodynamic quantities have been obtained both below and above T_c. in pure lattice gauge theories in $d = 3 + 1$ [8,9] and in $d = 2 + 1$ [10,11] dimensions. For $T < T_c$ the thermodynamics of these theories is very well described in terms of a gas of glueballs which are the only degrees of freedom of the theory in this regime. For $T > T_c$ the thermodynamics is well described by a gas of free gluons (and, accordingly, the thermodynamic observables scale as N^2). If glueballs were present also in the deconfined phase, they would give an additional contribution to the thermodynamic observables; while it is not possible to completely exclude the presence of this contribution, it seems unlikely given the precision of the available data.

These observations suggest that, with the current method to extract finite T glueball masses [4,5], one is probably measuring some other finite size scale of the model whose relation with the glueball spectrum is similar to the relation which exists between the spacelike string tension σ_s and the finite temperature string tension $\sigma(T)$. Indeed, also σ_s, which is extracted from spacelike Wilson loops [12], is almost constant for $T < T_c$, increases for $T > T_c$, and it is well known to be completely unrelated to the finite temperature string tension $\sigma(T)$ which is instead extracted from Polyakov loop correlators.

In this Letter, we propose an alternative prescription for evaluating finite T glueball masses, compatible with the above observations. The most direct way to ensure the expected finite T behavior is to construct an observable with the correct quantum numbers so as to be coupled in the continuum limit to the glueball states, using only Polyakov loops so as to have the correct dependence on the finite temperature string tension. The simplest proposal is to choose a pair $PP^\dagger$ of nearby Polyakov loops

$$M(x) = P(x)P^\dagger(x+a), \tag{2}$$

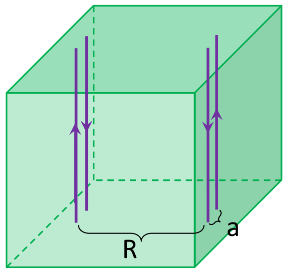

FIG. 1 (color online). The glueball correlator discussed in the text.

where a denotes the lattice spacing and $P(x)$ is the Polyakov loop located at the space point x. Then the glueball mass will be extracted looking at the large R behavior of the connected correlator of two $M(x)$ operators as depicted in Fig. 1

$$G(R,T) \equiv \langle M(0)M(R)\rangle - \langle M\rangle^2 \underset{R\to\infty}{\sim} e^{-m_0(T)R}. \quad (3)$$

The spacelike version (usually denoted as a "torelon pair") was used in [2,3] as part of the operator basis to obtain the $T=0$ glueball spectrum. In the proposed interpretation, this set up is new.

Let us also stress that this is not the only possible choice; for instance, in non-Abelian gauge theories, an equivalent interesting possibility would be the Wilson loop obtained joining the two Polyakov loops with two spacelike links at $t=0$ and $t=N_t$ (this choice is obviously equivalent to our proposal for Abelian LGT).

The nice feature of our proposal is that it has a natural interpretation in terms of the effective string model of pure gauge theories. It is the four point correlator of four closed effective strings (see Fig. 2). The external legs correspond to the four Polyakov loops (which are described as closed strings due to the compactification of the time direction), while the glueballs are the excitations of the closed string joining together the four legs. As mentioned in the introduction, this proposal is strongly based on our intuition of the glueball dynamics coming from the Isgur-Paton model. It might be useful to make this connection more explicit. If we could perform a section in the four strings function as depicted in Fig. 2, the effective string description of the flux distribution within the section would be given by the Isgur Paton model. Accordingly, we expect that all the glueballs (independently of their quantum numbers) would flow within the closed string, of which they would represent different radial or rotational excitations. In the large R limit, only the lowest mass survives, but in principle, looking at the subleading exponentials for lower values of R, one could also recover the remaining states of the spectrum. While the radius of the four external legs is fixed to be the inverse of the finite temperature, the radius of the internal closed string coincides with the glueball radius r_0 which is one of the parameters of the Isgur-Paton model.

