Stability of a Unitary Bose Gas

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We study the stability of a thermal ^{39}K Bose gas across a broad Feshbach resonance, focusing on the unitary regime, where the scattering length a exceeds the thermal wavelength λ. We measure the general scaling laws relating the particle-loss and heating rates to the temperature, scattering length, and atom number. Both at unitarity and for positive $a \ll \lambda$ we find agreement with three-body theory. However, for $a < 0$ and away from unitarity, we observe significant four-body decay. At unitarity, the three-body loss coefficient, $L_3 \propto \lambda^4$, is 3 times lower than the universal theoretical upper bound. This reduction is a consequence of species-specific Efimov physics and makes ^{39}K particularly promising for studies of many-body physics in a unitary Bose gas.

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The control of interactions provided by Feshbach resonances makes ultracold atomic gases appealing for studies of both few- and many-body physics. On resonance, the s-wave scattering length a, which characterizes two-body interactions, diverges. At and near the resonance a gas is in the unitary regime, where the interactions do not explicitly depend on the diverging a. Instead, a is replaced by another natural length scale. In a degenerate gas this length scale is set by the interparticle spacing; in a thermal gas it is set by the thermal wavelength $\lambda = h/\sqrt{2\pi m k_B T}$, where m is the particle mass and T is the temperature.

Over the past decade, there have been many studies of the unitary Fermi gas [1]. More recently, there has been an increasing interest in both universal and species-specific properties of a unitary Bose gas [2–15]. It is however an open question to what extent this state can be studied in (quasi-)equilibrium, since at unitarity three-body recombination leads to significant particle loss and heating [16]. The severity of this instability is not universal [10], as it depends on the species-specific few-body Efimov physics [8,18–28]. Characterizing and understanding the stability of a unitary Bose gas is thus important both from the perspective of Efimov physics and for identifying suitable atomic species for many-body experiments.

The per-particle loss rate due to three-body recombination is given by

$$\gamma_3 \equiv -\dot{N}/N = L_3 \langle n^2 \rangle, \quad (1)$$

where N is the atom number, L_3 is the three-body loss coefficient, n is the density, and $\langle \cdots \rangle$ denotes an average over the density distribution in a trapped gas. Away from unitarity, $L_3 \sim \hbar a^4/m$ [29,30], with a dimensionless prefactor exhibiting additional variation with a due to Efimov physics [19,27]. At unitarity L_3 should saturate at $\sim \hbar\lambda^4/m \propto 1/T^2$. Experimental evidence for such saturation was observed in [8,10,18]. More quantitatively, at unitarity we expect

$$L_3 \approx \zeta \frac{9\sqrt{3}\hbar}{m}\lambda^4 = \zeta \frac{36\sqrt{3}\pi^2\hbar^5}{m^3(k_B T)^2}, \quad (2)$$

where $\zeta \leq 1$ is a species-dependent, nonuniversal dimensionless constant [10] (see also Refs. [31–33]).

Similar scaling arguments apply to the two-body elastic scattering rate, γ_2, which drives continuous re-equilibration of the gas during loss and heating. Away from unitarity $\gamma_2 \propto \langle n \rangle \hbar a^2/(m\lambda)$; hence, at unitarity $\gamma_2 \propto \langle n \rangle \hbar\lambda/m$. The possibility to experimentally explore many-body physics of a quasiequilibrium unitary Bose gas depends on the ratio γ_3/γ_2. Remarkably, at a given phase-space density, $n\lambda^3$, this ratio depends only on the species-specific ζ.

Recently, $\zeta \approx 0.9$ was measured for ^{7}Li [10]. The gas was held in a relatively shallow trap, so that continuous evaporation converted heating into an additional particle loss, and the extraction of ζ relied on theoretically modeling this conversion and assuming the $1/T^2$ scaling of Eq. (2).

In this Letter, we study the stability of the ^{39}K Bose gas in the $|F, m_F\rangle = |1, 1\rangle$ hyperfine ground state, across a broad Feshbach resonance centered at 402.5 G [25]. We perform experiments in a deep trap and verify the predicted recombination-heating rate both at unitarity and for positive $a \ll \lambda$ [10,30]. At unitarity we measure $L_3 \propto T^{-1.7\pm0.3}$ and $\zeta \approx 0.3$, a value that makes ^{39}K particularly promising for studies of an equilibrium unitary gas. Additional measurements at $a < 0$, away from unitarity, reveal the importance of four-body processes [20,23], consistent with previous studies in ^{133}Cs [22], ^{39}K [25], and ^{7}Li [26].

