Quantum Information Processing with Hybrid Spin-Photon Qubit Encoding

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We introduce a scheme to perform quantum information processing that is based on a hybrid spin-photon qubit encoding. The proposed qubits consist of spin ensembles coherently coupled to microwave photons in coplanar waveguide resonators. The quantum gates are performed solely by shifting the resonance frequencies of the resonators on a nanosecond time scale. An additional cavity containing a Cooper-pair box is exploited as an auxiliary degree of freedom to implement two-qubit gates. The generality of the scheme allows its potential implementation with a wide class of spin systems.

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A classical computer is made of a variety of physical components specialized for different tasks. In the same way, a quantum computer will probably be a hybrid device exploiting the best characteristics of distinct physical systems. In this spirit, much work has recently been done to achieve strong coupling of high-quality factor coplanar-waveguide resonators with superconducting qubits, such as Cooper-pair boxes (CPBs) [1–3] and transmons [4] and/or spin ensembles (SEs) [5,6]. Superconducting qubits coupled to a microwave cavity field were proposed for quantum information processing (QIP) [7–9], using classical fields [7] or external voltages [10] as a manipulation tool. In recent years several theoretical works have considered the possibility of joining the fast processing of superconducting qubits to the long coherence times of SEs [11–15], which can naturally be exploited as quantum memories. Cavity photons can be used as a bus to transfer the quantum state from CPBs to spin ensembles, and to couple distant CPB qubits, leading to an effective interaction necessary to perform two-qubit gates [16]. Recently, it has been theoretically shown that a minimal architecture solely based on SEs can be exploited for full QIP [17], by employing a measurement-based scheme in which photons are still used as a quantum bus.

Here we introduce a qualitatively different approach based on a hybrid spin-photon encoding of the qubits. Our scheme differs from previous ones because here the spin and photon degrees of freedom enter on an equal footing in the definition of the qubit. This allows us to perform all the manipulations simply by tuning the resonance frequencies of microstrip line superconducting resonators (Fig. 1). Experimentally, the resonator frequencies have already been shown to be variable on a nanosecond time scale [18], and up to tenths of the fundamental-mode frequency ω_c^0 [19]. In addition, they can be assembled in a series of lumped elements, realizing large arrays of different geometries [20]. Such capability, along with the possibility of individually addressing the resonators, represents a prerequisite for scalability. As to the spins, the few requisites of the scheme can be fulfilled by a large variety of systems, ranging from diluted transition-metal or rare-earth ions to molecular nanomagnets [21]. In our hybrid encoding, each physical qubit is represented by a resonator mode and a SE. We describe the quantum gates in the elementary unit of a scalable setup, namely, two resonators containing different SEs, where each qubit is encoded in the state of a distinct SE-resonator pair. These are connected by an interposed cavity containing a CPB, which is exploited as an auxiliary degree of freedom to implement two-qubits gates.

Definition of the qubit.—We consider a resonant cavity containing a single photon in a mode of frequency ω_c, and an ensemble of N identical and noninteracting spins 1/2, initially prepared in the ground state $|\psi_0\rangle = |0\ldots0\rangle$. If the resonator mode is tuned to the gap of the two-level system ω_1, after some time the SE will collectively

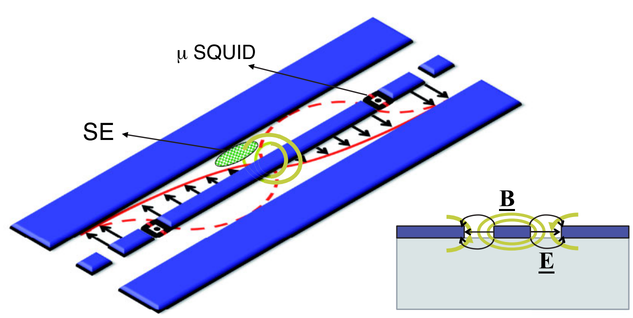

FIG. 1 (color online). Sketch of a tunable coplanar superconducting resonator in a microstrip line. The effective resonator length can be tuned by inductively coupled μ SQUIDs. The fundamental (third) harmonic of the resonator is schematically shown with a solid (dashed) line. The cavity-SE coupling is maximized at the magnetic field antinodes (rotational lines). Inset: cross section of the striplines on an insulating substrate.

