Sheath-Induced Instabilities in Plasmas with $\mathbf{E}_0 \times \mathbf{B}_0$ Drift

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It is shown that ion acoustic waves in plasmas with $\mathbf{E}_0 \times \mathbf{B}_0$ electron drift become unstable due to the closure of plasma current in the chamber wall. Such unstable modes may enhance both near-wall conductivity and turbulent electron transport in plasma devices with $\mathbf{E}_0 \times \mathbf{B}_0$ electron drift and unmagnetized ions. It is shown that the instability is sensitive to the wall material: a high value of the dielectric permittivity (such as in metal walls) reduces the mode growth rate by an order of magnitude but does not eliminate the instability completely.

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Introduction.—It has long been noted that collisional transport of magnetically confined plasma is strongly affected by the closure of the parallel (along the magnetic field) electron current in the chamber walls. This is the so-called Simon short circuit effect [1]; for recent work on this subject, see Refs. [2,3] and references therein. In this work we show that plasmas with $\mathbf{E}_0 \times \mathbf{B}_0$ electron drift and unmagnetized ions can be destabilized by the electron current admitted into the wall. The wall current, self-consistently determined from sheath boundary conditions, provides the positive feedback mechanism that renders the instability.

Devices with a stationary, externally applied, electric field $\mathbf{E}_0$, which is perpendicular to a moderate amplitude magnetic field $\mathbf{B}_0$, are common in magnetically controlled plasmas [4,5]. The electric field produces a stationary current due to the $\mathbf{E}_0 \times \mathbf{B}_0$ electron drift, while ions do not feel the magnetic field due to their large Larmor radius. High interest applications involve Penning-type plasma sources [6,7], magnetrons and magnetic filters [8,9], and electric space propulsion [4]. For typical parameters, classical (collisional) transport is too low to explain the experimental data on plasma current and heating in these devices, so that plasma instabilities [10] are suspected as a reason for anomalous electron transport [11,12]. The exact nature of the fluctuations responsible for this anomalous behavior, however, remains unknown. Significant experimental evidence [13] suggests that the electron cross-field transport is affected by the properties of the wall material (e.g., metal vs dielectric wall). This fact points to the near-wall conductivity due to electron collisions with the walls, which is sensitive to the sheath structure [14], as a possible mechanism responsible for the anomalous transport. On other hand, various unstable modes that may lead to the turbulent transport in bulk plasma have been identified in the past [15–20]. Some experimental data and numerical modeling suggest [11] that a combination of both bulk turbulence mobility and near-wall conductivity is required to satisfactorily describe the observed experimental behavior. The physical mechanism proposed in this Letter, which is based on the coupling of $\mathbf{E}_0 \times \mathbf{B}_0$ driven sound waves in bulk plasma with sheath fluctuations at the dielectric boundary, will result in fluctuations affecting both the turbulent mobility of bulk electrons and the near-wall conductivity.

Collisional destabilization of ion sound waves in plasmas with $\mathbf{E}_0 \times \mathbf{B}$ *electron drift.*—To underly the physical mechanism of the discussed instability, we first consider the simple case of ion sound modes in an infinite plasma with collisions, applied magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, and external electric field $\mathbf{E}_0 = E_0\,\hat{\mathbf{x}}$.

In absence of collisions, the electrons have large mobility along the magnetic in the $\hat{\mathbf{z}}$ direction and electron inertia can be neglected in the limit $\omega < k_z v_{Te}$, $v_{Te} = \sqrt{2T_e/m_e}$. This results in the Boltzmann distribution for the perturbed electron density

$$\tilde{n}_e = \frac{e\tilde{\phi}}{T_e} n_0, \tag{1}$$

where n_0 is the equilibrium plasma density. It is important to note that the electron density is not affected by the Doppler shift due to the $\mathbf{E}_0 \times \mathbf{B}_0$ drift. Unmagnetized cold ions with $T_i = 0$ have a local ion density response in the form

$$\tilde{n}_i = \frac{(k_y^2 + k_z^2)c_s^2}{\omega^2} \frac{e\tilde{\phi}}{T_e} n_0, \tag{2}$$

where $c_s = \sqrt{T_e/m_i}$ is the ion sound velocity, k_z and k_y are the wave vector components along the applied magnetic field and in the perpendicular direction, respectively. The quasineutrality condition, along with Eqs. (1) and (2), results in the dispersion relation for the ion sound waves

$$\omega^2 = (k_z^2 + k_y^2)c_s^2, \tag{3}$$

which is not affected by the magnetic field nor by the Doppler frequency shift (ions are not magnetized). It is assumed that $k_x = 0$.

