Challenge to Macroscopic Probes of Quantum Spacetime Based on Noncommutative Geometry

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Over the last decade, a growing number of quantum-gravity researchers has been looking for opportunities for the first ever experimental evidence of a Planck-length quantum property of spacetime. These studies are usually based on the analysis of some candidate indirect implications of spacetime quantization, such as a possible curvature of momentum space. Some recent proposals have raised hope that we might also gain direct experimental access to quantum properties of spacetime, by finding evidence of limitations to the measurability of the center-of-mass coordinates of some macroscopic bodies. However, I here observe that the arguments that originally led to speculating about spacetime quantization do not apply to the localization of the center of mass of a macroscopic body. And, I also analyze some popular formalizations of the notion of quantum spacetime, finding that when the quantization of spacetime is Planckian for the constituent particles, then for the center of mass of a composite macroscopic body the quantization of spacetime is much weaker than Planckian. These results suggest that the center-of-mass observables of macroscopic bodies should not provide good opportunities for uncovering quantum properties of spacetime. And, they also raise some conceptual challenges for theories of mechanics in quantum spacetime, in which, for example, free protons and free atoms should feel the effects of spacetime quantization differently.

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Introduction and motivation.—Traditionally, the quantum-gravity problem was studied as a mere technical exercise, assuming that it might be impossible to find experimental evidence of the minute effects produced by the characteristic length scale of quantum gravity, expected to be of the order of the Planck length $\ell_P \simeq 10^{-35}$ m. This changed over the last decade as a result of a growing number of studies (see, e.g., Refs. [1–11]) showing that evidence of Planck-length quantum properties of spacetime might be within our experimental reach if we exploit some candidate *indirect* manifestations of spacetime quantization. An intuitive example of candidate indirect manifestations of spacetime quantization is found in results showing that certain ways to introduce the Planck length as a scale of spacetime quantization admit a dual picture in which the Planck length also plays the role of the scale of curvature of momentum space, with implications for relativistic kinematics (see, e.g., Refs. [11,12]).

It would of course be important to also find opportunities for observing Planck-length spacetime quantization *directly*. And, according to the studies recently reported in Refs. [13,14], this might be possible, at least in the sense that we can achieve Planckian accuracy in measurements pertaining the center-of-mass coordinates of some macroscopic bodies The study reported by Pikovski *et al.* in Ref. [13] focuses on the center-of-mass motion of a mechanical oscillator, while the study reported by Bekenstein in Ref. [14] focuses on the center-of-mass motion of a macroscopic dielectric block traversed by a single optical photon. The study reported by Pikovski *et al.* in Ref. [13] focuses on the center-of-mass motion of a mechanical oscillator, while the study reported by Bekenstein in Ref. [14] focuses on the center-of-mass motion of a macroscopic dielectric block traversed by a single optical photon. (In addition, the more recent proposal reported in Ref. [15], of which I became aware during the last stages of finalization of this Letter, is similar to the proposals in Refs. [13,14] from the viewpoint here adopted: Ref. [15] seeks evidence of spacetime quantization by studying the longitudinal normal modes of a 3-meter long aluminum bar weighing 2.3 tons.)

In attempting to assess the likelihood of success of these proposals, I noticed that they involve small momentum transfer from a low-energy photon to a macroscopic body, the body being describable fully within the "nonrelativistic limit" (small velocities, where Galilean relativity holds). And, I find that the arguments that inspired quantum-gravity research on Planck-length spacetime quantization do not apply to such interactions. The current consensus among theorists (see, e.g., the reviews in Refs. [16,17]) is that spacetime quantization is needed because any attempt to localize a particle with Planckian accuracy requires concentrating energy of order of the inverse of the Planck length within a Planck-length-size region, and in such situations, our present understanding of gravitational phenomena suggests that a black hole should form, rendering the localization procedure meaningless. The procedures proposed in Refs. [13,14] for Planck-length accuracy in the control of the center-of-mass position of a macroscopic body evidently do not involve any particularly high concentration of energy in small regions.