absorb the photon and evolve into the state $|\psi_1\rangle = (1/\sqrt{N})\sum_{q=1}^{N}|0_1\ldots 1_q\ldots 0_N\rangle$. Transitions between $|\psi_0\rangle$ and $|\psi_1\rangle$ are described by $\hat{b}_1 = (1/\sqrt{N})\sum_{q=1,N}|0\rangle\langle 1|_q$ and $\hat{b}_1^\dagger$ [5,14]. In the low-excitation regime, the collective excitations of the SE can be described as a harmonic oscillator and $[\hat{b}_1, \hat{b}_1^\dagger] = 1$.

Within the single-excitation subspace of the system formed by a resonator mode and a SE, we introduce the hybrid encoding of the qubit μ:

$$|0\rangle_\mu \equiv |\psi_1^\mu, n_\mu = 0\rangle, \qquad |1\rangle_\mu \equiv |\psi_0^\mu, n_\mu = 1\rangle, \quad (1)$$

where n_μ is the photon occupation number of the cavity mode coupled to the SE. Thus, the logical state of the qubit depends on whether the excitation is stored within the SE or the quantized field of the resonator.

Description of the scalable setup.—To achieve universal QIP it is sufficient to perform a two-qubit quantum gate such as the controlled-Z (CZ), and arbitrary single-qubit rotations around two nonparallel axes [22]. Hereafter, we describe our scheme for the quantum-gate implementation in the basic unit of a scalable setup, i.e., a system of two qubits ($\mu = A, A'$), encoded in the hybrid states of two distinguishable SEs coupled to the modes of two different stripline resonators, as in Fig. 2(a). In its general form, the scheme can be implemented within any bipartite lattice of cavities [23]. Both SEs consist of effective $s = 1/2$ spins, but with different energy gaps: ω^A is coupled to the harmonic of frequency ω_c^A in cavity A, while $\omega^{A'}$ to the harmonic of frequency $\omega_c^{A'}$ in cavity A', as in the level scheme of Fig. 2(b). A third resonator B, which is not used to encode qubits, is located in between the cavities A and A'. It contains a CPB which we exploit to implement CZ. As we show in [24], for reasonable values of the Josephson and charge energies [1] the CPB is characterized by the anharmonic spectrum reported in the central part of Fig. 2(b) (hereafter we consider only the three lowest levels). In cavity B we consider two different harmonics, ω_c^B and $\omega_c^{B'}$, respectively close to the gaps ω^B and $\omega^{B'}$ of the CPB. In the idle configuration, the cavity modes $A - B$ and $B' - A'$ are detuned and qubits encoded in different cavities evolve independently from one another. Hence, single-qubit rotations can be implemented by varying the resonance frequency of the relevant resonator, while the CPB is left in its ground state and no photons are present in cavity B. The interaction between neighboring qubits is switched on when the two cavities A and A' are brought into resonance with B and photons can jump from cavities A and A' to B. As explained below, CZ gates can be obtained by exploiting a two-step Rabi oscillation of the CPB between its ground state $|\psi_0^B\rangle$ and the excited state $|\psi_2^B\rangle$ [Fig. 2(b)]. A three-level superconducting system was exploited for performing CZ gates in a different scheme in [25].

