New Probe of Naturalness

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(Received 5 June 2013; published 20 September 2013)

Any new scalar fields that perturbatively solve the hierarchy problem by stabilizing the Higgs boson mass also generate new contributions to the Higgs boson field-strength renormalization, irrespective of their gauge representation. These new contributions are physical, and in explicit models their magnitude can be inferred from the requirement of quadratic divergence cancellation; hence, they are directly related to the resolution of the hierarchy problem. Upon canonically normalizing the Higgs field, these new contributions lead to modifications of Higgs couplings that are typically great enough that the hierarchy problem and the concept of electroweak naturalness can be probed thoroughly within a precision Higgs boson program. Specifically, at a lepton collider this can be achieved through precision measurements of the Higgs boson associated production cross section. This would lead to indirect constraints on perturbative solutions to the hierarchy problem in the broadest sense, even if the relevant new fields are gauge singlets.

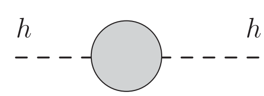
DOI: 10.1103/PhysRevLett.111.121803 PACS numbers: 12.60.Fr, 13.66.Fg, 14.80.Ec

Introduction.—The discovery of the Higgs boson at the LHC [1,2] and lack of evidence for physics beyond the standard model (SM) have heightened the urgency of the electroweak (EW) hierarchy problem. This motivates the focusing of experimental searches towards testing "naturalness from the bottom up" as broadly as possible. In practice, as natural theories are typically free of quadratic divergences, this means generalizing beyond the specifics of particular UV-complete models and instead constraining the additional degrees of freedom whose couplings to the Higgs boson are responsible for canceling the most pressing quadratically divergent standard model contributions to the Higgs boson mass. While these couplings may appear tuned from the perspective of the low-energy effective theory, we may assume that they are dictated by symmetries of the full theory, as found in supersymmetric or composite Higgs boson models. To a certain extent, this strategy is already being pursued in searches for top squarks in supersymmetry (SUSY) and t' fermions; however, the standard model gauge representations of top partners are not necessarily fixed by the cancellation of quadratic divergences. In twin Higgs boson models [3], the degrees of freedom protecting the Higgs mass are completely neutral under the standard model, whereas in folded supersymmetry [4] the scalar top partners are neutral under QCD and only carry electroweak quantum numbers, demonstrating that the Higgs mass may be protected by degrees of freedom with a variety of standard model gauge charges.

Direct searches for these additional degrees of freedom can be particularly challenging depending on the gauge charges. In this work we advocate an additional and complementary approach, concerned with exploring naturalness *indirectly*. In certain cases, this may be the most promising avenue for constraining additional degrees of freedom associated with the naturalness of the Higgs potential.

Specifically, we establish for the first time a quantitative connection between quadratically divergent Higgs boson mass corrections and new contributions to the Higgs boson wave-function renormalization in natural theories. The latter are physical and modify Higgs boson couplings.

To illustrate the possible indirect effects of natural new physics, consider a scenario where the Higgs boson is coupled to some new top-partner fields that cancel the one-loop quadratic divergences arising from top-quark loops. Equation (1) schematically indicates that, as well as the usual Higgs mass corrections, one will also in general have corrections to the Higgs boson wave-function renormalization [5]

$\delta Z_h, \delta m_h^2 \sim$  . (1)

At the Higgs boson mass scale we may write the full one-loop effective Lagrangian as

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{2}\delta Z_h(\partial_\mu h)^2 + \dots \tag{2}$$

where δZ_h is directly related to the new quadratic Higgs mass corrections, $\mathcal{L}_{\mathrm{SM}}$ is the full SM Lagrangian at one loop, and the ellipsis denote corrections to the Higgs mass, cubic, and quartic couplings coming from the new fields [6]. We canonically normalize the Higgs field by the rescaling $h \to (1 - \delta Z_h/2)h$. This rescales all Higgs boson couplings to weak gauge bosons and fermions by the same amount. This rescaling is physical: it can be moved around by rescaling other fields or couplings but cannot be removed from the theory. For canonically normalized fields, this rescaling will in general break the SM prediction for the relationship between the mass of a field and its coupling to the Higgs boson. This deviation from SM predictions can then be constrained with precision Higgs coupling measurements.

