

Hunting Mixed Top Squark Decays

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We point out that, in the irreducible natural supersymmetric spectrum, top squarks have comparable branching fractions to chargino-bottom and neutralino-top final states in the vast bulk of parameter space, provided only that both decay modes are kinematically accessible. The total top squark pair branching fractions into $t\bar{t} + \text{MET}$ (MET = missing transverse energy) can therefore be reduced to $\mathcal{O}(50\%)$, whereas $b\bar{b} + X$ branching fractions are typically much smaller, $\mathcal{O}(10\%)$, thus limiting the reach of traditional top squark searches. We propose a new top squark search targeting the asymmetric final state $\tilde{t}\tilde{t}^* \rightarrow t\chi^0\bar{b}\chi^- + \text{H.c.}$, which can restore sensitivity to natural top squarks in the 7 and 8 TeV LHC runs. In addition, we present a new variable, *topness*, which efficiently suppresses the dominant top backgrounds to semileptonic top partner searches. We demonstrate the utility of topness in both our asymmetric search channel and traditional $\tilde{t}\tilde{t}^* \rightarrow t\bar{t} + \text{MET}$ searches and show that it matches or outperforms existing variables.

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Introduction.—The recent discovery of what looks very much like a weakly coupled Higgs boson has made the question of the electroweak hierarchy acute. The weak-scale supersymmetry (SUSY) spectrum has long been a favored contender to protect the Higgs vacuum expectation value, but the impressive agreement of LHC data with standard model (SM) predictions has made it clear that most of the minimal supersymmetric standard model particle content must lie beyond the kinematic reach of the 7 and 8 TeV LHC if R parity is conserved. Only a small subset of the SM superpartners are immediately relevant for the naturalness of the Higgs potential, however [1]. In particular, only Higgsinos, with masses $m_H \lesssim 200$ GeV, and third-generation squarks $\tilde{t}_L, \tilde{t}_R, \tilde{b}_L$, with masses $m_{\tilde{Q}} \lesssim 400\text{--}500$ GeV, need to be light to keep electroweak symmetry breaking fully natural, though gluinos, winos, and binos must not be too far beyond current LHC energies [2]. Searching for a light top squark or three light third-generation squarks therefore becomes critical for understanding the extent to which SUSY is relevant for stabilizing the electroweak scale.

The purpose of the present work is twofold. First, we point out that, in a natural SUSY spectrum, the pure Higgsino nature of the light neutralinos and chargino can significantly weaken the reach in the traditional top squark search channel $\tilde{t}\tilde{t}^* \rightarrow t\bar{t} + \cancel{E}_T$, and we propose a novel search channel targeting the natural SUSY spectrum that can recover the lost sensitivity. Second, we introduce a new kinematic variable, *topness*, which efficiently suppresses the dominant backgrounds to top partner searches in semileptonic final states. We demonstrate its performance both in our novel search topology and in more traditional searches.

Top squark and bottom squark branching ratios.—In the natural SUSY spectrum, the lightest states are Higgsinos and third-generation squarks; the gauginos can be heavier, up to several hundreds of GeV. Mass mixing between the Higgsinos and the heavier EW gauginos splits the Higgsinos; for example, with $(M_1, M_2, \mu, \tan\beta) = (400 \text{ GeV}, 800 \text{ GeV}, 200 \text{ GeV}, 20)$, the resulting Higgsino masses are $(m_{\chi_1^0}, m_{\chi^\pm}, m_{\chi_2^0}) = (192 \text{ GeV}, 197 \text{ GeV}, 204 \text{ GeV})$.