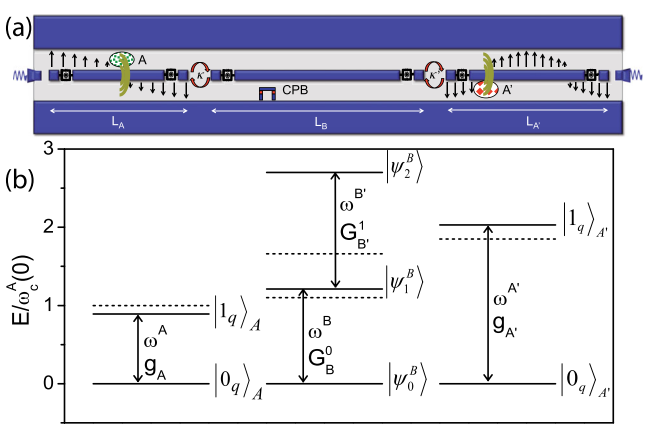

FIG. 2 (color online). (a) Representation of an elementary unit of the scalable setup. (b) Level diagram (solid line) of the two spin systems used to define the qubits and of the interposed CPB, which is used to implement CZ gates. The individual spin-photon coupling strengths are indicated as g_γ, corresponding to transition frequencies ω^γ between single-spin states $|0_q\rangle_\gamma$ and $|1_q\rangle_\gamma$ ($\gamma = A, A'$). The CPB-cavity couplings are indicated with capital letters. Dashed lines represent the frequencies of the cavity modes $\omega_c^\gamma(0)$.

The total Hamiltonian of the SEs + CPB coupled to the quantized cavity fields reads:

$$\hat{H} = \hat{H}_{\text{spin}} + \hat{H}_{\text{CPB}} + \hat{H}_{\text{ph}} + \hat{H}_{\text{int}} + \hat{H}_{\text{hop}}. \quad (2)$$

The first term describes the SEs, A and A', as independent harmonic oscillators [18] ($\hbar = 1$):

$$\hat{H}_{\text{spin}} = \omega^A \hat{b}_A^\dagger \hat{b}_A + \omega^{A'} \hat{b}_{A'}^\dagger \hat{b}_{A'}. \quad (3)$$

The time-dependent photonic Hamiltonian, which in our scheme is entirely responsible for the qubits manipulation, is

$$\hat{H}_{\text{ph}} = \sum_{\gamma=A,A',B,B'} \omega_c^\gamma(t)\hat{a}_\gamma^\dagger \hat{a}_\gamma, \quad (4)$$

where $\omega_c^\gamma(t) = \omega_c^\gamma(0) + \Delta_c^\gamma(t)$. Within the rotating-wave approximation, the spin-photon and CPB-photon couplings read:

$$\hat{H}_{\text{int}} = \sum_{\gamma=A,A'} \frac{\bar{G}_\gamma}{2}\hat{a}_\gamma^\dagger \hat{b}_\gamma + \sum_{\substack{\gamma=B,B' \\ j=0,1}} \frac{G_\gamma^j}{2}\hat{a}_\gamma^\dagger |\psi_j^B\rangle\langle\psi_{j+1}^B| + \text{H.c.} \quad (5)$$

The coupling constants $\bar{G}_\gamma$ are enhanced by a factor $\sqrt{N}$ with respect to their single-spin counterpart g_γ (see, e.g., Ref. [14]), while G_γ^j are referred to a single superconducting unit. Finally, the last term in Eq. (2) describes the photon hopping induced by the evanescent coupling between the modes $A - B$ and $A' - B'$ of the two neighboring cavities [26,27]:

$$\hat{H}_{\text{hop}} = -\kappa \hat{a}_A^\dagger \hat{a}_B - \kappa' \hat{a}_{A'}^\dagger \hat{a}_{B'} + \text{H.c.} \quad (6)$$

Other hopping terms can easily be made negligible by engineering the two different cavities. Hereafter, we will use the interaction picture, with $\hat{H}_0 = \hat{H}_{\text{spin}} + \hat{H}_{\text{CPB}} + \hat{H}_{\text{ph}}(t = 0)$.