In this model, the parallel electron current along the magnetic field does not appear explicitly, although obviously such current is finite. A silent feature of the electron parallel current is the Doppler phase shift with respect to the electron density perturbations

$$\tilde{J}_{\parallel e} = e(\omega - \omega_0)\tilde{n}_e/k_z. \tag{4}$$

The phase shift is due to the electron drift $\mathbf{v}_0 = c\mathbf{E}_0 \times \mathbf{B}_0/B_0^2$, $\omega_0 = \mathbf{k}\cdot\mathbf{v}_0$. In an infinite (along z) plasma and in the absence of collisions, the parallel electron current is not constrained and thus the electron flow $\mathbf{v}_0$ does not affect the wave dynamics. The situation changes when the feedback occurs between the parallel current and the electron density (and the electric potential fluctuations). The simplest feedback is created by electron collisions, which couple the Doppler shifted electron current to the potential and density fluctuations. The electron parallel momentum balance relates the parallel current to the potential and density fluctuations as follows:

$$k_z\left(\tilde{\phi} - \frac{T_e}{e}\frac{\tilde{n}_e}{n_0}\right) + \frac{m_e\nu_e}{e^2 n_0}\tilde{J}_{\parallel e} = 0. \tag{5}$$

The perturbed electron density, which is found from Eq. (5) and the electron continuity equation, now has a finite phase shift with respect to the potential fluctuation

$$\frac{\tilde{n}_e}{n_0}\left[1 - \frac{(\omega - \omega_0)\nu_e}{k_z^2 v_{Te}^2/2}\right] = \frac{e\tilde{\phi}}{T_e}. \tag{6}$$

Using the modified electron density response in Eq. (6), along with Eq. (2) and the quasineutrality condition, the dispersion relation for unstable ion sound waves is obtained in the form

$$\omega^2 = k^2 c_s^2 - i\frac{k^2 c_s^2}{k_z^2 v_{Te}^2}\nu_e(\omega - \omega_0), \tag{7}$$

where $k^2 = k_z^2 + k_y^2$. The instability growth rate from Eq. (7) is then $\gamma \simeq -kc_s\nu_e(\omega - \omega_0)/(2k_z^2 v_{Te}^2)$. The instability occurs for fluctuations moving with a phase velocity lower than the equilibrium electron flow, $\omega < \omega_0$. Fluctuations with $\omega < \omega_0$ have negative energy and are destabilized by dissipation due to collisions [21]. A similar mechanism is responsible for the enhancement of the plasma-beam instability due to collisions [22]. In a dynamical picture, the instability is due to the component of the parallel current that is directed into the regions of positive potential perturbation, and then reinforces the initial perturbation, as illustrated in Fig. 1(b). This current is created via a combination of collisions and the Doppler phase shift.

Boundary conditions and sheath impedance.—The above mechanism illustrates the role of the positive feedback loop between the parallel current and the density and potential fluctuations. In a finite length plasma, like the one shown in Fig. 1(a), a similar feedback is created by the