Our experimental setup is described in Ref. [34]. We start by preparing a weakly interacting ($\lambda/a \approx 35$) thermal gas in a harmonic optical trap. The trap has a depth of $U \approx k_B \times 30$ μK and is nearly isotropic, with the geometric mean of the trapping frequencies $\omega = 2\pi \times 185$ Hz. We then tune a close to a Feshbach resonance, by ramping an external magnetic field over 10 ms. At this point we have $N \approx 10^5$ atoms at $T \approx 1$ μK, corresponding to

$\lambda \approx 5 \times 10^3 a_0$, where a_0 is the Bohr radius. At the trap center $n \approx 3 \times 10^{12}$ cm^{-3} and $n\lambda^3 < 0.1$, so even at unitarity and assuming $\zeta = 1$, we still always have $\gamma_2 \gg \gamma_3$. We let the cloud evolve for a variable hold time, t, of up to 4 s, and then simultaneously switch off the trap and the Feshbach field (within ~100 μs [35]). Finally, we image the cloud after 5 ms of time-of-flight expansion.

Figure 1 shows the particle loss and heating in a resonantly interacting gas ($\lambda/a = 0$). Restricting our measurements to $T < 2$ μK ensures that evaporative losses and cooling are negligible. We have taken 19 similar data series, each at a fixed a, spanning the range $-12 < \lambda/a < 12$.

We first study the relationship between T and N during the evolution of the cloud. One expects three sources of heating related to three-body recombination [10,30]. (i) For any a, losses preferentially occur near the center of the cloud, where the atoms have lower potential energy. (ii) For $a > 0$, recombination results in a shallow dimer with binding energy $\varepsilon = \hbar^2/(ma^2)$, and the third atom carries away $(2/3)\varepsilon$ as kinetic energy. In all our experiments $\varepsilon < U$, so this atom remains trapped and increases the energy of the cloud. (iii) At unitarity, three-body recombination preferentially involves atoms that also have lower kinetic energy.

To a good approximation, in our experiments we can capture all these effects by a simple scaling law:

$$NT^{\beta} = \text{const}, \tag{3}$$

with the exponent β varying across the resonance. Ignoring unitarity effects, $\beta = 3$ for $a \leq 0$, and $\beta = 3/[1 + \lambda^2/(9\pi a^2)]$ for $a > 0$ (see also [30]). In the latter case β changes as the cloud heats, but in our measurements this variation is small enough that a constant $\beta = -d[\ln(N)]/d[\ln(T)]$ describes the data well (see inset of Fig. 2). At unitarity, a universal value of $\beta = 1.8$ was predicted in Ref. [10].

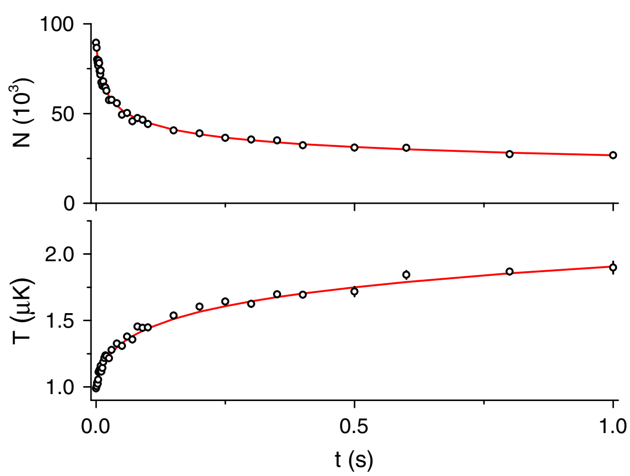

FIG. 1 (color online). Particle loss and heating in a resonantly interacting Bose gas ($\lambda/a = 0$). Each point is an average of 5 measurements and error bars show standard statistical errors. Solid red lines are fits based on Eqs. (5) and (3).

