Local Conservation Laws and the Structure of the Many-Body Localized States

Maksym Serbyn,[1] Z. Papić,[2] and Dmitry A. Abanin[3,4]

[1]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02138, USA*
[2]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*
[3]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
[4]*Institute for Quantum Computing, Waterloo, Ontario N2L 3G1, Canada*

We construct a complete set of local integrals of motion that characterize the many-body localized (MBL) phase. Our approach relies on the assumption that local perturbations act locally on the eigenstates in the MBL phase, which is supported by numerical simulations of the random-field *XXZ* spin chain. We describe the structure of the eigenstates in the MBL phase and discuss the implications of local conservation laws for its nonequilibrium quantum dynamics. We argue that the many-body localization can be used to protect coherence in the system by suppressing relaxation between eigenstates with different local integrals of motion.

Introduction.—Localization of eigenstates of a single particle in the presence of disorder is among the most remarkable consequences of quantum mechanics. Although the single-particle localization and localization-delocalization transition are well understood [1,2], much less is known about the nature of the eigenstates in interacting many-body disordered systems. The interest in the problem of the many-body localization was rekindled when recent works [3,4] suggested that the localized phase is stable with respect to weak interactions. This conjecture was also corroborated by numerical studies [5–16].

In the noninteracting localized phase, dynamics is simple because any initial wave function can be decomposed into a superposition of localized single-particle eigenstates. However, when interactions are introduced, the dynamics becomes notably richer [7,16–18]. Although particle transport is still expected to be blocked, the time evolution of initial product states in the interacting localized phase generates a universal slow growth of entanglement entropy [17]. Saturated entropy was established to be proportional to system size [7,16–18], and such growth of the entanglement was argued to reflect "partial thermalization" of the system. However, the type of the ensemble describing the many-body localized (MBL) phase is unknown.

On the experimental side, probing the dynamics of interacting disordered systems has become feasible due to the advances in the field of ultracold atomic gases [19,20]. In particular, nearly isolated quantum systems of cold atoms can now be engineered, prepared in a variety of initial states (including product states [21]), and studied during their subsequent time evolution. These opportunities call for developing a better understanding of the laws that govern the dynamics in the MBL phase.

Here we consider a many-body system whose eigenstates at all energies are localized, and show that they can be characterized by a large number of emergent local integrals of motion corresponding to multiple local conservation laws. These integrals of motion form a complete set, in the sense that their values completely determine the eigenstates. Local conservation laws strongly constrain the quantum dynamics in the MBL phase, preventing a complete thermalization of any given subsystem. Any initial state can be decomposed in terms of the eigenstates possessing definite values of the integrals of motion. During time evolution, the weights of different states cannot change. However, because of the exponentially weak interaction between distant degrees of freedom, the relative phases between the states with different values of local integrals of motion become randomized. Any local observable at long times is therefore determined only by the set of probabilities of local integrals of motion that affect the degrees of freedom in the region where the observable is measured. We refer to this as the local diagonal ensemble. The dephasing due to the interactions between distant subsystems is a distinct feature of the MBL phase compared to the noninteracting one, and underlies the slow growth of entanglement [16,17,22,23].

Integrals of motion.—First, we note that for the noninteracting case the local integrals of motion are simply given by $\hat{I}_i = c_i^\dagger c_i$, where $c_i^\dagger$ creates a localized single-particle state. For fermions, the possible eigenvalues of this integral of motion are $I_i \in \{0, 1\}$. In a system with K orbitals, there are 2^K eigenstates, which are uniquely labeled by the eigenvalues of K integrals of motion.

In order to explicitly construct local integrals of motion for an interacting system, we assume the following property of the localized phase: local perturbations lead only to local modifications of the eigenstates in the MBL phase. That is, if we act on a MBL eigenstate with a local perturbation, introduced either adiabatically or instantaneously, the degrees of freedom situated at a distance $L \gg \xi$

(here ξ is the localization length [24]) away from the support of the perturbation operator, are generally affected exponentially weakly. We will support this statement below by the numerical study of the random-field *XXZ* chain, also considered in Refs. [7,10,12,15,16].