The hope that the center of mass of a macroscopic body might be subject to the same Planck-length quantum properties of spacetime expected for fundamental particles is therefore evidently based on an implicit inductive argument: the necessity of Planck-length spacetime quantization arises exclusively in arguments involving fundamental particles, but once that is accommodated in the theory, perhaps by some (unproven and unknown) consistency criterion, the Planck-length quantum properties of spacetime would also affect the center of mass of a macroscopic body. To my knowledge, this huge extrapolation is not confirmed by any known results of quantum-spacetime research. On the contrary, I here provide a simple argument suggesting that this extrapolation is incorrect. I consider a few of the most popular models being studied in the quantum-spacetime literature, and I probe conceptually the issue here at stake by using a simplified characterization of the center of mass of a body composed of N constituent particles. I take as center-of-mass coordinates the observables X, Y, and Z, with

$$X = \frac{1}{N}\sum_{n=1}^{N} x_n, \qquad Y = \frac{1}{N}\sum_{n=1}^{N} y_n, \qquad Z = \frac{1}{N}\sum_{n=1}^{N} z_n \quad (1)$$

(where of course x_n, y_n, and z_n are the coordinates of the nth composing particle), and I take as center-of-mass momentum the observables P_x, P_y, and P_z, with

$$P_x = \sum_{n=1}^{N} p_{x,n}, \qquad P_y = \sum_{n=1}^{N} p_{y,n}, \qquad P_z = \sum_{n=1}^{N} p_{z,n} \quad (2)$$

(where of course $p_{x,n}$, $p_{y,n}$, and $p_{z,n}$ are the momentum components of the nth composing particle).

This simplified description of a macroscopic body is sufficient for my purposes since the relevant phenomenological opportunities are for the center of mass of macroscopic bodies in the nonrelativistic regime and my main objective is to provide a counterexample to the conjecture that Planck-length quantum properties of spacetime apply in an undifferentiated way to fundamental particles and to the center of mass of macroscopic bodies. I shall show that the conjecture is false by showing that it does not apply to macroscopic bodies whose center-of-mass motion is characterized by Eqs. (1) and (2). And, Eqs. (1) and (2) are appropriate for macroscopic bodies whose constituents all have the same mass and whose center-of-mass degrees of freedom decouple from the other degrees of freedom.

Results for classical spacetime and Lie-algebra quantum spacetime.—Let me first recall the mechanism through which the description (1) and (2) gives satisfactory results within ordinary quantum mechanics, in classical spacetime, where the only nontrivial commutator is Heisenberg's

$$[x, p_x] = i\hbar$$

(focusing for simplicity on the x direction).

Evidently, the Heisenberg commutator also applies to a body's center of mass, described by Eqs. (1) and (2):

$$[X, P_x] = \left[\frac{1}{N}\sum_{n=1}^{N} x_n, \sum_{m=1}^{N} p_{x,m}\right] = \frac{1}{N}\sum_{n=1}^{N}\sum_{m=1}^{N} \delta_{n,m} i\hbar = \frac{1}{N}\sum_{n=1}^{N} i\hbar = i\hbar. \quad (3)$$

My next application is already nontrivial and novel but nonetheless provides further elements in support of the usefulness of the conceptual probe I am using, centered on Eqs. (1) and (2). For this, I consider a class of quantum-spacetime pictures involving noncommutativity of coordinates of Lie-algebra type [18–20]:

$$[r^\alpha, r^\beta] = i\ell\theta^{\alpha\beta}_\gamma r^\gamma,$$

with $r^1 = x$, $r^2 = y$, and $r^3 = z$. (I focus on spatial noncommutativity, which suffices for establishing the issue for macroscopic bodies which is here of interest.)

This type of noncommutativity of coordinates is here particularly significant since it is the only case where the literature does provide preliminary evidence that macroscopic bodies might be affected by Planck-length features differently from their constituent particles. These are arguments focusing on the description of macroscopic bodies when momentum space is curved or anyway affected by nonlinearities (see Ref. [21] and references therein). Lie-algebra spacetimes are known to be dual to momentum spaces with curved geometry [11,12], and one of the implications is that the laws of conservation of momentum for fundamental particles are Planck-length deformed. Applying the relevant deformed conservation laws to the constituents of a macroscopic body can give a net result for collisions such that momentum conservation for macroscopic-body total momentum is affected by weaker corrections than momentum conservation for the particle constituents. Specifically, Ref. [21] focused on a situation such that before and after the momentum exchange, the bodies are composed of particles in exactly rigid motion and found that the curvature of momentum space was felt by the macroscopic body, not as set by the Planck length but rather as set by the Planck length divided by the number N of particle constituents.

