Dynamics of Thermalization and Decoherence of a Nanoscale System

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We study the decoherence and thermalization dynamics of a nanoscale system coupled nonperturbatively to a fully quantum-mechanical bath. The system is prepared out of equilibrium in a pure state of the complete system. We propose a random matrix model and show analytically that there are two robust temporal regimes in the approach of the system to equilibrium-an initial Gaussian decay followed by an exponential tail, consistent with numerical results on small interacting lattices [S. Genway, A. F. Ho, and D. K. K. Lee, Phys. Rev. Lett. **105**, 260402 (2010)]. Furthermore, the system decays towards a Gibbs ensemble in accordance with the eigenstate thermalization hypothesis.

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The origin of thermodynamics from a fully quantum-mechanical description has been the subject of much recent research [1–3]. Emergence of thermal behavior from the unitary evolution of a wave function on a generic closed system can be studied using concepts such as the eigenstate thermalization hypothesis [4] (ETH) and canonical typicality [5,6]. Local or few-body observables in a closed nonintegrable system are expected to "thermalize" at long times [7] in the sense that they converge to a thermal Gibbs distribution. This has been studied with various approaches [8–12] and for myriad systems [13–20]. Recent interest has turned to understanding the *dynamics* of the relaxation to the thermal state [20–26]. In the canonical model, one considers a composite of system and bath [6] and asks how the system relaxes and decoheres [27,28] to reach a thermal state at long times.

In previous work [20], we found numerically that a random matrix model provided a generic description of thermalization dynamics for (nonrandom) nanoscale Hubbard clusters. In this Letter, we provide an analytical framework for this random matrix model. We derive [Eqs. (2)–(4)] the relaxation dynamics of a generic quantum system over the whole temporal range from short to long times. We also confirm that the model produces a thermal state at long times in accordance with ETH. Random matrix methods have been employed to study nanoscale systems coupled to different environments [29–32]. However, they do not capture the full range of temporal behavior: there is no general account of the Gaussian decay towards thermalization that has been established [14,20] numerically as a generic feature for the relaxation of local observables in interacting systems.

We focus on a nanoscale system (S) with a discrete energy spectrum embedded in a nonintegrable interacting bath (B) with a quasicontinuous spectrum so that the average bath level spacing Δ_B is much smaller than the system level spacings. We will examine how the small system thermalizes with the bath via the unitary evolution of the quantum-coherent *composite* system using a banded coupling model. Previous authors studied a banded coupling [29,31] but were unable to access the regime where we see Gaussian decay (see below).

The model.—Suppose the system has N_s eigenstates $|s\rangle$ with energies ε_s and the bath has eigenstates $|b\rangle$ of energies ϵ_b. The Hamiltonian for the composite system is given by $H = H_0 + V$

$$H = \sum_{sb} E_{sb}|sb\rangle\langle sb| + \sum_{ss'bb'} |sb\rangle\langle sb|V|s'b'\rangle\langle s'b'|, \quad (1)$$

where $|sb\rangle \equiv |s\rangle \otimes |b\rangle$ are product states with energies $E_{sb} = \varepsilon_s + \epsilon_b$ for the decoupled system and bath, and V couples the system and the bath. The coupled system will have an average level spacing of $\Delta = \Delta_B/N_s$. Analogous to the classic random matrix theory of nuclear matter, we model the interacting bath with an energy spectrum that obeys Wigner-Dyson statistics. Note that the randomness does not arise from quenched disorder. We assume that the bath states $|b\rangle$ are random vectors with no special spatial structure, e.g., no spatial localization. This should be valid for generic interacting quantum systems at energies away from strongly correlated states near the bath ground state. The matrix elements of the coupling V in a basis involving these bath states should therefore also be random. We use a banded random matrix of bandwidth W and strength c. More precisely, the matrix element $\langle sb|V|s'b'\rangle$ is nonzero only if $|E_{sb} - E_{s'b'}| < W$, and each nonzero element is a Gaussian random variable with zero average and a mean-square value $\overline{|\langle sb|V|s'b'\rangle|^2} = c\Delta$. As we see below, this scaling with the level spacing Δ is consistent with a *local* coupling between system and bath.