Let us consider a MBL system described by a local Hamiltonian, and let us divide it into subsystems of size $l \gg \xi$. Without loss of generality, we consider a 1D system, although our conclusions apply to localized phases in any number of spatial dimensions. We number the subsystems by $i = 1, \ldots, N$ from left to right, assuming the number of degrees of freedom $\mathcal{M}$ in each subsystem is the same (e.g., for K spins, $\mathcal{M} = 2^K$). For the fixed subsystem i, we denote parts of the full system to the left and to the right of i by $\mathcal{L}_i$ and $\mathcal{R}_i$, respectively. The Hamiltonian can be written as

$$H = H_{\mathcal{L}} + H_i + H_{\mathcal{R}} + H_{\mathcal{L}i} + H_{\mathcal{R}i}, \qquad (1)$$

where $H_{\mathcal{L}}, H_{\mathcal{R}}, H_i$ act only on the degrees of freedom in $\mathcal{L}, \mathcal{R}, i$, while $H_{\mathcal{L}i}, H_{\mathcal{R}i}$ couple $\mathcal{L}, i$ and $\mathcal{R}, i$.

If the subsystems $\mathcal{L}, i, \mathcal{R}$ are disconnected from each other (i.e., $H_{\mathcal{L}i}, H_{\mathcal{R}i}$ are set to zero), the eigenstates are simple products: $|\alpha\beta\gamma\rangle_0 = |\alpha\rangle_{\mathcal{L}} \otimes |\beta\rangle_i \otimes |\gamma\rangle_{\mathcal{R}}$, where $\alpha \in \{1, \ldots, \mathcal{M}^{i-1}\}$, $\beta = \{1, \ldots, \mathcal{M}\}$, $\gamma = \{1, \ldots, \mathcal{M}^{N-i}\}$. Once the subsystems are connected, the eigenstates of the full Hamiltonian (1) are obtained from the product states $|\alpha\beta\gamma\rangle_0$ by nearly local rotations. We label the resulting eigenstates by their "ancestors," omitting the "0" subscript,

$$|\alpha\beta\gamma\rangle = \hat{O}_{\mathcal{L}i}\hat{O}_{\mathcal{R}i}|\alpha\rangle_{\mathcal{L}} \otimes |\beta\rangle_i \otimes |\gamma\rangle_{\mathcal{R}}. \qquad (2)$$

Operator $\hat{O}_{\mathcal{L}i}$ is a unitary many-body rotation which strongly transforms only the degrees of freedom within a distance $\sim \xi$ away from the boundary between $\mathcal{L}$ and i (similarly for $\hat{O}_{\mathcal{R}i}$). The commutator of $\hat{O}_{\mathcal{L}i}$ and $\hat{O}_{\mathcal{R}i}$, as well as the action on the degrees of freedom far away, decays exponentially. We note that the assignment (2), which links the eigenstates of the system to the eigenstates of subsystems, is not unique, and assume that a certain one-to-one correspondence is chosen.

We now define the integral of motion for subsystem i

$$\hat{I}_i = \sum_{\beta=1}^{\mathcal{M}} \beta \sum_{\alpha=1}^{\mathcal{M}^{i-1}} \sum_{\gamma=1}^{\mathcal{M}^{N-i}} |\alpha\beta\gamma\rangle\langle\alpha\beta\gamma|. \qquad (3)$$

Being a linear combination of projectors onto the exact eigenstates, $\hat{I}_i$ necessarily commutes with the Hamiltonian and assumes eigenvalues $1, \ldots, \mathcal{M}$. Intuitively, states with the same eigenvalue of $\hat{I}_i$ look nearly identical within the subsystem i at distances larger than ξ away from the boundaries with subsystems $\mathcal{L}, \mathcal{R}$.

Sums of projectors onto the eigenstates are integrals of motion by construction; however, generally such operators are nonlocal and affect all degrees of freedom of the system. The operator in Eq. (3) is special in that it is local;

i.e., it weakly affects the degrees of freedom in $\mathcal{L}$ or $\mathcal{R}$ at a distance $x \gg \xi$ away from the boundaries with the *i*th subsystem. The locality of $\hat{I}_i$ follows directly from the locality of operators $\hat{O}_{\mathcal{L}i}, \hat{O}_{\mathcal{R}i}$, which implies that the sum of projectors becomes very close to the identity operator far away from the boundaries. Below, we will test the locality of the operator in Eq. (3) in a specific model.