The total branching ratios for top squark decay into the neutral channel $\tilde{t}_i \rightarrow t\chi_{1,2}^0$ and the charged channel $\tilde{t}_i \rightarrow b\chi_1^\pm$ depend only on minimal supersymmetric standard model Yukawa couplings and phase space. Provided the mass splitting between top squarks and Higgsinos is large enough that the top-Higgsino mode is open, the top squark branching ratios into charged and neutral Higgsino modes are comparable in magnitude over the large majority of parameter space [3,4]. This leads to a sizeable pairwise branching ratio into the *mixed* final state $\tilde{t}\tilde{t}^* \rightarrow tb\chi^0\chi^\pm$. Figure 1 shows top squark pair branching ratios to

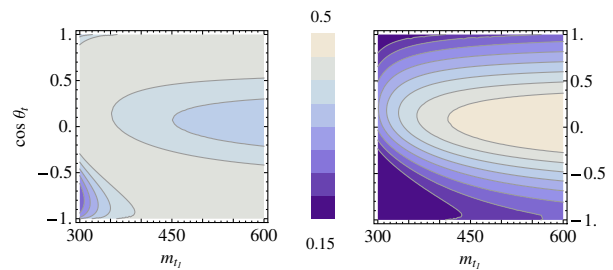


FIG. 1 (color online). Top squark pair branching fractions into (left) the mixed mode $t + \chi^0, b + \chi^\pm$; (right) $\tilde{t}\tilde{t} + 2\chi^0$, for $\tan\beta = 20$ and degenerate Higgsinos with mass $\mu = 100$ GeV.

both the mixed channel and the usual search channel $t\bar{t} + 2\chi^0$ in the $m_{\tilde{t}}\text{-cos}\theta_t$ plane, in the limit $m_{\chi^0} = m_{\chi^\pm}$. The total $\sigma \times \text{BR}$ into the $t\bar{t} + \cancel{E}_T$ final state can easily be suppressed by a factor of 2, whereas the $\sigma \times \text{BR}$ into the mixed mode is often comparable or larger. The bottom-squark-like signal $\tilde{t}^* \rightarrow b\bar{b} + 2\chi^\pm$ makes up a distant third, except in regions where the neutral decay channel is suppressed by available phase space.

Assuming the gluino is inaccessible, the most stringent collider bounds on this minimal natural SUSY spectrum are the Large Electron-Positron Collider chargino bounds [5]. We assume for simplicity that the nondegenerate Large Electron-Positron Collider bound $m_{\chi^\pm} > 103.5$ GeV applies, though for sufficiently small splittings between the chargino and the (N)LSP the limits could be mildly relaxed. Limits from direct top squark searches [6,7] have some reach in the region of interest ($m_{\chi^0} > 100$ GeV, $m_{\tilde{t}} > m_{\chi^0} + m_t$) but still leave most of the region unconstrained, particularly when the signal is reduced by a branching ratio $\text{BR}(\tilde{t}\tilde{t}^* \rightarrow t\bar{t} + \cancel{E}_T) \sim 0.5$. In principle, the CMS razor analysis [8] could also place interesting limits if analyzed subject to this signal hypothesis.

In what follows, we use a reference Higgsino sector where the mass splitting is taken to be $(m_{\chi_1^0}, m_{\chi^\pm}, m_{\chi_2^0}) = (m_{\chi_1^0}, m_{\chi_1^0} + 6 \text{ GeV}, m_{\chi_1^0} + 12 \text{ GeV})$. This splitting is large enough to reflect the possible influence of nearby gauginos but small enough that all Higgsinos produced in the decay of 300–600 GeV top squarks appear primarily as missing energy: the additional soft daughters produced in the decay of heavier Higgsinos down to the (N)LSP χ_1^0 are not boosted enough to pass selection cuts for hard isolated objects. We use a reference top squark pair branching fraction of $\text{BR}(\tilde{t}\tilde{t}^* \rightarrow tb + \cancel{E}_T + X) = 0.5$ and show results for a reference signal point with $m_{\tilde{t}} = 500$ GeV, $m_{\chi_1^0} = 200$ GeV, $\chi_1^0 = \tilde{h}_u^0$ unless otherwise specified. In all signal points, we take $\tilde{t}_1 = \text{cos}\theta_t \tilde{t}_R + \text{sin}\theta_t \tilde{t}_L$ with mixing angle $\text{cos}\theta_t = 0.833$.