We can motivate this banded coupling model in the context of ultracold atoms in optical lattices. A small cluster of sites (system) is initially isolated from the rest of the lattice (bath) by a high tunneling barrier. The

coupling is introduced by lowering this barrier to allow particles to hop between the cluster and the lattice. This particle exchange only couples bath states with an energy difference of the order of the single-particle bandwith. This produces a dense banded matrix with bandwidth W (see Fig. 20 of Ref. [7] on the Hubbard model description of this setup). This is the motivation of our banded coupling V. Our scaling of the coupling with the level spacing Δ is also motivated by the *local* quench in this lattice example: $\mathrm{Tr}V^2 \propto dNJ_h^2$ where N is the number of states in the composite system and there are d links with hopping integral J_h. Since there are $2NW/\Delta$ nonzero matrix elements, this corresponds [7] to $c \sim dJ_h^2/W$. Unlike in conventional statistical mechanics, we do *not* assume a weak system-bath coupling so that we can study local observables in a homogeneous optical lattice. Such local measurements are becoming experimentally accessible [33]. Effects of time-reversal symmetry can be studied by trap rotation or artificial gauge fields [34].

Central result.—At time $t = 0$, we prepare the total system in a separable initial state $|\Psi(0)\rangle = |S\rangle \otimes |B\rangle$, for a general system state $|S\rangle$. The bath state $|B\rangle$ is restricted only by the requirement that it should have a small energy uncertainty. This means $|\Psi(0)\rangle$ has significant overlap only with eigenstates of H centered around a total energy $E_0 = \langle\Psi(0)|H|\Psi(0)\rangle$. The system evolves as $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle = \sum_A e^{-iE_At}|A\rangle\langle A|\Psi(0)\rangle$ where $|A\rangle$ are the exact eigenstates of the composite system with energies E_A ($\hbar = 1$). We study the reduced density matrix (RDM) obtained by tracing out the bath: $\rho_{ss'}(t) \equiv \sum_b \langle sb|\Psi(t)\rangle \times \langle\Psi(t)|s'b\rangle$. Our main result is the full temporal evolution of the RDM in the limit of a large bath ($\Delta \ll c, W$), for times $t \ll 1/\Delta$

$$\rho_{ss}(t) \simeq \rho_{ss}(\infty) + [\rho_{ss}(0) - \rho_{ss}(\infty)]e^{-2\Lambda(0,t)}, \quad (2)$$

$$\rho_{ss'}(t) \simeq \rho_{ss'}(0)e^{-i(\varepsilon_s - \varepsilon_{s'})t}e^{-2\Lambda(0,t)} \quad (s' \neq s), \quad (3)$$

$$\Lambda(t', t) = \int_{-\infty}^{\infty} \frac{c(E)R(E)}{E^2}(e^{iEt'} - e^{iE(t-t')})dE. \quad (4)$$

Here, $c(E)$ is the profile for the banded coupling matrix: $c(E) = c$ for $|E| < W$ and zero otherwise. The symmetries of the random matrix model enter via the level repulsion [35], expressed by $R(E) \propto |E|$ or E^2 for systems with or without time-reversal symmetry, respectively, for $|E| \lesssim \Delta$, and tending to unity for $|E| \gg \Delta$. Note that the thermalization dynamics discussed below is insensitive to time-reversal symmetry because thermalization occurs over time scales shorter than the time scale $1/\Delta$ over which the system is sensitive to level repulsion.

In this limit of a large bath, we find that the diagonal elements of the RDM decay to reach a steady-state value expected from the Gibbs distribution $\rho_{ss}(\infty) = \nu_b(E_0 - \varepsilon_s)\Delta$ where ν_b is the bath density of states [36]. Moreover, decoherence has the same dynamics as thermalization, with the off-diagonal elements $\rho_{ss'}(t)$ tending to zero at long times on the same time scales [37].

Most importantly, we establish that the relaxation towards the thermal state has two temporal regimes [as seen in our numerics [20]]. The RDM is controlled by $\Lambda(0, t) \simeq t^2 \int_{-\infty}^{\infty} c(E)dE = cWt^2$ for $t \ll W^{-1}$ and $c(E \to 0)\pi t$ for $W^{-1} \ll t \ll \Delta^{-1}$. So, the RDM has a Gaussian decay with a decay rate of $2\sqrt{cW}$ for $t < W^{-1}$ but has an exponential tail at longer times with decay rate $2\pi c$. For weak coupling ($c \ll W$), the decay is predominantly exponential, as expected from Fermi's Golden Rule and perturbative Lindblad theory. For stronger coupling [38] ($c \gg W$), the Gaussian form dominates with thermalization completed by the crossover time W^{-1}. We stress that the existence of the Gaussian and exponential regimes is robust as our results apply to a general $c(E)$, and the rates are controlled by only two quantities $\int_{-\infty}^{\infty} c(E)dE \propto \mathrm{Tr}V^2$ and $c(E \to 0)$.