Having defined the integral of motion for the subsystem i, we can similarly define $N - 1$ integrals of motion for the remaining $N - 1$ subsystems, such that in total we have N integrals $\hat{I}_i$, $i = 1, \ldots, N$. Different $\hat{I}_i$ commute with each other $[\hat{I}_i, \hat{I}_j] = 0$ since they are sums of projectors onto the exact eigenstates of the full system. Each $\hat{I}_i$ has $\mathcal{M}$ possible eigenvalues; thus, the full description of the system via integrals of motion requires $\mathcal{M}^N$ parameters, which coincides with the dimensionality of the Hilbert space. An operator $\hat{I}_i$ can also be viewed as the z component $\hat{I}_i = \hat{S}_{iz}$ of a "spin" $S = (\mathcal{M} - 1)/2$. Raising and lowering operators can then be used to construct the entire set of eigenstates, starting from any given eigenstate $|I_1 I_2 \ldots I_N\rangle$ characterized by the integrals of motion $I_1, I_2, \ldots, I_N$. Therefore, specifying the eigenvalues of all integrals of motion defined above completely determines the eigenstates of the system.

Hamiltonian and its relation to integrals of motion.—The Hamiltonian takes an especially simple form when written in terms of the integrals of motion:

$$H = \sum_i^N \sum_{I_i=1}^{\mathcal{M}} E_{I_i} \hat{\mathcal{P}}_{I_i}^i + \sum_{i \neq j}^N \sum_{I_i, I_j = 1}^{\mathcal{M}} E_{I_i I_j} \hat{\mathcal{P}}_{I_i}^i \hat{\mathcal{P}}_{I_j}^j$$
$$+ \sum_{i<j<k}^N \sum_{I_i, I_j I_k = 1}^{\mathcal{M}} E_{I_i I_j I_k} \hat{\mathcal{P}}_{I_i}^i \hat{\mathcal{P}}_{I_j}^j \hat{\mathcal{P}}_{I_k}^k + \cdots, \qquad (4)$$

where $\hat{\mathcal{P}}_{I_i}^i$ is the projector onto the subspace for which the eigenvalue of the *i*th integral of motion is equal to I_i. In the above equation, E_{I_i} can be roughly viewed as the energy of the *i*th subsystem for the sector I_i, $E_{I_i I_j}$ is the interaction energy between i and j subsystems, etc. There are interactions between any given n subsystems; however, they are exponentially small. Generally, we expect that energies E_{I_i} are proportional to l, the size of the subsystems. $E_{I_i I_j}$ are proportional to ξ when $i = j \pm 1$, and are suppressed as $\xi e^{-l(|i-j|-1)/\xi}$ otherwise (the interactions between the neighboring subsystems are limited to the boundary and are therefore proportional to ξ). The above representation of the Hamiltonian gives us a way to describe the dynamics in the MBL phase for various kinds of initial states [7,10–12,16,17].

Dynamics.—As a first step, we consider the dynamics of an eigenstate which is perturbed locally. We assume a sudden action of the local unitary operator $\hat{U}$ on the eigenstate $|\Psi_0\rangle = |I_1 I_2 \ldots I_N\rangle$. Operator $\hat{U}$ acts only on the degrees of freedom in subsystem 1, and its support is

situated far from the boundary between subsystems 1 and 2. The initial wave function $|\Psi(t = 0)\rangle$ can be decomposed in terms of the eigenstates:

$$|\Psi(t = 0)\rangle = \hat{U}|\Psi_0\rangle = \sum_{I_1'} U_{I_1 I_1'} |I_1' I_2 \ldots I_N\rangle + \cdots. \quad (5)$$