We have checked that the modes $\tilde{t} \rightarrow t\chi_1^0$ and $\tilde{t} \rightarrow t\chi_2^0 \rightarrow t\chi_1^0 + X$ pass our selection cuts with efficiencies differing by $\mathcal{O}(10\%)$, with the single largest difference in efficiency coming from isolated lepton acceptance. Thus, to good approximation the asymmetric signal is insensitive to the details of the neutralino mixings. We have chosen a relatively large mass splitting for our reference Higgsino sector; in more degenerate spectra, the difference between χ_1^0 and χ_2^0 would be negligibly small.

Topology.—The dominant backgrounds to semileptonic top squark searches are dileptonic $t\bar{t}$ where one of the leptons is either too soft or too forward to be identified and $t\bar{t}$ events with one lepton and one unidentified τ . Although three of the six partons in these final states are missing or unidentified, these backgrounds still contain a large amount of kinematic information that can be used to identify events consistent with top quark pair production.

Much literature has been devoted to kinematic variables that can identify particle masses in the presence of multiple invisible particles. Two of the most studied variables are the transverse mass M_{T2} [9] and the contranverse mass M_{CT} [10]. Both of these variables admit straightforward extensions to the asymmetric decay chains that appear in top backgrounds with missing leptons, as was studied for M_{T2} in Ref. [11].

We propose here a novel alternative. Dileptonic top events are reconstructible when both leptons are identified: the mass-shell conditions provide enough constraints to completely solve for the unmeasured components of the neutrino momenta, up to discrete combinatoric and quadratic ambiguities. Once one of the leptons is lost, this is no longer true: the missing particles are (by assumption) now a neutrino and a W , and one of the mass-shell conditions is lost along with the lepton, leaving an underconstrained system.

We replace the missing mass-shell condition with the condition that *the reconstructed center-of-mass energy of the event be minimized*. As the parton distribution functions fall off steeply with \sqrt{s} , this provides a good approximation to the true event kinematics. We construct a function S that quantifies how well an event can be reconstructed subject to the dileptonic top hypothesis

$$S(p_{Wx}, p_{Wy}, p_{Wz}, p_{vz}) = \frac{(m_W^2 - p_W^2)^2}{a_W^4} + \frac{[m_t^2 - (p_{b_1} + p_\ell + p_\nu)^2]^2}{a_t^4} + \frac{[m_t^2 - (p_{b_2} + p_W)^2]^2}{a_t^4} + \frac{[4m_t^2 - (\sum_i p_i)^2]^2}{a_{\text{CM}}^4}, \quad (1)$$

where in the last term the sum runs over all five assumed final state particles. We have imposed transverse momentum conservation as well as the mass-shell conditions $p_\nu^2 = 0$, $p_W^2 = m_W^2$ to fix E_W , E_ν , $p_{\nu x}$, and $p_{\nu y}$ in terms of the four remaining undetermined variables. The denominators a_k determine the relative weighting of the mass-shell conditions and should not be smaller than typical resolutions; we take $a_W = 5$ GeV, $a_t = 15$ GeV, and $a_{\text{CM}} = 1$ TeV. The value of S at its minimum quantifies how well an event can be reconstructed according to the dileptonic top pair hypothesis. The inputs to S are two jets, a lepton, and the \vec{p}_T . To find the best possible reconstruction, we sum over both possible pairings of jets with reconstructed W bosons and keep the pairing that minimizes $\min S$. When the event contains two identified b jets, we use them as input to S ; when the event contains only one identified b , we consider the two hardest untagged jets with $|\eta| < 2.5$ and $p_T > 20$ GeV and use the pair (b, j) , which yields the minimum value for $\min S$. We define *topness* as