In the case where the new fields are not gauge singlets, one expects additional corrections beyond the wave-function renormalization. Some of these corrections involve the gauge sector alone and can be constrained via the Peskin-Takeuchi parameters [7,8] and their generalization [9]; other corrections may also directly correct the Higgs-weak boson vertices. Although this situation is more involved, the wave-function renormalization typically dominates [10]. Hence, we see that if the hierarchy problem is resolved by new physics then it may leave its footprint through indirect signatures in SM processes via modified Higgs couplings, even in situations where it is difficult to observe the new physics directly.

To render these effects quantitative, we must commit to a concrete, calculable set up. We will construct a general scenario based solely on the naturalness criterion: a "weak-scale effective theory of naturalness," restricted only by the simplifying assumption that the new fields canceling the top quadratic divergence are scalars [11]. We describe how, guided by *naturalness alone*, one is led to very specific quantitative predictions for Higgs boson coupling corrections within this effective natural theory, with the only free variables being the number of fields and their masses. We will demonstrate that the generic parameter space of natural theories can be thoroughly explored through percent-level precision Higgs boson coupling measurements at a lepton collider (LC) or potentially at the LHC.

Weak-scale effective theory of naturalness.—Assuming that the leading natural degrees of freedom are scalar top partners, we can define the perturbative effective natural theory as

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i (|\partial_\mu \phi_i|^2 - m_i^2|\phi_i|^2 - \lambda_i |H|^2|\phi_i|^2), \qquad (3)$$

where without loss of generality we take the scalars to be complex, and we use the EW symmetry-breaking conventions $H \to v + h/\sqrt{2}$ with $v \approx 174$ GeV and $m_{\phi_i}^2 = m_i^2 + \lambda_i v^2$, leading to a trilinear coupling $\mathcal{L} \supset \sqrt{2}\lambda_i v h|\phi_i|^2$ [12]. Here, the index $i = 1, \ldots, n_\phi$ counts the number of fields coupled to H, which may be related by gauge or global symmetries. For example, in SUSY, $n_\phi = 6$ counts the two top squarks transforming as triplets under $SU(3)_c$, whereas in folded SUSY $n_\phi = 6$ counts the two top squarks transforming as triplets under a distinct SU(3) gauge group.

In the SM, the most pressing quadratic divergence arises at one loop from a top quark, and hence the most basic requirement in any natural theory is the cancellation of this leading divergence up to a scale of a few TeV [13]. In order to cancel one-loop quadratic Higgs mass corrections from the top quark alone, it is simply required that

$$\sum_i \lambda_i = 6\lambda_t^2, \qquad (4)$$

where λ_t is the top Yukawa coupling and in known natural theories this coupling value is always enforced by a symmetry, such as SUSY or global symmetries [14]. For simplicity, we can make the further assumption that all n_ϕ scalars have the same mass m_ϕ and the same coupling λ_ϕ. As we will show, this extremely simple effective theory of naturalness is broad enough to capture the dominant indirect corrections to Higgs physics even though we have not specified the gauge representations and are agnostic as to the UV completion of the model. Throughout we only assume this theory is valid up to scales of a few TeV as around this scale a full UV completion is required to ensure naturalness up to higher scales.

From this point, we can define a measure of naturalness. Although the theory so far is renormalizable, we should choose an energy scale Λ at which the theory is UV completed. We can then calculate corrections to the high-scale Higgs boson mass m_H due to logarithmic running from Λ down to the weak scale. At one loop, this correction is

$$\delta m_H^2 = -n_\phi \frac{\lambda_\phi}{8\pi^2} m_\phi^2 \log\left(\frac{\Lambda}{m_\phi}\right), \qquad (5)$$

$$= -\frac{6\lambda_t^2}{8\pi^2} m_\phi^2 \log\left(\frac{\Lambda}{m_\phi}\right), \qquad (6)$$

where we have imposed the cancellation of quadratic divergences in the final line and m_ϕ is the top partner mass above the EW-breaking scale. Following standard conventions used in SUSY literature, we can define the fine tuning measure as a function of the logarithmic corrections to the high-scale Higgs mass m_H and the physical Higgs mass m_h as [15]