This form of the decomposition is dictated by the fact that the values of the integrals of motion $I_2, \ldots, I_N$ can be changed only with an exponentially small probability; hence, the terms with other values of $I_2, I_3, \ldots$ in Eq. (5) are represented by ellipses. Neglecting these terms, the subsequent dynamics becomes trivial:

$$|\Psi(t)\rangle = \sum_{I_1'} U_{I_1 I_1'} e^{-iE_{I_1' I_2 \ldots I_N} t} |I_1' I_2 \ldots I_N\rangle, \quad (6)$$

where $E_{I_1' I_2 \ldots I_N}$ is the energy of the state $|I_1' I_2 \ldots I_N\rangle$. Generally, we expect a finite number of different I_1' which have significant matrix elements $U_{I_1 I_1'}$, typically comparable to the dimensionality of a subsystem of size $\sim \xi$. Therefore, the time evolution (6) describes coherent oscillations that involve a finite number of states. Any local observable in region 1 would therefore oscillate at a number of frequencies, showing revivals but no dephasing. This situation changes if the state $|\Psi_0\rangle$ is not an eigenstate, but a superposition of several eigenstates which involve different values of $I_2, I_3, \ldots, I_k$. In this case, exponentially slow dephasing arises, suppressing the revivals and oscillations of local observables in the long-time limit. The values of observables at long times are determined by the probabilities $|U_{I_1 I_1'}|^2$.

Second, we describe the global evolution of states which differ from the eigenstates everywhere, not just locally. For definiteness, consider an initial product state of subsystems $1, 2, \ldots, N$:

$$|\Psi\rangle = \otimes_{i=1}^{N} \left(\sum_{\alpha_i=1}^{\mathcal{M}} A_{\alpha_i} |\alpha_i\rangle \right), \quad (7)$$

where $|\alpha_i\rangle$ is an eigenstate of the Hamiltonian H_i. Modern experimental techniques allow for the preparation and manipulation of such states in optical lattices [21].

Each component $\otimes_{i=1}^{N} |\alpha_i\rangle$ of the product state (7) can be related to the eigenstate of the whole system, $|I_1 I_2 \ldots I_N\rangle$, by the set of local rotations acting near the boundaries between different subsystems. The dynamics corresponding to this effect will be limited to the boundaries between pairs of subsystems. However, for each wave function, degrees of freedom at a distance $x \gg \xi$ away from the boundary will remain undisturbed. Such dynamics therefore does not generate long-range entanglement.

More importantly, since we are dealing with a superposition of different product states $\otimes_{i=1}^{N} |\alpha_i\rangle$, the degrees of freedom in the subsystem i will be in a superposition of states with different values of the integral of motion I_i. Different states entering this superposition are eigenstates; therefore, their relative weights cannot change under time evolution. However, their phases will become random due to the interactions with distant subsystems, as is evident from the Hamiltonian (4). Such dephasing, though exponentially slow, will produce long-range entanglement, and thus give rise to the entanglement entropy that is extensive in the system size and determined by the participation ratios of different eigenstates [25], as discussed in detail in Ref. [16].

Numerical simulations.—Although our construction is general, we now test the validity of our basic assumption using exact diagonalization of a particular model—the random-field XXZ spin chain. We consider a chain of L spins with open boundary conditions, exchange $J_\perp = 1$, and interaction strength $J_z = V$, while the random onsite magnetic field is uniformly distributed in the interval $\pm W$. The total z component of the spin is conserved, and calculations are restricted to the $S_z = 0$ sector. For $V = 0$ the model is equivalent to free fermions with disorder, and all states are localized. Because of the limits on the accessible system sizes in exact diagonalization, we restrict ourselves to the case of the symmetric bipartite division of the full system $\mathcal{LR}$ into the left ($\mathcal{L}$) and right ($\mathcal{R}$) half.

First, we study the averaged entanglement entropy S_{ent} of the $\mathcal{L}$ subsystem in the eigenstates of $\mathcal{LR}$, illustrated in Fig. 1(a). For strong disorder, S_{ent} saturates to a value of the order 1 with increasing system size, indicating short-range entanglement in the MBL eigenstates, which is consistent with our basic assumption.