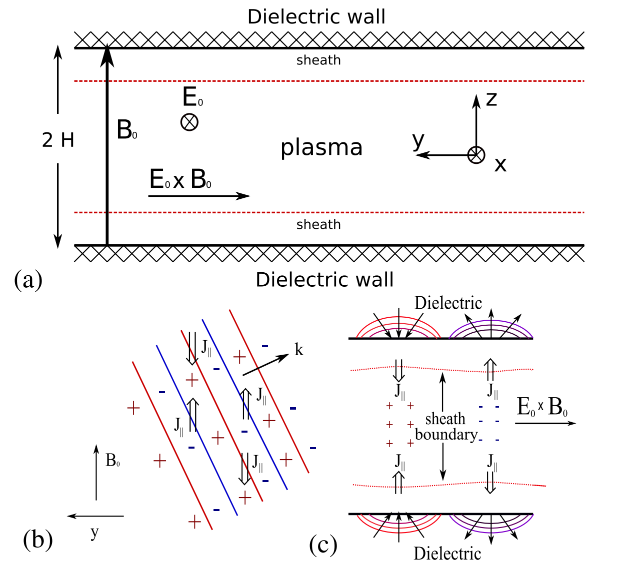

FIG. 1 (color online). (a) Geometry of the plasma bound by the dielectric walls; (b) Perturbed parallel current in infinite plasma. The wave fronts are oblique with respect to the magnetic field, $\mathbf{k} = (k_y, k_z)$; (c) Perturbed current in bound plasma. In (b) and (c) the instability is driven by the component of the perturbed parallel current that is directed into the regions of positive charge (shown with +), thus enhancing the initial perturbation.

sheath boundary conditions, which couple the parallel current with the potential and density fluctuations. We consider a plasma between two symmetric material walls. The equilibrium magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ is normal to the wall, and the y axis is in the direction parallel to the wall. The equilibrium electric field $\mathbf{E}_0 = E_0\,\hat{\mathbf{x}}$ is in the axial direction, so that the electrons drift along the azimuthal direction y.

The plasma parallel current is closed by the current into the sheath, Fig. 1(b), so that $J_\parallel$ has to be matched to the sheath current in each component (electron and ion). The sheath current is determined by the perturbation of the standard ion Bohm current $J_{i0} = en_0c_s$ and the current of the electrons in the tail of the distribution function with energies above the potential drop across the sheath: $J_{e0} = n_0 v_{Te}\exp[-e(\phi_p - \phi_w)/T_e]/(2\sqrt{\pi})$, where ϕ_p is the plasma potential at the plasma-sheath edge, ϕ_p is the wall potential, and $v_{Te}^2 = 2T_{e0}/m_e$. In the stationary state, the total current is zero: $J_{0e} + J_{0i} = 0$. For perturbations, the current is finite. The perturbed ion current in the sheath is related to density perturbation, $\tilde{J}_{i\parallel} = e\tilde{n}c_s$; the electron current is $\tilde{J}_{e\parallel} = -e\tilde{n}c_s + \tilde{J}_{\rm sh}$, and $\tilde{J}_{\rm sh}$ is the total sheath current in the perturbed state

$$\tilde{J}_{\rm sh} = \frac{e^2 n_0 c_s}{T_e}(\tilde{\phi}_p - \tilde{\phi}_w). \tag{8}$$

For an ideal metal wall, $\tilde{\phi}_w = 0$, and the total sheath current appears as a dissipation for the bulk plasma. This was called sheath resistivity [23–26]. For the dielectric wall, $\tilde{\phi}_w \neq 0$. The dielectric wall boundary conditions can be derived similarly to Refs. [24,27]. From current conservation, it follows that the plasma conductive current at the sheath edge is equal to the displacement current in the dielectric $\tilde{J}_d = (4\pi)^{-1}\varepsilon\partial E_{zD}/\partial t$, where E_{zD} is the electric field at the dielectric boundary (inside), ε is the dielectric constant. The displacement current is closed inside the dielectric. From the Laplace equation, $\nabla^2\phi = 0$, one finds for the potential in the thick dielectric: $\phi = \phi_w \exp(ik_y y)\exp(-|k_y|z)$, and $E_{zD} = |k_y|\phi_w \exp(ik_y y)$, where ϕ_w is the potential at the wall. Equation (8) together with the condition $\tilde{J}_{\rm sh} = \tilde{J}_d$ determine the amplitude of the potential fluctuations at the wall

$$\tilde{\phi}_w = \tilde{\phi}_p \frac{1}{1 - i\varepsilon\omega|k_y|c_s/\omega_{pi}^2}. \tag{9}$$