$$\Delta \equiv \frac{2\delta m_H^2}{m_h^2}, \qquad (7)$$

which quantifies the degree of fine tuning required in the Higgs potential. Since we do not know the details of the UV completion, it is sensible to assume a low UV-completion scale, so we set $\Lambda = 10$ TeV, although we note that in concrete models this scale can in principle be much higher, exacerbating the fine tuning. With this measure we can consider some benchmark tuning points

$$\Delta^{-1}(m_\phi = 350 \text{ GeV}) = 25\%, \qquad (8)$$

$$\Delta^{-1}(m_\phi = 605 \text{ GeV}) = 10\%, \qquad (9)$$

which illustrate that scalar top partners should not lie too far from the weak scale in a natural theory [16].

Although irrelevant to the one-loop corrections to the Higgs mass, the fields ϕ may be charged under various representations of the standard model gauge group and may be searched for directly at the LHC. For example, states charged under $SU(3)_c$ are primarily produced via QCD interactions and are efficiently constrained by LHC top squark searches or, if stable, searches for R hadrons. Similarly, states charged under $SU(2)_L$ and/or $U(1)_Y$ are primarily produced via Drell-Yan processes and are constrained by LHC searches for electroweak final states or, if stable, CHAMP searches.

States neutral under the standard model are much more challenging to constrain through direct searches. Although ϕ appears in the invisible decay products of the Higgs boson when $m_\phi < m_h/2$, the coupling λ_ϕ is typically large enough that this invisible partial width vastly exceeds the standard model width and is ruled out by current limits on the invisible width of the Higgs boson [17] unless m_ϕ is finely tuned to lie at the kinematic threshold for pair production. For $m_\phi > m_h/2$, the primary means of observing ϕ at the LHC involves pair production through an off-shell Higgs boson. The most promising channels are vector boson fusion $qq \to V^\star V^\star qq \to \phi\phi^* qq$ and vector associated production $q\bar{q} \to V^\star \to V\phi\phi^*$, where $V = W$, Z. However, the small production cross sections and the challenges of triggering and pileup for the relevant final states render these direct search channels unpromising at the LHC. Although the lightest ϕ_i could constitute a dark matter candidate if absolutely stable and neutral under the SM, its thermal relic abundance is typically too small due to the large coupling to the Higgs boson [18]. If this issue is circumvented via a nonthermal production mechanism and the top partner saturates the observed DM abundance, then direct detection constraints rule out such large couplings [18].

Note that even states carrying standard model gauge charges are exceptionally difficult to discover if the kinematics of their decays are unfavorable. Colored scalars decaying to nearly degenerate neutral states are challenging to distinguish from standard model di-jet backgrounds. Electroweak scalars whose mass is close to the W boson are difficult to discover underneath W^+W^- backgrounds, whereas decays to nearly degenerate neutral states are challenging for standard triggers.

Thus, it is entirely possible that the mass of the Higgs boson is rendered completely natural by top partners whose kinematic properties or quantum numbers make them difficult to discover at the LHC, and perhaps the most promising avenue for discovery lies in *indirect* searches.

A new probe of naturalness.—An efficient indirect phenomenological test of naturalness depends on the precision with which Higgs boson properties can be measured. An indirect search for natural physics is therefore best performed in the clean phenomenological environment offered by a future LC. Higgs boson associated production at a precision instrument such as a LC provides an extremely sensitive tool to analyze the Higgs boson, as at $\sqrt{s} = 250$ GeV it is the dominant Higgs production mode for $m_h \simeq 125$ GeV [19]. The clean and fully reconstructible final state allows for precise measurements of the Higgs boson couplings and properties [20,21]. A particular strength lies in the fact that the hZZ coupling can be determined independent of Higgs decays, removing uncertainties in the total width and other Higgs couplings. The program to reach a theoretically precise understanding of $e^+e^- \to hZ$ in the standard model dates back to the beginning of the large electron-positron collider era [22–26]. Recent analyses of associated production cross section measurements indicate that uncertainties as low as $O(0.5\%)$ can be achieved [20,21,27–29].