In Fig. 2 we show our measured values of β. For $\lambda/a \gg 1$ we find agreement with the nonunitary prediction shown by the red dashed line. However, approaching unitarity we see gradual deviation from this theory. On resonance, we measure $\beta = 1.94 \pm 0.09$, close to the unitary prediction of $\beta = 1.8$ (indicated by the red star), and far from the nonunitary $\beta = 3$.

Moving away from unitarity into the $a < 0$ region (open symbols in Fig. 2, corresponding to $-2000 < a/a_0 < -400$), β rises further, but does not reach the expected nonunitary limit. By analyzing the dynamics of the particle loss, $N(t)$, we find that in this region four-body decay is also significant (see Fig. 3); in this case our prediction for β is not applicable. Previously, indirect evidence for four-body decay in this region was seen in Ref. [25], but not in Ref. [28], where the initial cloud density was significantly lower.

We fit the $N(t)$ data by numerically evolving a loss equation featuring both three- and four-body decay [22],

$$\dot{N} = -L_3\langle n^2\rangle N - L_4\langle n^3\rangle N, \tag{4}$$

where L_3 and L_4 are fitting parameters and we use the measured $T(t)$ to evaluate the thermal density averages. To obtain purely three- (four-) body fits we fix L_4 (L_3) to zero.

In Fig. 3 we show $N(t)$ for $a = -850a_0$. The model including both L_3 and L_4 provides an excellent fit to the data, with $\chi^2 \approx 1$. In comparison, pure four- and three-body fits have $\chi^2 \approx 5$ and 7, respectively. We observe four-body effects for all our data with $-2000 < a/a_0 < -400$. However, we find that they are relevant only at densities $\gtrsim 10^{12}$ cm^{-3}, which reconciles the observations of

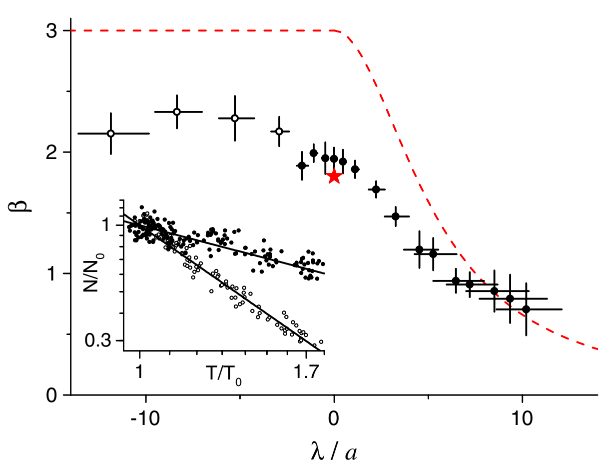

FIG. 2 (color online). Heating exponent β, as defined in Eq. (3). The red dashed line is a result of nonunitary three-body theory, while the red star indicates the predicted value of 1.8 at unitarity. Open symbols indicate the region where four-body decay is significant (see text and Fig 3). Note that $\lambda \approx 5 \times 10^3 a_0$ and horizontal error bars reflect its variation during a measurement sequence at a fixed a. Vertical error bars show fitting uncertainties. Inset: Log-log plots of N vs T (scaled to their values at $t = 0$) for the data series at $\lambda/a \approx -5.3$ (open) and 8.5 (solid).

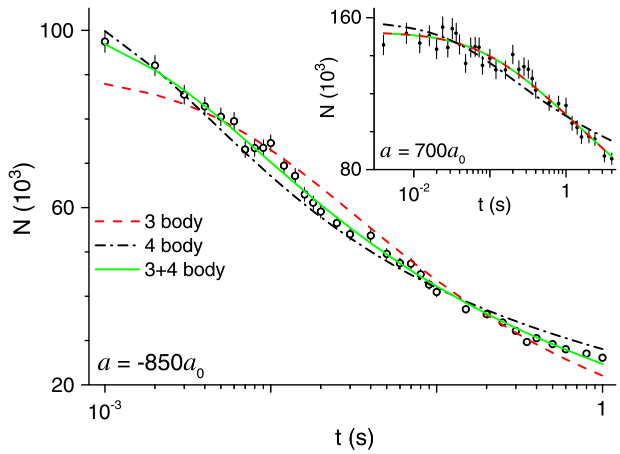

FIG. 3 (color online). Three- vs four-body decay for $a < 0$ (away from unitarity). N decay at $a = -850a_0$ is fitted to a model including both three- and four-body losses (green solid line), as well as to pure three- and four-body models (red dashed and black dot-dashed line, respectively). Inset: For comparison, at $a = 700a_0$, the solid green and the dashed red lines are indistinguishable, showing that four-body decay does not play a detectable role.