Even though they applied only to rather special contexts [rigid motion is an assumption stronger than the ones required by my Eqs. (1) and (2)] and they concerned momentum-space nonlinearities rather than spacetime fuzziness, these previous arguments could already hint at the possibility that in Lie-algebra spacetimes, the effective Planck length should be rescaled for macroscopic bodies. My simple "conceptual probe" produces for the noncommutativity of coordinates results which are indeed consistent with the intuition emerging from those previous studies on the momentum-space side. To see this, let me consider the case of a commutator of type

$$[x, y] = i\ell r^\alpha,$$

with α taking any value among 1, 2, 3 (so that essentially I consider at once cases of the type $[x, y] = i\ell x$ and of the type $[x, y] = i\ell z$).

Applying $[x, y] = i\ell r^\alpha$ to the constituent particles of a macroscopic body, one then finds for the center-of-mass coordinates described in Eq. (1) the result

$$[X, Y] = \left[\frac{1}{N}\sum_{n=1}^{N} x_n, \frac{1}{N}\sum_{m=1}^{N} y_m\right] = \frac{1}{N^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\delta_{n,m} i\ell r_n^\alpha$$
$$= \frac{1}{N^2}\sum_{n=1}^{N} i\ell r_n^\alpha = i\frac{\ell}{N}R^\alpha, \tag{4}$$

where of course $R^\alpha \equiv N^{-1}\sum_{n=1}^{N} r_n^\alpha$.

Evidently, Eq. (4) shows that the effects of Lie-algebra coordinate noncommutativity for the center of mass of macroscopic bodies are scaled down by a factor of $1/N$. While this could be expected intuitively on the basis of the dual momentum-space picture described in Ref. [21], it is noteworthy that my approach provides a consistent picture of the quantum-spacetime aspects.

Results for other quantum-spacetime pictures.—I shall now show that my perspective on center-of-mass degrees of freedom of macroscopic bodies has applicability that goes beyond the specific context of Lie-algebra spacetime noncommutativity. My next example is "Moyal noncommutativity," with coordinate-independent commutators, such as

$$[x, y] = i\ell_M^2. \tag{5}$$

This is perhaps the most studied candidate scenario for the quantization of spacetime [22,23], and there is no result in the literature anticipating that macroscopic bodies should be affected by Moyal noncommutativity differently from their constituents. (Note however that about a decade ago, as his novel approach to quantum gravity [24] started to earn wider appreciation, Volovik raised the possibility that this might be the case in the context of private discussions with advocates of Moyal noncommutativity.) The applicability of my thesis to Moyal noncommutativity is easily checked by using (1) for center-of-mass coordinates with the constituents governed by noncommutativity (5):

$$[X, Y] = \left[\frac{1}{N}\sum_{n=1}^{N} x_n, \frac{1}{N}\sum_{m=1}^{N} y_m\right] = \frac{1}{N^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\delta_{n,m} i\ell_M^2$$
$$= \frac{1}{N^2}\sum_{n=1}^{N} i\ell_M^2 = i\left(\frac{\ell_M}{\sqrt{N}}\right)^2. \tag{6}$$

Therefore, also for the Moyal case, the noncommutativity of center-of-mass coordinates should be weaker than the noncommutativity of the coordinates of the constituents. Specifically, the Moyal noncommutativity length scale ℓ_M gets reduced by a factor of $1/\sqrt{N}$.

The results I found in Eqs. (4) and (6) show that for pictures of quantum spacetime based on spacetime noncommutativity the center of mass of a macroscopic body must have quantum-spacetime properties different from those of its constituents. It would be interesting to check whether the same holds in the popular picture of quantum spacetime given by the Loop-Quantum-Gravity approach, but such an analysis is not within the reach of our present understanding of that complex formalism [25]. However, I can verify the applicability of my thesis to another much-studied class of quantum-spacetime pictures, not based on spacetime noncommutativity. This is the one centered on the possibility that the Planck length intervenes in modifications of the Heisenberg commutator of the general type [26,27]

$$[x, p] = i\hbar(1 - \lambda' p + \lambda^2 p^2). \tag{7}$$

Even with commuting coordinates, these modifications of the Heisenberg commutator produce spacetime quantization. The key role for this is played by the parameter λ^2 of the quadratic term. The standard Heisenberg commutator still allows localizing a particle sharply at a point ($\delta x \to 0$) if $\delta p \to \infty$, i.e., if all information on the conjugate momentum is given up. But, for $\lambda^2 \neq 0$, Eq. (7) produces a seesaw formula [26,27] such that δx receives a novel contribution proportional to δp in addition to the standard Heisenberg term going like $1/\delta p$, in such a way that the coordinate x cannot ever be measured sharply, as required for a quantum-spacetime picture.