Next, we use the inverse participation ratio (IPR) as an intuitive, albeit somewhat indirect, test of the locality of operators $\hat{O}_{\mathcal{L}i}$ from Eq. (2), which, when acting on products of eigenstates of systems $\mathcal{L}$, $\mathcal{R}$, give eigenstates of $\mathcal{LR}$. The IPR for some state $|\Psi\rangle$ over a complete basis $|\alpha_i\rangle$ is defined as $\text{IPR}(|\Psi\rangle) = (\sum p_i^2)^{-1}$, where $p_i = |\langle \Psi | \alpha_i \rangle|^2$ represents the probability of finding a state $|\alpha_i\rangle$. Defined in such a way, the IPR takes values between 1 and the Hilbert space dimension, and effectively tells us how many
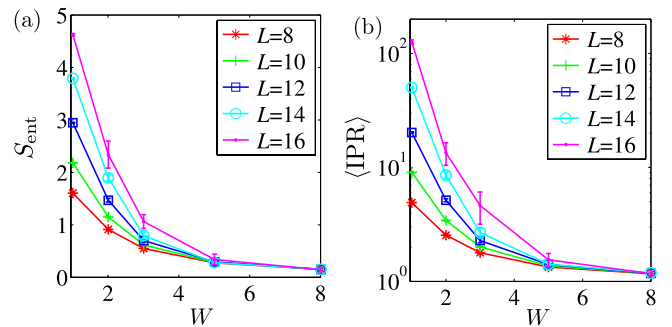

FIG. 1 (color online).   (a) Averaged entanglement entropy is of the order 1 and varies weakly with L for strong disorder, indicating that eigenstates are short-range entangled. Interaction strength is $V = 1$. (b) Inverse participation ratios for the product of two eigenstates of the $\mathcal{L}$ and $\mathcal{R}$ subsystems are close to 1 and do not depend on L for strong disorder.

components have nonzero weight in the decomposition of the given state over the chosen complete basis. Figure 1(b) shows the average IPR for the product $|\alpha\rangle \otimes |\beta\rangle$ of two random eigenstates of the $\mathcal{L}$ and $\mathcal{R}$ subsystems over the eigenstates $|\lambda\rangle$ of $\mathcal{LR}$. The value of the IPR at strong disorder is very close to 1, indicating that the product of eigenstates of $\mathcal{L}$ and $\mathcal{R}$ is "close" to the eigenstate of the full system $\mathcal{LR}$. Furthermore, the IPR does not grow with L for strong disorder, suggesting that the product of eigenstates of $\mathcal{L}$ and $\mathcal{R}$ differs from the eigenstate of the full system only near the boundary.

To provide further support for our construction of the integrals of motion, we numerically implemented the projector operator similar to the one defined in Eq. (3). Every eigenstate $|\lambda\rangle$ of $\mathcal{LR}$ is labeled by its "ancestor" in $\mathcal{L}$ as in Eq. (2). To find the ancestor, we calculate the density matrix $\hat{\rho}_\lambda$ for the $\mathcal{L}$ subsystem from $|\lambda\rangle$. Using $\hat{\rho}_\lambda$, we extract the probabilities of all eigenstates of $\mathcal{L}$ as $p_\alpha = |\langle\alpha|\hat{\rho}_\lambda|\alpha\rangle|^2$. In the limit of very strong disorder the typical value of the largest p_α is close to 1 [26]. Thus, the "ancestor" for $|\lambda\rangle$ is defined to be an eigenstate of $\mathcal{L}$ with the largest probability p_α.

Although we do not assign labels for the right subsystem, such labeling is sufficient to implement the operator $\hat{P}_\alpha = \sum_\beta \hat{P}_{\alpha\beta}$ as a projector onto the subspace of all eigenstates with the same label α for the $\mathcal{L}$. As a simple test, we study the locality of the projector $\hat{P}_\alpha$: by construction it must have trivial action in the right subsystem. To test this property, we perturb some eigenstate with label α, $|\lambda^\alpha\rangle$, at the right boundary $|\psi^\alpha\rangle = (1/2 + 2\mathbf{S}_L \cdot \mathbf{S}_{L-1})|\lambda^\alpha\rangle$. Because we are interested in the weight of $|\psi^\alpha\rangle$ in the subspace with the same label α, we plot the averaged $\langle\psi^\alpha|\hat{P}_\alpha|\psi^\alpha\rangle$ as a function of disorder in Fig. 2. For strong disorder, even when the interaction strength is $V = 1$, the perturbed state $|\psi^\alpha\rangle$ has almost all of its weight in the subspace with index α, indicating that the degrees of