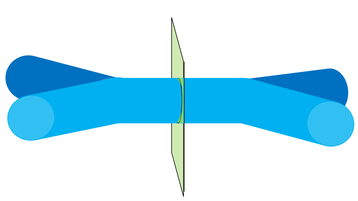

FIG. 2 (color online). Effective string description of our proposal.

Despite all these interesting features, there is apparently a major problem with this proposal. In fact, due to dimensional reduction, one expects that any mass scale extracted from an observable of this type should scale in the vicinity of the deconfinement transition as $m_s(T) \sim \sigma(T)/T$ which, as it is easy to see, is pretty different from the expected scaling behavior of Eq. (1).

We shall see below, in a concrete example, how this problem can be addressed. Indeed, as we shall see, for any $T < T_c$, a (glueball) mass with the correct scaling behavior is always present in the spectrum of $G(R,T)$, but as the deconfinement transition is approached, this mass becomes subleading and the large R behavior is dominated by a different mass scale (whose physical meaning we shall discuss below) with the "wrong" scaling behavior $m_s(T) \sim \sigma(T)/T$. However, even in the vicinity of T_c, there is always a suitable range of values of R in which the glueball mass, even if subleading, can be unambiguously observed.

Test in the 3D gauge Ising model.—In order to test our proposal, we computed the mass of the lightest glueball in the 3D gauge Ising model. This choice has two relevant advantages.

First, very precise estimates exist for the zero temperature spectrum [13] with which we can compare our results in the low T limit. In particular, we know that in the range of β values that we study in this Letter, $m_0 = 3.15(5) \times \sqrt{\sigma(0)}$ [13].

Second, using dimensional reduction [14] and the fact that the 3D gauge Ising model has a second order deconfinement transition in the same universality class of the 2D Ising magnetization transition, we can predict the behavior of the correlator $G(R,T)$ in the vicinity of T_c using results borrowed from the exact solution of the 2D Ising model. In this limit, our observable becomes equivalent to the energy-energy correlator in the high temperature phase of the 2D Ising spin model. From the exact solution of the model, we know that the large R behavior of the function should be dominated by a new mass scale m_s which is exactly twice the fundamental mass of the model, which, from dimensional reduction, is known to be $\sigma(T)/T$ (see, for instance, the discussion in Sec. 2.3 of [15]). Thus, as anticipated, we expect in this limit

$$m_s(T) \sim 2\sigma(T)/T. \quad (4)$$

We performed three sets of simulations at different values of the gauge coupling (corresponding to $1/T_c = 5.67a$, $8a$, and $12a$, respectively, [16]) in order to test

TABLE I. For each of the three β values, we report the corresponding critical temperature T_c and the values of L_s, N_t, and R that we studied.

β	$1/T_c$	L_s	N_t	R
0.743543	5.67 a	90	7,8,9	$6 \leq R \leq 20$
0.751805	8 a	90	9,10,11,12,13, 14,20,56,64	$8 \leq R \leq 22$
0.756427	12 a	120	20	$12 \leq R \leq 33$

scaling corrections. For each value of β, we chose a value of the lattice size in the spatial direction L_s large enough to make finite size effects negligible, and studied various values of compactified time direction $N_t \equiv 1/T$ in the range $T < T_c$. For each value of T, we evaluated the correlator $G(R, T)$ for several values of R. We also evaluated, for each T in a separate simulation, the finite temperature string tension $\sigma(T)$ (using the methods discussed in [17]) so as to be able to construct the scaling functions Eqs. (1) and (4). A few details on the simulations are reported in Table I.

We found two different behaviors. For low values of T (in our simulations the threshold was $T \lesssim 0.6T_c$) the data were perfectly fitted by the following expression:

$$G(R, T) = a_0(T)\frac{e^{-m_0(T)R}}{\sqrt{R}}, \tag{5}$$

with good χ^2 values in the whole range of values of R that we considered. The data were so precise that we could also confirm the presence of the expected $1/\sqrt{R}$ prefactor. The values of $m_0(T)$ extracted from the fits are reported in the last few lines (for each β) of Table II and in Fig. 3 and turned out to follow exactly the expected behavior Eq. (1), with a value of the glueball mass in good agreement with the $T = 0$ value $m_0 \sim 3.15\sqrt{\sigma(0)}$ obtained in [13].