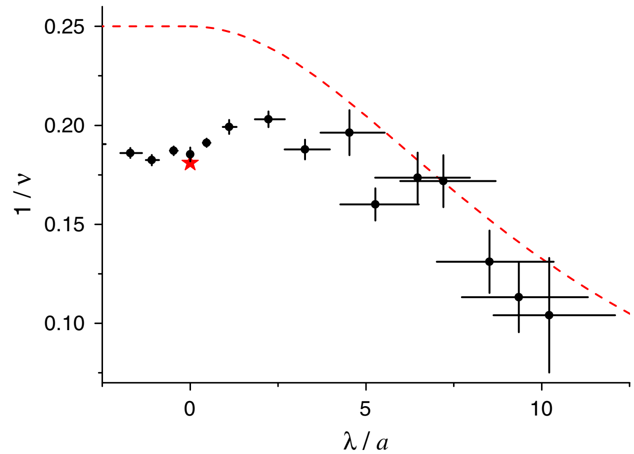

FIG. 4 (color online). Particle-loss exponent ν, as defined in Eq. (5). The red dashed line shows the nonunitary theory, $\nu = 3 + 3/\beta$, assuming nonunitary β values. The red star shows the unitary prediction, $\nu = 3 + 5/\beta$, corresponding to $L_3 \propto 1/T^2$ and the measured β. Error bars are analogous to those in Fig. 2.

Refs. [25,28]. A more detailed study of this region, including any four-body resonances [22], is outside the scope of this Letter.

For $a > 0$ the same analysis does not reveal any four-body decay (see inset of Fig. 3). In this case the pure three-body fit and the fit including both L_3 and L_4 are indistinguishable, with $\chi^2 \approx 1$, and give the same L_3 (within the 10% fitting errors), while the pure four-body fit has $\chi^2 \approx 2$. This strongly excludes L_4 as a relevant fit parameter. Using a similar procedure, we have also checked that for both positive and negative a we do not detect any five-body decay.

We henceforth focus on the three-body decay dynamics at unitarity, using the $a > 0$ nonunitary regime for comparison. Invoking Eq. (3), in both regimes the particle loss should be described by:

$$\dot{N} = -AN^{\nu}, \tag{5}$$

where A and ν are constants. Here, ν absorbs all the N and T dependence of L_3 and $\langle n^2 \rangle$. Integration gives a fitting function $N(t) = [A(\nu - 1)t + N(0)^{1-\nu}]^{1/(1-\nu)}$. For $a \ll \lambda$ we expect $\nu = 3 + 3/\beta$, whereas at unitarity $L_3 \propto 1/T^2$ implies $\nu = 3 + 5/\beta$. To test this hypothesis in an unbiased way, we analyze our data using ν as a free parameter.

Note that here we invoke Eq. (3) merely to anticipate the validity of Eq. (5) and the ν values; experimentally, our analysis of $N(t)$ and ν is decoupled from the measurements of $T(t)$ and β. The validity of our approach is seen in Fig. 1, where the fit of $N(t)$ is based on Eq. (5). The fit of $T(t)$ is then obtained by inserting the fitted $N(t)$ and β into Eq. (3).

Our fitted values of ν are summarized in Fig. 4. We see a crossover from nonunitary to unitary behavior as the resonance is approached, confirming the appearance of a temperature-dependent L_3. Now combining our measurements of β and ν, at unitarity we get $L_3 \propto T^{-1.7\pm0.3}$, in agreement with the expected $1/T^2$ scaling.