Brownian model.—We use the Dyson Brownian technique [39,40], which enables us to calculate the ensemble-averaged effects of the random coupling V by building it up as a sum of uncorrelated random perturbations

$$V \to V_{\mathrm{Br}}(\tau) = \int_0^{\tau} \xi(\tau')d\tau' \quad \text{with} \quad \tau = 1. \quad (5)$$

It can be pictured as a random walk in fictitious time τ in the space of random Hamiltonians. At $\tau = 0$, the system and bath are decoupled. Dyson observed that the $\tau = 1$ case corresponds, after ensemble averaging, to the model defined in Eq. (1) with $H = H_0 + V$. More precisely, at each fictitious time step $\delta\tau$, a small perturbation $\xi(\tau)\delta\tau$ is added to the Hamiltonian $H(\tau) = H_0 + V_{\mathrm{Br}}(\tau)$ which has exact eigenstates $|A(\tau)\rangle$. This perturbation can be written in the basis of these eigenstates as $\langle A(\tau)|\xi\delta\tau|B(\tau)\rangle = \sqrt{c_{AB}}\xi_{AB}$. The banded coupling profile, defined after Eq. (4), is mimicked by $c_{AB} \equiv c(E)\Delta$, with $E = E_A - E_B$. (See the Discussion for the validity of this approach.) Restricting ourselves to time-reversal-invariant systems, we model the randomness by the independent Gaussian random variables $\xi_{AB}(= \xi_{BA})$ with the stochastic properties $\overline{\xi_{AB}} = 0$, and $\overline{\xi_{AB}\xi_{CD}} = (\delta_{AC}\delta_{BD} + \delta_{AD}\delta_{BC})\delta\tau$. It can be shown [41] from perturbation theory that we have Langevin processes for the eigenstates and eigenenergies

$$\delta X_A^{sb} = \sum_{B \neq A}\left(\frac{\sqrt{c_{AB}}\xi_{AB}}{E_{AB}}X_B^{sb} - \frac{c_{AB}\delta\tau}{2E_{AB}^2}X_A^{sb}\right), \quad (6)$$

$$\delta E_A = \sqrt{c_{AA}}\xi_{AA} + \sum_{B \neq A}\frac{c_{AB}\delta\tau}{E_{AB}}, \quad (7)$$

where $E_{AB} \equiv E_A - E_B$, and the overlap $X_A^{sb}(\tau) \equiv \langle sb|A(\tau)\rangle$ is a component of the eigenstate in the decoupled basis. The initial ($\tau = 0$) condition is $X_A^{sb}(0) = \langle sb|A(0)\rangle$. ($|A(0)\rangle$ is a product state of system and bath eigenstates.) The perturbations for the overlaps and the energy levels

involve independent (off-diagonal and diagonal) elements of ξ_{AB}. So we can replace the sum over energies in Eq. (6) with statistical averages over the well-known energy level distribution. Fluctuations should be small owing to the rigidity of the spectrum. The second moment of the overlap $\overline{|X_A^{sb}|^2}$ is the "local density of states" (LDOS) in energy space. Its Brownian motion has been studied [41] for an unbanded coupling matrix. We have extended the theory to obtain the fourth moments of the overlap that are needed for the RDM.

Derivation.—We will now describe our analytical calculation in more detail. We focus first on the diagonal elements of the RDM and consider, for brevity, the case of an initial product state $|\Psi(0)\rangle = |s_0 b_0\rangle$. (It is straightforward to generalize to other initial product states.) It is useful to express the RDM in terms of the overlaps between the exact eigenstates and the decoupled product states $X_A^{sb} \equiv \langle sb|A\rangle$, which is of random sign over the ensemble of random couplings. It can be shown that

$$\rho_{ss}(t) = \sum_{ABb} \langle A|s_0 b_0\rangle\langle s_0 b_0|B\rangle\langle B|sb\rangle\langle sb|A\rangle e^{-iE_{AB}t}. \quad (8)$$