The total current into the sheath can finally be written in terms of the amplitude of the plasma potential fluctuations

$$\tilde{J}_{\rm sh} = -\frac{e^2 n_0 c_s}{T_e}\tilde{\phi}_p \frac{iK}{1 - iK}. \tag{10}$$

In the case of the dielectric wall, the sheath response becomes reactive for low frequency perturbations, $K \equiv \varepsilon\omega|k_y|c_s/\omega_{pi}^2 < 1$; for $K \gg 1$ this boundary condition becomes equivalent to the case of a metal wall [24] with dissipative sheath response. The regime $K \gg 1$ occurs for high frequency short wavelength fluctuations, $k_y\lambda_D \geq 1$, or metal walls ($\varepsilon \to \infty$).

Sheath induced modes in the long-wavelength regime.—It is instructive to consider first the simpler case of long-wavelength modes with weak variations along the magnetic field (global modes), $H\partial/\partial z \ll 1$. In this limit, one can use averaged (along the magnetic field lines) equations with the sheath current as boundary conditions that couple plasma potential, current, and electron and ion densities. Together with the quasineutrality condition, these constraints result in a component of the parallel current that is directed into the regions of the positive charge, thus feeding the initial perturbation and leading to the instability, as schematically illustrated in Fig. 1(c).

Consider the magnetic field line tube between $z = -H$ and $z = H$ and introduce the averaged potential and density $\bar{n} \equiv (2H)^{-1}\int_{-H}^{H}\tilde{n}dz$, $\bar{\phi} \equiv (2H)^{-1}\int_{-H}^{H}\tilde{\phi}dz$. The integrated electron and ion continuity equations take the form $-i(\omega - \omega_0)\bar{n} - J_{\parallel e}(H)/(eH) = 0$, and $-i\omega\bar{n} + ien_0k_y^2\bar{\phi}/\omega m_i + J_{\parallel i}(H)/(eH) = 0$, where $J_{\parallel e}(H)$ and $J_{\parallel i}(H)$ are the electron and ion currents at the sheath boundary, described in the section titled "Boundary conditions and sheath impedance," and which depend on the density and potential fluctuations. Thus, the electron density response acquires a phase shift due to the Doppler frequency: $\bar{n}_e/n_0 = (e\bar{\phi}/T_e)\nu_{\rm sh}K/[(1 - iK)(\omega - \omega_0 + i\nu_{\rm sh})]$, where $\nu_{\rm sh} = c_s/2H$ [compare with Eq. (6)]. The ion response is insensitive to the electron $\mathbf{E}_0 \times \mathbf{B}_0$ drift: $\bar{n}_i/n_0 = (e\bar{\phi}/T_e)k_y^2c_s^2/[\omega(\omega + i\nu_{\rm sh})]$. Applying the quasineutrality condition results in the following dispersion relation (for $K < 1$)

$$\omega^2(\omega + i\nu_{\rm sh}) = \frac{|k_y|c_s\omega_{pi}^2}{\nu_{\rm sh}}(\omega - \omega_0 + i\nu_{\rm sh}), \tag{11}$$

which has an unstable root for $\omega < \omega_0$, $\gamma \simeq \omega_r \simeq (-\omega_0|k_y|c_s\omega_{pi}^2/\varepsilon\nu_{\rm sh})^{1/3}$. The unstable mode has the real part of the frequency smaller than ω_0, $\omega < \omega_0$, corresponding to the reactive instability of the negative energy mode [21]. For the metal wall, $K \gg 1$, in the limit of large $\omega_0 > \nu_{\rm sh}$, the instability becomes of the dissipative type, with the complex frequency given by $\omega = \pm(i\omega_0k_y^2c_s^2/\nu_{\rm sh})^{1/2}$.