Next, using the fitted A and ν, for each data series at a particular a, and for any evolution time t, we extract

$$L_3(t) = 3\sqrt{3}\left(\frac{2\pi k_B T(t)}{m\omega^2}\right)^3 N(t)^{\nu-3}A. \tag{6}$$

Combining all our data series, we reconstruct $L_3(a, T)$.

In Fig. 5 (main panel) we show L_3 at a fixed $T = 1.1\ \mu$K, scaled to the theoretical upper bound $L_3^{\mathrm{M}}(T)$,

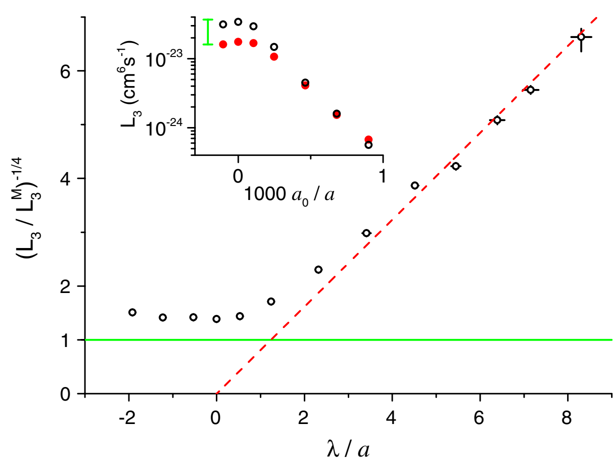

FIG. 5 (color online). Three-body loss coefficient. Main panel: $(L_3/L_3^{\mathrm{M}})^{-1/4}$ (see text) at $T = 1.1\ \mu$K. Horizontal green line marks the theoretical upper bound on L_3, while the red dashed line is a guide to the eye showing the $L_3 \propto a^4$ nonunitary scaling. At unitarity, $L_3/L_3^{\mathrm{M}} \approx 0.27$. Inset: L_3 at 1.1 μK (open symbols) and 1.7 μK (solid symbols). The expected ratio between the two unitary plateaux is indicated by the green vertical bar.

obtained by setting $\zeta = 1$ in Eq. (2). Plotting $(L_3/L_3^{\mathrm{M}})^{-1/4}$ versus λ/a clearly reveals two key effects. First, for $\lambda/a \gtrsim 3$, we see the nonunitary scaling $L_3 \propto a^4$ [37]. Second, close to the resonance, L_3 saturates at $\approx 0.27 L_3^{\mathrm{M}}$.

In the inset of Fig. 5 we focus on the region close to the resonance and compare L_3 for two different temperatures, $T = 1.1\ \mu\mathrm{K}$ and $1.7\ \mu\mathrm{K}$. Away from the resonance, L_3 does not show any T dependence. At unitarity, the ratio of the two saturated L_3 values is close to the expected $1/T^2$ scaling.

Finally, to refine our estimate of ζ, we fix $\nu = 3 + 5/\beta$ (i.e., $L_3 \propto 1/T^2$) and reanalyze the three data series taken closest to the resonance, for which $|\lambda/a| < 0.6$ at all times. This gives us a combined estimate of $\zeta = 0.29 \pm 0.03$, while the systematic uncertainty in ζ due to our absolute atom-number calibration [38,39] is about 30%. Writing $L_3 = \lambda_3/T^2$, this corresponds to $\lambda_3 \approx 4.5 \times 10^{-23}\ (\mu\mathrm{K})^2\,\mathrm{cm}^6\,\mathrm{s}^{-1}$. In the context of Efimov physics, $\zeta = 1 - e^{-4\eta}$ [10], where η is the Efimov width parameter [40]. We deduce $\eta = 0.09 \pm 0.04$ (see also [25]).

In conclusion, we have fully characterized the stability of a ^{39}K gas at and near unitarity. We have experimentally verified the theoretically predicted general scaling laws characterizing particle loss and heating in the unitary regime, confirmed the relevance of four-body decay on the negative side of the Feshbach resonance, and measured the species-specific unitarity-limited three-body loss coefficient, $L_3 \propto 1/T^2$. The unitary value of L_3, 3 times lower than the universal theoretical upper bound, makes ^{39}K a promising candidate for experimental studies of many-body physics in a unitary Bose gas.

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Note added.—Recently, a study of a degenerate unitary ^{85}Rb gas was reported [41].

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