$$t = \ln(\min S). \quad (2)$$

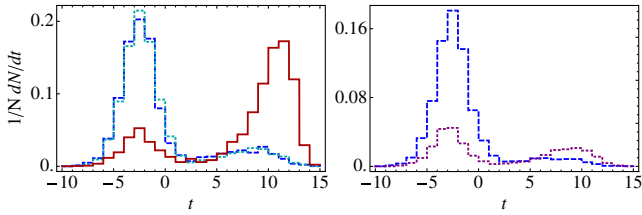


FIG. 2 (color online). Left: unit-normalized topness distributions for events passing preselection cuts as described in the text (signal, red solid line; dileptonic top, blue dashed line; one ℓ , one τ top, cyan dotted line). Right: topness distributions for the dileptonic background broken down into samples with two truth b jets (blue dashed line) and one truth b jet (purple dotted line).

Minimization of S is a nontrivial computational problem. In our implementation, we use 10 iterations of the Nelder-Mead algorithm per event. In general, this is not sufficient to find the global minimum; however, it will find a minimum that is sufficiently close to the global minimum that cuts and distributions are insensitive to any difference. We show distributions of topness for the major dileptonic and one- ℓ -one- τ top backgrounds as well as signal in Fig. 2. In the left panel of Fig. 3, we compare the performance of topness to both the asymmetric implementation of M_{CT} , which we find most effective, and M_{T2}^W , the M_{T2} variant identified as most effective in Ref. [11]. The events shown here have passed preselection cuts as described in the text below.

Our present interest is in the asymmetric top squark decay mode, but we emphasize that topness is useful in any search where the background is dominated by dileptonic tops with a missed lepton, most notably top squark searches in other channels. The right panel of Fig. 3 shows the relative performance of topness on the traditional top squark signal $\tilde{t}\tilde{t}^* \rightarrow t\bar{t} + 2\chi_1^0$ followed by semileptonic top decay, with backgrounds given by missed-lepton $t\bar{t} + 2j$ events. Events shown here have passed object selection cuts analogous to those in Ref. [12], including the requirement that $m_T(\ell, \cancel{E}_T) > 150$ GeV. Topness performs

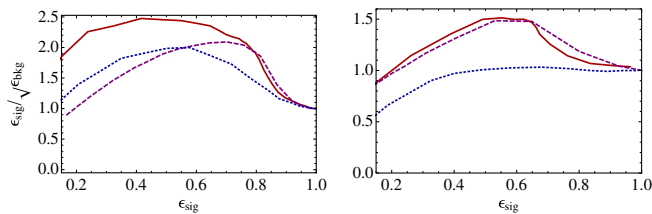


FIG. 3 (color online). Performance comparison of topness (red solid lines) to the variables M_{CT} (blue dotted lines) and M_{T2}^W (purple dashed lines) for the asymmetric signal (left) and the symmetric signal $t\bar{t} + 2\chi_1^0$ (right). The quantity plotted is the gain in signal significance, assuming Gaussian statistics, as a function of signal efficiency. Results are shown for $m_{\tilde{t}} = 500$ GeV and $m_{\chi_1^0} = 200$ GeV in both channels.

comparably to M_{T2}^W in the symmetric search channel, realizing (by a slim margin) the largest gain in significance among all three variables. The dilution in the efficacy of topness in the $t\bar{t} + 2\chi_1^0$ channel, relative to the $tb + \cancel{E}_T$ channel, is because the extra jets in the events give more possibilities for signal to accidentally reconstruct as a top-like event. We do comment, however, that our implementation of topness was not optimized for the $t\bar{t} + \cancel{E}_T$ signal.