For high values of T (i.e., in our case $T \gtrsim 0.6T_c$) it turned out to be impossible to fit the data using Eq. (5). Reasonable χ^2 values could only be obtained discarding the low R values of the correlators and using a different fitting function

$$G(R, T) = a_s(T)\frac{e^{-m_s(T)R}}{R^2}. \tag{6}$$

We use the notation m_s to stress the fact that this mass was obtained using a different fitting function. The prefactor $1/R^2$ is exactly what one would expect for the energy-energy correlator in the 2D Ising model and the mass m_s extracted form this fit scales exactly as suggested by Eq. (4). In full agreement with the expectation of dimensional reduction, not only the T dependence but also the fact that m_s is exactly twice the fundamental mass of the model is perfectly reproduced by the data (see Table II and Fig. 3).

This explains the large R behavior of $G(R, T)$; however, in order to also include the small R data in the fit, it turned out to be mandatory to use a two exponentials fitting function

$$G(R, T) = a_0(T)\frac{e^{-m_0(T)R}}{\sqrt{R}} + a_s(T)\frac{e^{-m_s(T)R}}{R^2}. \tag{7}$$

It is well known that this type of fit is very delicate. In our case, we used the following procedure: we fit introducing a Gaussian prior for m_s, centered at its value at large R with width comparable with its uncertainty, and we left, as free parameters only, $a_0(T)$, $a_s(T)$, and $m_0(T)$. In this way, we could fit all the data with a reduced χ^2 of order unity. It is important to stress that the identification of the subleading exponential was facilitated by the very different behavior of the two prefactors, by the wide range of values of R that we used in the fit, and by the fact that these data were not cross correlated since, due to the algorithm that we used (see [17]), each value of R was obtained in an independent simulation. These observations should be taken into account when trying to reproduce our results in other

TABLE II. Values of $\sigma(T)$, $m_s(T)/\sigma(T)$, $m_s(T)/\sqrt{\sigma(T)}$, and $m_0(T)/\sqrt{\sigma(T)}$.

β	T/T_c	$\sigma(T)$	$(m_s(T)T)/(\sigma(T))$	$m_s(T)/\sqrt{\sigma(T)}$	$m_0(T)/\sqrt{\sigma(T)}$
0.743543	0.8	0.009 61	2.04(3)	1.40(2)	2.8(1)
0.743543	0.7	0.013 15	1.9(1)	1.74(9)	3.0(3)
0.743543	0.62	0.015 42	1.7(2)	1.9(2)	3.22(8)
0.751805	0.89	0.002 68	2.1(2)	0.99(8)	3.0(3)
0.751805	0.8	0.004 44	1.94(8)	1.29(5)	3.0(2)
0.751805	0.73	0.005 66	1.76(5)	1.46(4)	3.1(4)
0.751805	0.67	0.006 54			3.3(1)
0.751805	0.62	0.007 20			3.23(3)
0.751805	0.57	0.007 71			3.29(5)
0.751805	0.4	0.009 22			3.25(4)
0.751805	0.14	0.010 37			3.14(3)
0.751805	0.125	0.010 40			3.21(2)
0.756427	0.6	0.003 26			3.29(6)