FIG. 2 (color online). The weight of the perturbed eigenstate $|\lambda^\alpha\rangle$ in the subspace with index α. For strong disorder, the action of the projector is contained within the subspace α, irrespective of the interaction: the case of no interaction $V = 0$ is shown in (a), and $V = 1$ in (b). The weight increases with system size. For weak disorder ($W < 3$), the presence of interactions causes the weight to decrease with the system size, suggesting the onset of the delocalized phase.

freedom in the subsystem $\mathcal{L}$ are not affected by the perturbation acting on the subsystem $\mathcal{R}$. It is evident from Fig. 2(b) that the weight within the subspace α grows as a function of system size at $W > W_*$, and decreases at $W < W_*$, where $W_* \approx 3$. Thus, W_* gives an estimate of the MBL transition location in agreement with Ref. [10]. We note that the construction described above allows for more explicit tests to be done, which will be presented in future work [27]. Additional numerical verifications of our central assumption can be found in Ref. [26].

Discussion.—We established that the MBL phase is characterized by a number of local integrals of motion, supporting the hypothesis put forward in Ref. [18]. This implies that the MBL phase does not thermalize, and only partial thermalization of initial product states, constrained by the local conservation laws, is possible.

It should be noted that there are many ways to define local integrals of motion. For example, in certain problems [28] it might be helpful to label the integrals of motion by a set of 1/2 pseudospins. Then, $\mathcal{M} = 2^K$ possible values of a given integral of motion $\hat{I}_i$ can be viewed as states of K pseudospins σ_i^η, $\eta = 1, \ldots, K$. The z projections of these pseudospins form a complete set of integrals of motion, and the Hamiltonian only involves σ_{iz}^η operators and their products. Operators σ_i^η can be viewed as effective degrees of freedom, in terms of which the dynamics becomes trivial: up-down states of spins are eigenstates, so time evolution can only lead to the dephasing between them.

Another implication of our work concerns the structure of the MBL eigenstates: they are short-range entangled, obey the area law, and can be generally represented as a product of eigenstates of the subsystems of size $\gg \xi$ which have been locally "corrected" near the boundaries with neighboring subsystems. This suggests an efficient numerical procedure for describing the MBL eigenstates in terms of matrix-product states. Starting from the product of eigenstates of decoupled blocks of size $\gg \xi$, entanglement between the blocks is introduced by the repeated action of the boundary terms in the Hamiltonian. The boundary terms generate only a finite-dimensional space; thus, diagonalizing the boundary Hamiltonian for each finite-dimensional subspace, it should be possible to find the eigenstates of two coupled blocks, etc.

Finally, our picture suggests a realistic route to extending coherence times in nearly isolated quantum systems, where decoherence is induced by interactions. Examples of such systems, in addition to systems of ultracold atoms, include nuclear spins and NV centers in diamond [29]. Assuming that one could induce strong static disorder leading to the many-body localization, the coherence time of a subsystem can be made very long. To achieve this, one needs to prepare a subsystem of size $\gg \xi$ (e.g., subsystem 1 in the above example), as well as its immediate neighborhood (e.g., subsystem 2) in some eigenstate. Then, local operations on the subsystem's degrees of
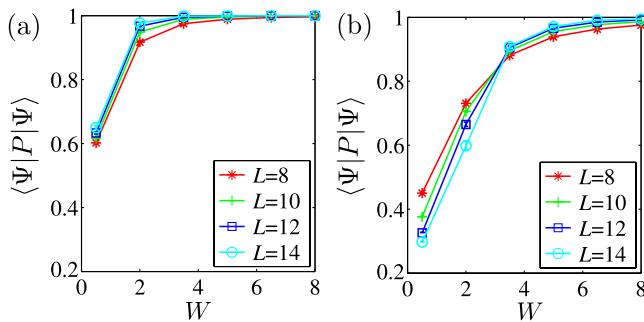
freedom would couple states with different integrals of motion I_1, but with fixed values of I_2. Therefore, even though the rest of the system is in some complicated superposition state, it will only give rise to an exponentially weak dephasing, with the rate proportional to $\exp(-l/\xi)$.

Note added.—During the completion of this manuscript, we became aware of a related work [28] discussing the existence of local integrals of motion in the MBL phase.

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