Hunting asymmetric top squarks.—In this section, we discuss a search strategy for top squark pair production in the mode $\tilde{t}\tilde{t} \rightarrow tb + \text{MET}$ (MET = missing transverse energy). We target semileptonic top decay, so the final state of interest contains one ℓ , two b jets, and missing energy. We impose the following cuts at *preselection*: exactly one lepton satisfying $p_\mu > 20$ GeV, $p_e > 25$ GeV, $|\eta_\ell| < 2.5$; $\cancel{E}_T > 200$ GeV, with $m_T(\ell, \vec{p}_T) > 150$ GeV; and at least two jets with $p_T > 20$ GeV and $|\eta| < 2.5$, at least one of which must be b tagged. The cut on m_T suppresses all backgrounds where the \cancel{E}_T arises from a single W , in particular semileptonic $t\bar{t}$ and the enormous $W + \text{jets}$, which is further suppressed by the b -tag requirement. The major remaining background is therefore dileptonic top pair events where one of the leptons is not identified, either because it falls outside acceptance or because it is a nonidentified τ . A secondary background is the associated production of a top with a W boson, again with doubly dileptonic decays and a missed lepton. All major SM backgrounds can be reduced by identifying softer leptons in the event; we thus reject events containing identified hadronic taus with $|\eta| < 2.5$ and $p_T > 20$ GeV or additional (isolated) leptons with $|\eta| < 2.5$ and $p_T > 15$ GeV. Importantly, the additional soft decay products of the heavier Higgsinos in signal events have negligible impact on the ability of signal to pass the veto. More aggressive vetos, as in Ref. [6], would further reduce the backgrounds at little cost to signal.

In addition to cuts on the hardness of final state particles and \cancel{E}_T , we add three novel variables that discriminate signal and background. First and by far the most important is topness, discussed in the previous section. Another useful variable can be constructed by exploiting the asymmetric signal kinematics. The b jet coming from the decay

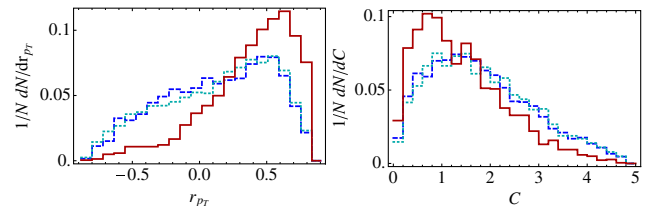


FIG. 4 (color online). Unit-normalized distributions of the variables r_{pr} (left) and C (right), for events passing preselection cuts as described in the text. The signal is a red solid line; the dileptonic top is a blue dashed line; the one ℓ , one τ top is a cyan dotted line.

TABLE I. Cut flow for the reference signal point, with $m_{\tilde{t}} = 500$ GeV, $m_{\chi} = 200$ GeV, and assuming a 50% branching ratio into the $t\bar{b} + \cancel{E}_T$ final state. All cross sections are measured in fb, and significances are shown for $\mathcal{L} = 20 \text{ fb}^{-1}$ at 8 TeV.

	σ_{sig}	$\sigma_{t\bar{t}}$	σ_{tW}	S/B	σ
Preselection	2.1	57	5.3	0.034	1.2
Lepton veto	2.1	43	3.9	0.045	1.4
$b_1 p_T > 125$	1.5	21	1.6	0.065	1.4
$r_{p_T} > -0.1$	1.4	20	1.5	0.067	1.4
$C < 4.0$	1.4	19	1.4	0.069	1.4
$t > 9.0$	0.89	0.62	0.37	0.90	3.5

$\tilde{t} \rightarrow b\chi^+$ is typically much harder than the daughters of the top. The p_T asymmetry

$$r_{p_T} = \frac{p_{Tb_1} - p_{T\ell}}{p_{Tb_1} + p_{T\ell}} \quad (3)$$

of the lepton and the highest p_T b jet is thus useful for distinguishing signal and background. We also employ a *centrality* variable,

$$C = \max(|\Delta\eta_{j_1, j_2 + \ell + \vec{p}_T}|, |\Delta\eta_{j_2, j_1 + \ell + \vec{p}_T}|), \quad (4)$$

formed from the two highest p_T jets j_1 and j_2 in the event as well as the lepton and missing momentum. Centrality is typically larger for backgrounds than for signal, reflecting both the larger signal masses and the different kinematics of top versus top squark pair production [13]. Distributions of r_{p_T} and C are shown in Fig. 4. This particular cut on rapidity separations is most useful when used in conjunction with topness, because background events with large topness have often selected an ISR jet in place of one of the b jets, and this ISR jet is distinct in rapidity from the other objects in the event.