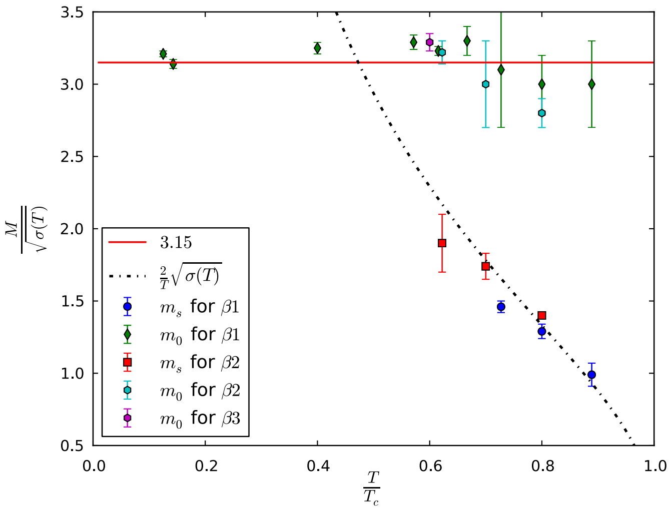

FIG. 3 (color online). $m_0/\sqrt{\sigma(T)}$ and $m_s/\sqrt{\sigma(T)}$ plotted as a function of T/T_c for $\beta_1 = 0.743\,543$, $\beta_2 = 0.751\,805$ and $\beta_3 = 0.756\,427$. The two curves correspond to the two expected scaling behaviors: $m_0(T) \sim 3.15\sqrt{\sigma(T)}$ and $m_s(T) = 2\sigma(T)/T$.

LGTs. The results of our fits are reported in Table II and Fig. 3. The subleading mass m_0 in the $T > 0.6T_c$ region turned out to be the natural continuation of the glueball mass that we had identified for $T < 0.6T_c$. As it is easy to see, looking at Fig. 3, m_0 follows Eq. (1) up to the highest temperatures that we studied with a value $m_0 \sim 3.15$ in agreement with the $T = 0$ estimate.

It is important to understand the effective string interpretation of this new scale m_s. We have no rigorous proof, but it is likely that the crossover that we observe between m_0 and m_s is due to a competition between the two minimal surfaces bounded by the four Polyakov loops which are compatible with the topology of the lattice and of our observable. As the temperature increases, it becomes less and less costly for the flux tube to wind around the periodic boundary conditions leading to a minimal surface composed by two parallel flux tubes as depicted in Fig. 4. This crossover is controlled by the competition of two scales: the compactification radius $1/T$ and the glueball radius r_0. As T increases, r_0 also increases (since it is due to the flux tube width which is known to increases with T); thus, a crossover value of T it will certainly exist, above which $r_0 > 1/T$, which in the 3D gauge Ising model that we studied, turns out to be around $T_c/2$.

Concluding remarks.—The main message of our analysis is that the mass of the lowest glueball (and thus, likely the whole glueball spectrum) scales at finite temperature as $\sqrt{\sigma(T)}$ and, thus, is a decreasing function of T/T_c and vanishes at $T = T_c$. Our results also suggest that the Isgur-Paton model is also valid at finite temperature and that its predicted scaling behavior $[m_0 \sim \sqrt{\sigma(T)}]$ can be conciliated with the different scaling behavior predicted by the Svetitsky-Yaffe conjecture [14] thanks to the appearance of a new mass scale ($m_s \sim \sigma(T)/T$), which, in the vicinity of T_c, dominates the large R behavior of the correlator. This is likely to be a general mechanism. For instance, a similar phenomenon was also observed a few years ago in the finite T behavior of the monopole spectrum of the random percolation gauge theory (see Fig. 2 of Ref. [18]). It is also interesting to notice that this new mass scale strongly resembles the "spurious states" observed in [2,3] which, in fact, were characterized by a large overlap with the torelon pair states.

It would be interesting to understand the physical meaning of m_s. Preliminary simulations show that, at high enough temperatures, the picture we have discussed holds almost unchanged even if we increase the distance between the two nearby Polyakov loops up to a few lattice spacings. In this limit, our observable describes the interaction of two mesons and, according to the effective string picture discussed above, the mass scale m_s should measure the attractive interaction between quarks and antiquarks belonging to different mesons. Our results show that this interaction becomes the dominant contribution in the meson-meson correlator as T_c is approached from below. This agrees with the intuitive picture of deconfinement as a "melting" of mesons into individual quarks. In our framework, this melting transition would be driven by the interaction mediated by the mass scale m_s. It would be really interesting to extend our study to a non-Abelian gauge theory. We expect these correlators to be very noisy; however, using an exponential error reduction algorithm and a reasonable computational power, we believe it is now possible to extract the finite temperature glueball mass also in these theories.

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FIG. 4 (color online). Minimal surface associated with the m_s mass.

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