This involves the fourth moments of the overlaps. Let us start with the second moments $J_A^{\alpha\beta}(\tau) \equiv \overline{X_A^\alpha X_A^\beta}$ ($\alpha \equiv (ra)$, $\beta \equiv (sb)$). Using Eq. (6), we can write down the Langevin equation for $\delta(X_A^\alpha X_A^\beta) = X_A^\alpha \delta X_A^\beta + X_A^\beta \delta X_A^\alpha + \delta X_A^\alpha \delta X_A^\beta$. Averaging over the noise ξ gives

$$\partial_\tau J_A^{\alpha\beta} = \sum_{B\neq A} \frac{c_{AB}}{E_{AB}^2}(J_B^{\alpha\beta} - J_A^{\alpha\beta}) \quad (9)$$

with $J_A^{ra,sb}(0) = \langle sb|A(0)\rangle\delta_{ra,sb}$. The sum is in the form of a convolution, and so this differential equation can be simplified in the time domain in terms of $J^{\alpha\beta}(t,\tau) = \int J_A^{\alpha\beta}(\tau)e^{-iE_A t}dE_A/\Delta$. It simply becomes $\partial_\tau J^{\alpha\beta}(t,\tau) = \Lambda(0,t)J^{\alpha\beta}(t,\tau)$, and the solution is

$$J^{\alpha\beta}(t,\tau) = \delta_{\alpha\beta}e^{-iE_\beta t}e^{-\tau\Lambda(0,t)}. \quad (10)$$

This is nonzero only if $\alpha = \beta$ because, upon averaging over the random couplings $V_{\mathrm{Br}}(\tau)$, there should be no correlations between different components of $|A\rangle$ in the decoupled basis. (A test is to consider the terms that survive under an average over random gauge transformations of the set of the basis states $|sb\rangle$.)

We note that $J^{\beta\beta}(t,\tau=1)$ is the Fourier transform of the local density of states $\overline{|X_A^{sb}|^2}$. From the behavior of $\Lambda(0,t)$ at short and long times as discussed after Eq. (4), we see [42] that the LDOS is a function of $\omega = E_A - E_{sb}$, which is a Lorentzian of width πc for $\omega \ll W$, and is cut off at $\omega \gg W$ by a Gaussian of width $\sqrt{2cW}$. The LDOS can also be obtained in large-N diagrammatics for the random coupling where N corresponds to the number of bath states. For an unbanded matrix ($W \to \infty$), the leading result corresponds to a self-consistent Born approximation, giving the Lorentzian broadening to the LDOS [31]. However, the Gaussian tail for a banded matrix is more difficult to capture in such an approximation.

The result (10) demonstrates analytically ETH [4], which gives a sum rule for the LDOS, i.e., the projection of an eigenstate A onto a system state s: $\sum_b \overline{|\langle A|sb\rangle|^2} = \sum_b J_A^{sb,sb} \propto \nu_b(E_A - \varepsilon_s)$. Using Eq. (10), we see that

$$\sum_b \overline{|\langle A|sb\rangle|^2} = \Delta\int_{-\infty}^{\infty}\frac{dt}{2\pi}e^{-\tau\Lambda(0,t)}\sum_b e^{i(E_A-E_{sb})t}. \quad (11)$$

This is a sum of the Fourier transform $r_1(\omega)$ of $e^{-\tau\Lambda(0,t)}$ at frequencies $\omega = E_A - \varepsilon_s - \epsilon_b$ over all ϵ_b. Recall that $e^{-\tau\Lambda(0,t)}$ is mainly Gaussian decay with a rate of $\sqrt{c\tau W}$ for $c\tau \gg W$ and mainly exponential with rate $\pi c\tau$ for $c\tau \ll W$. Therefore, $r_1(\omega)$ should be a function centered at $\omega = 0$ with width $\sim \min[c\tau, \sqrt{c\tau W}]$. Assuming that the bath density of states ν_b varies slowly over this width, we find agreement with ETH,

$$\sum_b \overline{|\langle A|sb\rangle|^2} \simeq \nu_b(E_A - \varepsilon_s)\Delta\int r_1(E_A - \varepsilon_s - \epsilon)d\epsilon$$
$$= \nu_b(E_A - \varepsilon_s)\Delta e^{-\tau\Lambda(0,0)} = \nu_b(E_A - \varepsilon_s)\Delta. \quad (12)$$