Quantum gates.—In order to perform one- and two-qubit gates, we exploit the absorption (emission) of the photons entering the hybrid encoding [Eq. (1)]. These processes can be straightforwardly controlled by tuning the frequencies of the cavity modes by a quantity Δ_c^γ for suitable time intervals. We will refer to such variations of the resonator frequencies as *pulses*. In order to make the manipulation experimentally easier, we choose [see Fig. 2(b)] $\omega_c^B(0)$ to be intermediate between $\omega_c^A(0)$ and ω^B, while $\omega_c^{A'}(0)$ is close to $\omega_c^{B'}(0)$ and $\omega_c^{B'}(0)$ is close to $\omega^{B'}$. In the idle configuration $\Delta_c^\gamma(t) = 0$, the cavity-mode frequencies are significantly detuned from the spin energy gaps $[|\omega_c^\gamma(0) - \omega^\gamma| \gg \bar{G}_\gamma]$ and $\hat{H}_{\text{int}}$ is ineffective. In addition, the cavities A and A' are far detuned from B $(|\omega_c^A - \omega_c^B| \gg \kappa, |\omega_c^{A'} - \omega_c^{B'}| \gg \kappa')$ and the effect of intercavity coupling is negligible. Hence, single-qubit gates can be performed independently on each cavity.

In particular, a rotation $\hat{R}_z(\varphi)$ of an arbitrary angle φ about the z axis of the Bloch sphere is performed by off-resonance pulses. These induce a phase difference between the $|0\rangle_\mu$ and $|1\rangle_\mu$ states of the hybrid qubits [Eq. (1)], given by $\int_{t_0-\tau/2}^{t_0+\tau/2} \Delta_c^\gamma(t)dt$ (τ is the pulse duration). For simplicity we assume steplike pulses, $\Delta_c^\gamma(t) \equiv \delta_c^\gamma \theta(\tau/2 - |t - t_0|)$, so that a generic $\hat{R}_z(\varphi)$ rotation is obtained by setting $\delta_c^\gamma \tau = \varphi$. Conversely, $\hat{R}_x(\varphi)$ rotations are obtained by tuning the frequency of the cavity mode to match the corresponding energy gap of the SE $[\delta_c^\gamma = \omega^\gamma - \omega_c^\gamma(0)$, with $\gamma = A, A']$ for the time $\tau = \varphi/\bar{G}_\gamma$. The additional phase $\phi = \delta_c^\gamma \tau$ of the $|1\rangle_\mu$ qubit state is straightforwardly corrected by an $\hat{R}_z(-\phi)$ rotation. A simulation of this gate is reported in Fig. 3(a) in terms of the overlaps $c_{ij}(t) = \langle i_A j_{A'} | \psi(t) \rangle$ between the system state $|\psi(t)\rangle$ and the logical two-qubits states $|i_A j_{A'}\rangle = |i_A\rangle \otimes |j_{A'}\rangle$ $(i, j = 0, 1)$.

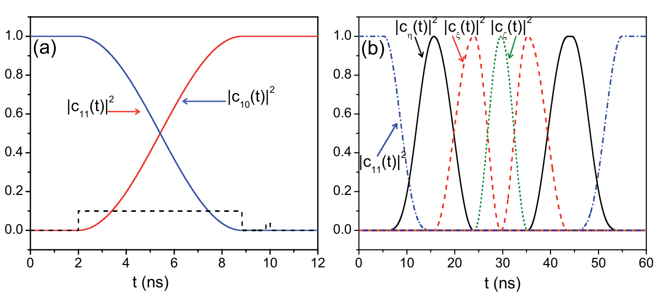

FIG. 3 (color online). (a) Calculated time dependence of the components of $|\psi(t)\rangle$ in a single-qubit rotation $R_x(\pi)$. Dashed black lines represent $\Delta_c^\gamma(t)/\omega_c^\gamma(0)$. Negligible components are not shown. (b) Time dependence of the main components of $|\psi(t)\rangle$ in a CZ gate. Auxiliary states $|\eta\rangle$, $|\xi\rangle$, and $|\zeta\rangle$ are described in Eq. (7).