Beyond the standard model-modified NLO electroweak corrections to associated production are typically larger than the projected $O(0.5\%)$ uncertainty [10]. Thus, Higgs boson coupling measurements can constrain natural new physics for generic top partners even when they are neutral under the SM gauge group. To see the relevant effects clearly, consider the theory of Eq. (3) when all scalar top partners ϕ_i are gauge singlets. In the limit $m_\phi \gg v$, we may integrate out the ϕ_i and express their effects in terms of an effective Lagrangian below the scale m_ϕ involving only standard model fields with appropriate higher-dimensional operators. At one loop, integrating out the ϕ_i leads to shifts in the wave-function renormalization and potential of the Higgs doublet H as well as operators of dimension six and higher. Most of these shifts and operators are irrelevant from the perspective of low-energy physics, except for one dimension-six operator in the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{c_H}{m_\phi^2}\left(\frac{1}{2}\partial_\mu |H|^2 \partial^\mu |H|^2\right) + \ldots \qquad (10)$$

where the ellipses include additional higher-dimensional operators that are irrelevant for our purposes. Matching to the full theory at the scale m_ϕ, we find $c_H(m_\phi) = n_\phi |\lambda_\phi|^2/96\pi^2$. Although this operator may be exchanged for a linear combination of other higher-dimensional operators using field redefinitions or classical equations of motion, the physical effects are unaltered. Below the scale of electroweak symmetry breaking, Eq. (10) leads to a shift in the wave-function renormalization of the physical scalar h as in Eq. (2), with $\delta Z_h = 2c_H v^2/m_\phi^2$. Canonically normalizing h alters its coupling to vectors and fermions, leading to a measurable correction to, e.g., the hZ associated production cross section

$$\delta\sigma_{Zh} = -2c_H \frac{v^2}{m_\phi^2} = -\frac{n_\phi |\lambda_\phi|^2}{48\pi^2}\frac{v^2}{m_\phi^2}, \qquad (11)$$

where we have defined $\delta\sigma_{Zh}$ as the fractional change in the associated production cross section relative to the

SM prediction, which vanishes for the SM alone. Since $n_\phi|\lambda_\phi|^2$ must be large to cancel the top quadratic divergence, this effect may be observable in precision measurements of σ_{Zh} despite arising at one loop.

While this effective Lagrangian approach makes the physical effect transparent, naturalness dictates that $m_\phi \sim v$ and threshold corrections to Eq. (10) may be large, and a complete calculation is required. In the on-shell renormalization scheme, the Higgs self-energy enters through the counterterm part of the renormalized $e^+e^- \to hZ$ amplitude. Thus, the hG^0Z and hZZ vertices receive corrections from the Higgs boson wave-function renormalization [30]. For scalar top partners the Higgs boson wave-function renormalization arises at one loop through scalar trilinear couplings, which gauge invariance relates to the quartic vertices, which are in turn directly relevant for the cancellation of the quadratic divergences in δm_h^2.

At one loop, the effective theory of naturalness defined in Eq. (3) leads to a correction to the associated production cross section of the form [10]

$$\delta\sigma_{Zh} = n_\phi \frac{|\lambda_\phi|^2 v^2}{8\pi^2 m_h^2}(1 + F(\tau_\phi)), \tag{12}$$

$$= \frac{9\lambda_t^2 m_t^2}{2\pi^2 n_\phi m_h^2}(1 + F(\tau_\phi)), \tag{13}$$

where in the last line we have again imposed the cancellation of quadratic divergences and $\tau_\phi = m_h^2/4m_\phi^2$. $F(\tau)$ is given by

$$F(\tau) = \frac{1}{4\sqrt{\tau(\tau-1)}} \log\left(\frac{1 - 2\tau - 2\sqrt{\tau(\tau-1)}}{1 - 2\tau + 2\sqrt{\tau(\tau-1)}}\right). \tag{14}$$

Equation (13) contains the full one-loop correction for gauge singlet top-partners. Additional corrections should also be included in the case where the top partners carry electroweak quantum numbers. However, these corrections have been calculated in Ref. [10] where it was found that the one-loop corrections are still dominated by Eq. (13). This follows from the fact that the square of the top Yukawa coupling is greater than the square of electroweak couplings. Thus, Eq. (13) applies equally well to generic scalar top partners, irrespective of gauge charges.