Let us now turn to the fourth moments needed for the evaluation of the RDM, $M_{AB}^{s\alpha}(\tau) \equiv \sum_b \overline{X_A^\alpha X_A^\beta X_B^\beta X_B^\alpha}$ and $N_{AB}^{s\alpha}(\tau) \equiv \sum_b[\overline{X_A^\alpha X_A^\alpha X_B^\beta X_B^\beta} + (A \leftrightarrow B)]/2$, and their associated time-domain functions $M^{s\alpha}(t,\tau) \equiv \iint M_{AB}^{s\alpha}e^{-iE_{AB}t}dE_A dE_B/\Delta^2$ and similarly for $N^{s\alpha}$. It can be shown [Supplemental Material [43]] from the Langevin equation (6) that

$$\hat{D}_\tau M^{s\alpha}(t,\tau) = 2\Delta\int\frac{dt'}{2\pi}\Lambda(t',t)[M^{s\alpha}(t',\tau) + N^{s\alpha}(t',\tau)], \quad (13)$$

$$\hat{D}_\tau N^{s\alpha}(t,\tau) = 4\Delta\int\frac{dt'}{2\pi}\Lambda(t',t)M^{s\alpha}(t',\tau), \quad (14)$$

where $\hat{D}_\tau \equiv \partial_\tau + 2\Lambda(0,t)$ and the initial conditions are $M^{s\alpha}(t,0) = \delta_{rs}$ and $N^{s\alpha}(t,0) = \sum_b \cos[(E_\alpha - E_{sb})t]$. The latter sums over *all* bath states. It is strongly peaked at $t = 0$ with a width of the inverse bath bandwidth and is approximately $(2\pi/N_s\Delta)\delta(t)$.

We will now proceed to a solution of these equations of motion for a large bath ($\Delta \to 0$). Consider the t' integrations over $M^{s\alpha}$ in the above equations. We can divide up $M^{s\alpha}$ into its transient part and its steady-state value $M_\infty^{s\alpha}(\tau)$ at long times. Anticipating that the transient part decays exponentially at long times and does not scale with $1/\Delta$ [see Eq. (2)], we expect that its contribution to the integral should vanish with Δ. The contribution of the steady-state part $M_\infty^{s\alpha}(\tau)$ is proportional to

$$M_\infty^{s\alpha}(\tau)\int_{-\infty}^{\infty}\Lambda(t',t)dt' \propto \int_{-\infty}^{\infty}dEf(E,t)R(E)\delta(E), \quad (15)$$

where $f(E,t) = c(E)(1 - e^{iEt})/E^2$. This integral vanishes since $R(E) \sim |E|$ as $E \to 0$. Hence, we find that N is not coupled to M in this limit of $\Delta \to 0$ and that the solution to Eq. (14) is simply $N^{s\alpha}(t,\tau) = e^{-2\tau\Lambda(0,t)} N^{s\alpha}(t,0)$. Thus, the right side of Eq. (13) becomes

$$\frac{\Delta}{\pi}\int dt' dE \sum_b f(E,t) e^{-2\tau\Lambda(0,t') - iEt'} \cos[(E_{s\alpha} - \epsilon_b)t'] = \frac{\Delta}{2\pi}\int d\epsilon dE f(E,t)\nu_b(\epsilon) \sum_{\eta=\pm 1} r_2(\epsilon - E_{s\alpha} + \eta E), \tag{16}$$

where $E_{s\alpha} = E_\alpha - \varepsilon_s$ and $r_2(\omega)$ is the Fourier transform of $e^{-2\tau\Lambda(0,t)}$, which is peaked at zero with width $\sim \min[c\tau, \sqrt{c\tau W}]$. For a smooth ν_b, $\nu_b(\epsilon) \simeq \nu_b(\epsilon = E_{s\alpha})$ for the ϵ range over which r_2 contributes to the ϵ integration. Then, the right-hand side of Eq. (16) becomes $2\Lambda(0,t)\nu_b(E_{s\alpha})\Delta$. The equation of motion simplifies to

$$[\partial_\tau + 2\Lambda(0,t)]M^{s\alpha}(t,\tau) = 2\Lambda(0,t)\nu_b(E_{s\alpha})\Delta. \tag{17}$$

From Eq. (8), $\rho_{ss} = M^{s,\alpha=s_0b_0}$ for an initial state $|s_0b_0\rangle$ of energy $E_0 = E_{s\alpha} + \varepsilon_s$. The solution at $\tau = 1$ for Eq. (17) is indeed our result of Eq. (2) with $\rho_{ss}(0) = \delta_{ss_0}$. We can perform an analogous calculation for $\rho_{s\neq s'}(t)$. The dominant contributions [Supplemental Material [44]] come from terms that are positive definite in the sum over bath states

$$\rho_{ss'}(t) \simeq \rho_{ss'}(0)\sum_{AB} \overline{|\langle A|sb_0\rangle|^2 |\langle B|s'b_0\rangle|^2}\, e^{-iE_{AB}t} \simeq \rho_{ss'}(0) J^{sb_0,sb_0}(t) J^{s'b_0,s'b_0}(-t). \tag{18}$$

With Eq. (10), this gives our result of Eq. (3) for decoherence.