We normalize signal [14] and background $t\bar{t}$ [15] and tW [16] processes to inclusive NLO + N(N)LL 8 TeV cross sections. Events are generated in MADGRAPH [17], showered in PYTHIA [18], and clustered in FASTJET using the anti- k_T algorithm with $R = 0.4$ [19]. In generating $tW + 1j$ events, we forbid $t\bar{t}$ events from contributing when the momentum in one of the internal top propagators lies in the

TABLE II. Signal significances and best cuts for $\mathcal{L} = 20 \text{ fb}^{-1}$ at 8 TeV. All masses and energies are in GeV. For the signal point ($m_{\tilde{t}} = 400$, $m_{\chi_1^0} = 100$), the quoted significance is Gaussian.

$m_{\tilde{t}}$	$m_{\chi_1^0}$	MET	$b_1 p_T$	C	r_{p_T}	Topness	σ	S/B	σ_{sig} (fb)
400	100	200	125	3.5	-0.1	8.0	11	2.2	2.7
400	200	150	50	3.5	-0.1	7.5	2.7	0.31	1.4
500	100	200	150	4.0	-0.1	9.0	4.9	1.6	1.1
500	200	200	125	4.0	-0.1	9.0	3.5	0.90	0.89
600	100	300	150	3.5	-0.3	10.5	2.9	2.2	0.32
600	200	250	175	4.0	-0.1	10.0	2.3	1.2	0.30

window $|p^2 - m_i| < 15\Gamma_i$ [20,21]. Leptons are declared isolated if the scalar sum p_T deposited in a cone of radius $R_{\text{iso}} = 0.2$ around a lepton is less than $r_{\text{iso}} = 0.2$ times the lepton p_T . The isolation threshold is thus 4 GeV for a lepton with $p_T = 20$ GeV, which matches well with the experimentally employed criteria at threshold. We consider a hadronic τ to be identifiable when the scalar sum p_T deposited in a cone of radius $R_{\text{iso}} = 0.4$ around the visible hadronic τ is less than $r_{\text{iso}} = 0.3$ times the τp_T . We then apply a probabilistic tagging algorithm to identifiable τ 's to arrive at a 50% overall efficiency for identifying hadronic τ 's with $p_T = 20$ GeV coming from the W in $t\bar{t}$ events, comparable to experimental working points. Finally, we assume a uniform probability $\mathcal{P}_b = 0.7$ to tag a b jet with $p_T > 20$ GeV and $|\eta| < 2.5$. We then generate more than enough events to ensure our final results are insensitive to statistical fluctuations in our Monte Carlo simulations.

In Table I, we show the detailed cut flow for our reference signal working point as further specialized cuts are imposed. As the total number of expected signal and background events is small, we use Poisson statistics to evaluate signal significance. Table II shows optimal cuts for other signal mass points. We find in general that the region of best significance is relatively flat as a function of the cuts; that is, a broad range of cut combinations yield similar final significances.

To conclude, top squarks in natural SUSY spectra will generically have large branching fractions to the mixed final state $\tilde{t}\tilde{t}^* \rightarrow t\bar{b} + \cancel{E}_T + \text{soft}$, substantially diluting the rate into the traditional search channel $\tilde{t}\tilde{t} \rightarrow t\bar{t} + \cancel{E}_T$. Despite the large $t\bar{t}$ backgrounds in the $t\bar{b} + \cancel{E}_T$ final state, good search reach is achievable with the use of the novel variables introduced here, recovering LHC sensitivity to the natural region of parameter space. The novel variable topness is critical to this search and is useful in more generic contexts, matching or outperforming such kinematic variables as M_{T2}^W and M_{CT} .

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