In Fig. 1 we show the extent to which the parameter space of natural theories can be indirectly explored at a lepton collider. For measurements of the associated production cross section at the estimated accuracy of $O(0.5\%)$, natural theories tuned at the 25% level or greater can be probed, depending on the number of degrees of freedom. Optimistically, if the measurement accuracy were improved further to $O(0.1\%)$ then natural theories could be probed if they were tuned up to the 10% level, even if they

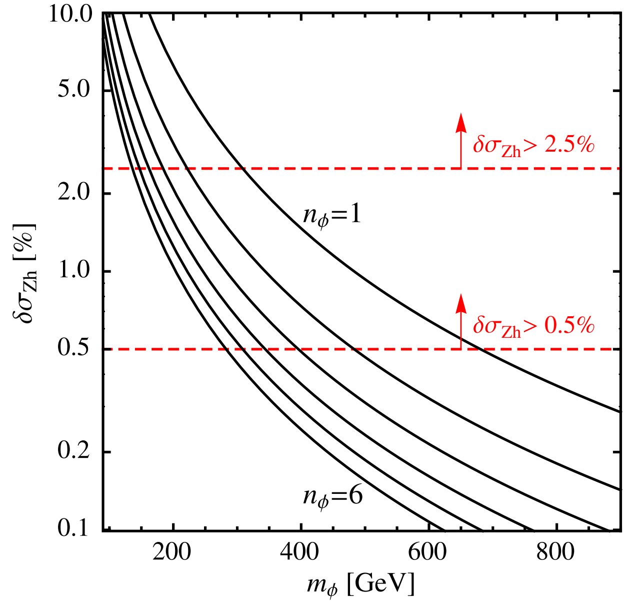


FIG. 1 (color online). Scalar top-partner corrections to the Higgs boson associated production cross section at a 250 GeV lepton collider as a function of the top-partner mass m_ϕ in the effective theory of naturalness of Eq. (3). Corrections are shown for $n_\phi = 1, \ldots, 6$ top partners. Estimates for the measurement precision of 2.5% [20,21] and 0.5% [27] are also shown. Note that expected SM theory errors are included in these numbers. It is remarkable that with current precision estimates a large portion of model-independent parameter space for Higgs boson naturalness can be probed. In particular, if one compares this with the tuning estimates of Eq. (9), this broadly corresponds to probing 10% tuned regions for a single scalar top partner and close to 25% tuned regions for $n_\phi = 6$ scalar top partners as in SUSY. Optimistically, if the precision could be improved to $\delta\sigma_{Zh} \sim 0.1\%$, then virtually all parameter space for generic natural scalar theories with up to $\sim$10% tunings could be probed.

contained only gauge singlets. These results apply to the broad class of effective natural theories described here, regardless of the top-partner gauge charges and, hence, contain SUSY theories as a subset. They also apply to scenarios where top partners are difficult to directly discover or constrain due to their kinematic properties or quantum numbers. In cases with neutral top partners, this presents the most promising prospect for discovery or exclusion. If the top partners carry color or electroweak quantum numbers, direct searches may be more powerful; however, such cases can always be concealed at hadron colliders, in which case measurements of the associated production cross section represent a complementary orthogonal test of naturalness.

If precision measurements of the Higgs boson associated production cross section at a lepton collider show deviations from SM expectations at the level of $O(1\%)$, then this would constitute strong indirect evidence for new physics in the Higgs boson sector and would be suggestive of a solution to the hierarchy problem. Alternatively, if no deviations are observed then such measurements could put the compelling notion of electroweak naturalness under strain.

We would like to thank Ayres Freitas, Anson Hook, Markus Klute, Dmytro Kovalskyi, Michael Peskin, Michael Rauch, Jesse Thaler, and Scott Thomas for useful conversations. N. C. and M. M. acknowledge the hospitality of the Weinberg Theory Group at the University of Texas, Austin, where this work was initiated.

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