Discussion.—Brownian motion produces a random matrix $V_{\rm Br}(\tau = 1)$ that has identical statistical properties to V only for an unbanded random matrix. For a banded coupling, this is only approximate. This is because the coupling matrix is banded in the eigenstate basis of $H(\tau)$ at each Brownian step, instead of being banded in the eigenbasis of $H(\tau = 0)$. For finite W, we can show [45] that $V_{\rm Br}(\tau)$ has a broadened profile $c'(E)$ for its matrix elements with increasing τ. As discussed after Eq. (4), the features of $c'(E)$ relevant to the physics here are the integrated profile $\int c'(E)dE$ and small-E limit of $c'(E)$. The former gives $\mathrm{Tr}V_{\rm Br}^2$, which has been fixed at $\mathrm{Tr}V^2$ [implying that the Brownian model reproduces the short-time expansion correctly, $e^{-iHt} \simeq 1 - iHt$, giving $\rho_{s_0s_0} \simeq 1 - 2(cWt)^2$]. So, the broadening of $c'(E)$ compared to $c(E)$ means that $c'(0) < c(0)$. Thus, we overestimate the exponential decay rate, but this is only significant when $c \gg W$ so that Gaussian decay dominates and the exponential tail is negligible. As we show in Fig. 1, our analytical results for $H = H_0 + V_{\rm Br}(\tau = 1)$ agree with the dynamics for $H = H_0 + V$.

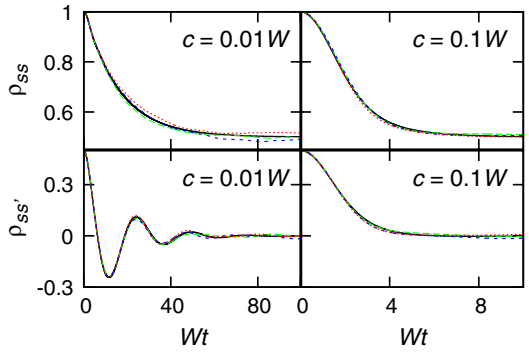

FIG. 1 (color online). Comparison of Brownian motion result (solid line) with exact diagonalization (dotted line) of 3 random realizations with 2 system states and 7000 bath states. Top: diagonal RDM elements ρ_{ss} for initial state $|sa\rangle$ with state a near the center of the bath spectrum (high effective temperature). Bottom: off-diagonal RDM elements $\mathrm{Re}(\rho_{ss'})$ for initial state $(|sa\rangle + |s'a\rangle)/\sqrt{2}$ ($\varepsilon_s - \varepsilon_{s'} = W/4$, spacing $\Delta = W/4000$). Weak coupling (left) shows predominantly exponential decay, whereas stronger coupling (right) shows the early Gaussian regime.

To summarize, we have used a random matrix model to describe the nonequilibrium dynamics of a system coupled to a fully quantum-mechanical bath. In contrast with studies employing an effective scattering approach [46] with a non-Hermitian random Hamiltonian, we study the full Hilbert space of a system with an interacting quantum bath. This provides an analytical demonstration of the eigenstate thermalization hypothesis. (Many previous works provided only numerical support.) We also find that thermalization and decoherence both follow the same dynamical behavior, with Gaussian decay at short times and exponential decay at long times. We should point out that these two regimes have been qualitatively anticipated in works based on semiclassical dynamics of energy wave packets [47]. Also, a short-time Gaussian regime was found [48] for a global quench that switches on a random two-body interaction among all particles [49]. That Gaussian decay originates from the interactions generating a Gaussian density of states for the total energy spectrum. In contrast, our *local* quench does not alter drastically the spectrum of the total system, and so we argue that the Gaussian regime in our problem has a completely different physical origin. More recently, Gaussian decay has been found for a small system coupled to a classical bath in a slow local quench [50], with a decay time controlled by the correlation time in the bath. The quench rate can be mimicked in our formalism by the width W. Our model has a short correlation time in the bath. Incorporating bath correlations is the goal of future work.

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[44] See Section II of the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.111.130408 for the derivation of the equations of motion satisfied by the fourth moments of the wavefunction overlaps needed for the decoherence dynamics of the off-diagonal elements of the reduced density matrix.
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