Small scale modes.—In general, the sheath modes have an eigenmode structure that depends both on the z (along magnetic field) and on y (perpendicular) coordinates. A general solution for density and potential perturbations can be obtained as a sum of the mode with the z-dependent field structure, $\sim\phi_0\exp(-i\omega t + ik_y y + ik_z z)$, and the boundary induced mode which has only y dependence, $\sim\phi_b\exp(ik_y y)$,

$$n(z, y, t) = \frac{e}{T_e}n_0\left[\phi_0 e^{ik_z z} + \phi_b\frac{k_y^2c_s^2}{\omega^2}\right]e^{i(k_y y - \omega t)}, \tag{12}$$

$$\phi = [\phi_0 e^{ik_z z} + \phi_b]e^{i(k_y y - \omega t)}. \tag{13}$$

The frequency ω in these expressions has to satisfy the sound wave dispersion relation in Eq. (3), where k_y is a free parameter, defined by periodic boundary conditions, but k_z is an eigenvalue that has to be determined as a solvability condition for the system of the ion and electron continuity equations closed with the sheath currents and the quasineutrality condition. This solvability condition has the form

$$\begin{aligned}\frac{\tan(k_zH)}{k_zH}\Bigg[\omega_0 - \frac{k_y^2c_s^2}{\omega} - \frac{i\omega(\omega - \omega_0)}{\nu_{\rm sh}}\left(1 - \frac{k_y^2c_s^2}{\omega^2}\right) \\ + i\omega\left(\frac{\omega^2}{k_y^2c_s^2} - 1\right)\frac{K}{1 - iK}\Bigg] + \Bigg[\omega - \omega_0 + \frac{\nu_{\rm sh}}{1 - iK} \\ - \nu_{\rm sh}\frac{\omega^2}{k_y^2c_s^2}\frac{K}{1 - iK}\Bigg] = 0.\end{aligned} \tag{14}$$

Solving this last equation, together with the dispersion relation in Eq. (3), one finds the eigenvalues for ω.

In the long-wavelength limit, $k_zH \ll 1$, local variations in ϕ and n can be neglected, $\bar{\phi} = \phi(H)$ and $\bar{n} = n(H)$, and one then recovers the dispersion relation in Eq. (11).

The growth rate and real part of the frequency of the unstable modes in Eqs. (11) and (14) are shown in Fig. 2(a) as a function of ω_0, for $T_e = 10$ eV and xenon plasma, length scale $H = 1$ cm, and the lowest wave vector $k_y = 0.2$ rad/cm. The real part of the frequency is negative, which corresponds to a rotation in the direction of the

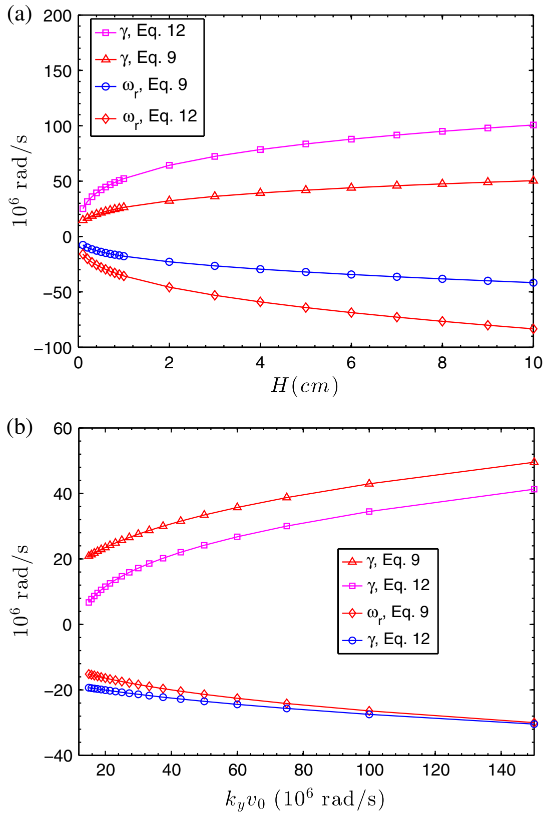

FIG. 2 (color online). Growth rate and real part of the frequency for the global and local modes are shown: (a) as a function of the length H, $k_y = 0.2\ \mathrm{cm}^{-1}$; (b) as a function of the absolute value of $k_y v_0$, $H = 1$ cm.