The implementation of two-qubit gates requires the coupling between the degrees of freedom that are used to encode the two qubits. The CZ gate is performed with a two-step Rabi oscillation of the CPB between $|\psi_0^B\rangle$ and $|\psi_2^B\rangle$ accompanied by the absorption and emission of the two photons entering the definition of the two qubits. A multistep pulse sequence involving the auxiliary states

$$\begin{aligned}|\eta\rangle &= |\psi_0^A, n_A = 0\rangle|\psi_0^B, n_B = 1, n_{B'} = 1\rangle|\psi_0^{A'} n_{A'} = 0\rangle,\\ |\xi\rangle &= |\psi_0^A, n_A = 0\rangle|\psi_1^B, n_B = 0, n_{B'} = 1\rangle|\psi_0^{A'} n_{A'} = 0\rangle,\\ |\zeta\rangle &= |\psi_0^A, n_A = 0\rangle|\psi_2^B, n_B = 0, n_{B'} = 0\rangle|\psi_0^{A'} n_{A'} = 0\rangle\end{aligned} \tag{7}$$

is adopted. We illustrate it considering the two qubits initially in $|1_A 1_{A'}\rangle$. The first step corresponds to the transfer of the photon of mode A in cavity A into mode B in cavity B, by means of a π pulse that brings the two modes into resonance, $\delta_c^A = \omega_c^B(0) - \omega_c^A(0)$. Simultaneously, the photon of mode A' is transferred into mode B' by varying $\delta_c^{A'}$. This induces the transition $|1_A 1_{A'}\rangle \to |\eta\rangle$. As a second step, the photon of frequency ω_c^B is absorbed by the CPB, after a π pulse corresponding to $\delta_c^B = \omega^B - \omega_c^B(0)$, bringing the system into state $|\xi\rangle$. Then, a 2π pulse corresponding to $\delta_c^{B'} = \omega_c^{B'}(0) - \omega^{B'}$ brings the mode $\omega_c^{B'}$ into resonance with the $|\psi_1^B\rangle \leftrightarrow |\psi_2^B\rangle$ transition of the CPB, thus inducing a complete Rabi flopping between the states $|\xi\rangle$ and $|\zeta\rangle$. As a result, a phase π is added to $|\xi\rangle$. Finally, the repetition of the first two steps brings the state back to $|1_A 1_{A'}\rangle$ with an overall phase π [28]. The time evolution of the two-qubit state $|\psi(t)\rangle$ induced by this pulse sequence is reported in Fig. 3(b). Conversely, the other basis states do not acquire any phase and therefore this sequence implements the CZ gate. Indeed, the basis states $|0_A 1_{A'}\rangle$ and $|1_A 0_{A'}\rangle$ contain only one photon and thus the full two-step Rabi oscillation cannot occur [24], while the state $|0_A 0_{A'}\rangle$ is completely unaffected.

These simulations have been performed by assuming fundamental frequencies $\omega_c^{0A}/2\pi = 22$, $\omega_c^{0A'}/2\pi = 21$, and $\omega_c^{0B}/2\pi = 12.5$ GHz [6], and using the first and second harmonic (second and third harmonics) for the A and A' (B) cavities. With this choice, photon hopping between modes other than those included in Eq. (6) is negligible. As a figure of merit for CZ, we compute the *fidelity loss* λ corresponding to the basis states $|i_A j_{A'}\rangle$:

$$\lambda \equiv \max_{i,j}\{1 - \mathcal{F}_{ij}^2\} = \max_{i,j}\{1 - \langle i_A j_{A'}|\hat{\rho}|i_A j_{A'}\rangle\}, \tag{8}$$

where $\hat{\rho} = |\psi(t_f)\rangle\langle\psi(t_f)|$ is the final system state, when the qubits are initialized in $|i_A j_{A'}\rangle$. We found $\lambda = 8 \times 10^{-4}$ for CZ and less for the other gates, demonstrating the accuracy of the operations. Analogous values of λ are obtained for initial states corresponding to linear superpositions of the basis states. Smaller values of λ can be obtained by using larger frequencies or larger detunings.