$\mathbf{E} \times \mathbf{B}$ drift. Figure 2(b) shows the convergence of the eigenvalues for the small scale modes to the long-wavelength limit of Eq (11). Figure 3, where the growth rate and frequencies are plotted as a function of ε, demonstrates that with an increase in ε, both the frequency (whose absolute value is shown) and the growth rates decrease, reaching asymptotic values that correspond to the case of a metal wall ($\varepsilon \longrightarrow \infty$).

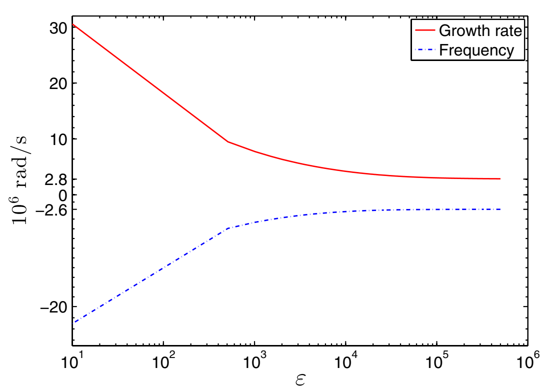

FIG. 3 (color online). Growth rate and real part of the frequency of the unstable modes as a function of ε, $k_y = 0.2\ \mathrm{cm}^{-1}$, $H = 1$ cm. The metal regime is $\varepsilon \longrightarrow \infty$.

Summary.—The $\mathbf{E}_0 \times \mathbf{B}_0$ flow of magnetized electrons is a powerful source of free energy in plasmas with unmagnetized ions. The electron perturbations with phase velocities below $\mathbf{v}_0 = c\mathbf{E}_0 \times \mathbf{B}_0/B_0^2$ reduce the total energy of the system resulting in dissipative and reactive instabilities of negative energy modes. We have derived the sheath current closure relations for the dielectric wall and have shown that the sheath current boundary conditions lead to the positive feedback that destabilizes the ion sound waves in plasmas with $\mathbf{E}_0 \times \mathbf{B}_0$ electron drift. It follows from Eqs. (11) and (14) that the frequency and growth rate of the unstable mode increase with electron temperature. The sheath boundary conditions derived in this Letter show that the unstable modes are sensitive to the nature of the wall material. A generic dielectric wall provides a reactive response, while the response is dissipative for the metal wall. Unstable modes are shown to exist in the form of coupled oscillations of the bulk plasma and the sheath. If present, such fluctuations would strongly influence both bulk plasma transport and near wall conductivity. Sheath oscillations have been observed in some simulations [28,29], although direct experimental detection of sheath fluctuations in Hall plasma devices is difficult. Experimental observations in Ref. [30] demonstrate the strong sensitivity of the amplitude of the fluctuations to the wall material. In particular, it was shown [30] that metal walls suppress the fluctuations. Our analytical results show a similar result, namely that the instability growth rates for the metal wall are reduced by an order of magnitude, which is generally consistent with experimental observations [30]. The saturation of streaming instabilities driven by $\mathbf{E}_0 \times \mathbf{B}_0$ flow is likely to lead to strong turbulence regimes. In the latter case, as a simple estimate of the effective collision frequency due to turbulent fluctuations, one can use the growth rate of the instability [31]. For typical plasma parameters [30], the unstable modes have $\gamma \simeq 10^8$ rad/s (see results in Fig. 3), which is a typical value used in empirical simulations of anomalous effects [32].

A number of previous studies dealt with instabilities of $\mathbf{E}_0 \times \mathbf{B}_0$ Hall plasmas, caused by the equilibrium electron flow in combination with density and magnetic field gradients [15,20,33], resistive effects [18,19], and electron cyclotron resonances [16]. These instabilities may also be affected by sheath closure effects.

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