The frequency is varied up to $\pm 0.1\ \omega_c^0$ for the fundamental harmonic, and proportionally for the others. Similar detunings have been experimentally shown in Ref. [19]. We have assumed realistic values of the CPB-cavity $G_\gamma^j/2\pi = 60$–90 MHz and SE-cavity coupling rates $\bar{G}_\gamma/2\pi = 60$ MHz, the latter corresponding to $N \sim 10^{12}$ spins [5], and tunneling rate $\kappa/2\pi = 25$ MHz, which has already been shown experimentally [27]. Larger values of G_γ and κ would reduce the gating times but would also increase the fidelity loss λ, unless compensated by larger detunings.

Differently from alternative approaches (see, e.g., [29]), the main source of decoherence within our hybrid qubit encoding is determined by photon leakage due to cavity modes losses, since spin coherence times can be very long (see below). We have checked that cavity loss rates in the 10 kHz range [30,31], i.e., photon lifetime of tens of microseconds, result only in a few photons lost over a thousand within the gate's time scale. We report full numerical simulations including these effects in [24], confirming the robustness of our scheme. We notice that pure dephasing of cavity modes is negligible in such resonators [32]. As far as the CPBs are concerned, in the present scheme they are always in the ground state, except during the implementation of CZ gates. Using a CPB as a nonlinear element instead of a transmon or a phase qubit increases the anharmonicity of the system, while reducing the coherence times [4]. However, it is sufficient that the coherence time of the CPB is much longer than the CZ gating time. This can be easily achieved with present technology [33]. At last, the tuning of the resonator frequency could introduce extra dissipation to the system. These effects have been analyzed experimentally (see, e.g., Ref. [19]) and found to be small. They are mainly attributed to thermal noise and they could be further reduced with high frequency resonators, as the ones assumed in this work, and working at very low temperatures where Q factors are known to be larger and more stable.

Physical implementation.—Initialization and readout can be achieved by exploiting the CPBs [30,34]. Several classes of systems can be exploited as SEs for the implementation of our scheme. A simple possibility would be to exploit the different gyromagnetic factors of $3d$ and $4f$ ions, in order to obtain suitable level schemes in an applied magnetic field. The embedding of these ions in a nonmagnetic crystalline matrix allows us to reduce harmful dipolar interactions, and to enhance the decoherence time up to the millisecond range [35]. Another promising class of spin systems is represented by molecular nanomagnets [21]. These spin clusters form crystals in which the molecules behave as identical and noninteracting magnetic units, and can be diluted by using nonmagnetic analogues. Additionally, molecular nanomagnets present degrees of freedom, such as spin chirality, that can be manipulated by the electric-field component of the cavity field [36] and are substantially protected from the magnetic environment [37].

In summary, we have developed a scheme for quantum information processing with spin ensembles in superconducting stripline resonators, exploiting a hybrid spin-photon encoding of the qubits. Our scheme is qualitatively different from previous ones because the spin and photon degrees of freedom enter the definition of the qubit on an equal footing. In this way, the evolution can be induced simply by tuning the cavity mode frequencies to the spin-energy gaps. Arbitrary single- and two-qubit gates are implemented, over much shorter times than typical decoherence times of cavity photons and spin ensembles. Promising candidates for the spin degrees of freedom are diluted magnetic ions or molecular nanomagnets. The application of this scheme to an $ABAB\ldots$ array of cavities enables general quantum algorithms as well as quantum simulators [20,38] to be implemented.

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