Of some interest for my thesis is also the perspective given in Ref. [27], advocating the specific choice of $\lambda' = \lambda$ in Eq. (7), partly because of its consistency (in the sense of Jacobi identities) with commutativity of coordinates among themselves and of momenta among themselves.

Keeping these facts in mind, it is then interesting to look at the properties of a center of mass described by Eqs. (1) and (2) when the constituents are governed by Eq. (7):

$$[X, P_x] = \left[\frac{1}{N}\sum_{n=1}^{N} x_n, \sum_{m=1}^{N} p_{x,m}\right]$$
$$= \frac{1}{N}\sum_{n=1}^{N}\sum_{m=1}^{N}\delta_{n,m} i\hbar(1 - \lambda' p_{x,m} + \lambda^2 p_{x,m}^2)$$
$$= i\hbar\left[1 - \frac{\lambda'}{N}P_x + \frac{\lambda^2}{N^2}P_x^2 + \frac{\lambda^2}{N}\sum_{n=1}^{N}\left(p_{x,n}^2 - \frac{P_x^2}{N^2}\right)\right]$$
$$\simeq i\hbar\left(1 - \frac{\lambda'}{N}P_x + \frac{\lambda^2}{N^2}P_x^2\right), \tag{8}$$

where for the last approximate equality, I restricted my attention to macroscopic bodies in (quasi-)rigid motion, as with those of interest for the mentioned experimental proposals put forward in Refs. [13,14], so that one can expect for every n that $p_{x,n} \simeq P_x/N$. Evidently, at least in this rigid-motion limit, also for quantum spacetimes characterizable in terms of Eq. (7), I am finding that the center of mass of a macroscopic body should be affected more weakly than its constituents by spacetime quantization.

Notably, my argument suggests that in the rigid-motion limit, the prescription $\lambda' = \lambda$ advocated in Ref. [27] could apply both to fundamental particles and to the center of mass of a macroscopic body (but in the macroscopic case, both λ' and λ are reduced by $1/N$).

Implications and outlook.—The analysis I here reported should put to rest any further temptations of relying on the unquestioned assumption that the center-of-mass degrees of freedom of a macroscopic body be affected by quantum-spacetime effects just as much as the microscopic constituents of the body. I have provided counterexamples to that assumption which, because of the nature of my conceptual probe centered on Eqs. (1) and (2), are robust at least for the center of mass of bodies in quasirigid motion (like a solid at low temperatures). Let me also stress that it does not take a particularly macroscopic system for my concerns to be applicable. Think of just bound systems of two identical particles, with coordinate vectors $\vec{r}_1$ and $\vec{r}_2$ and with bounding potential $V(|\vec{r}_1 - \vec{r}_2|)$ affecting only the relative motion: for such systems, Eqs. (1) and (2) are correct, with $N = 2$.

While I am proposing that simplistic assumptions about the properties of macroscopic bodies in a quantum spacetime must be abandoned for good, I believe it would be incorrect to give up on the idea of discovering quantum-spacetime effects through observations of macroscopic bodies. After all (if only the development of observational techniques had had a different history), quantum mechanics itself could have been discovered by studying white dwarfs, rather than through observations at atomic and subatomic scales. There might be an opportunity out there for uncovering a manifestation of quantum spacetime through studies of some specific macroscopic bodies. But, in order for us to capitalize from such opportunities, it will be necessary to move much beyond simple-minded assumptions about general properties of center-of-mass degrees of freedom. Macroscopic bodies have a huge variety of properties, and only some special ones among them under some suitable special conditions (and for observables not necessarily linked to the center-of-mass degrees of freedom) could manifest quantum-spacetime properties tangibly.

One could try with macroscopic bodies for which the center-of-mass degrees of freedom do not fully decouple from the internal degrees of freedom. In such cases, the arguments I here reported would be inapplicable, but of course this does not mean that some naive guess work is then allowed. One should handle the tough challenge of modeling such bodies and figure out under which conditions the Planck-scale effects could be tangible. And, it will be necessary to achieve rigorous quantifications of the implications for a given macroscopic body of interest: in phenomenology, negative results are also important since they allow us to set limits on the parameters of candidate new theories, but that is only possible if the quantification of predicted effects is rigorously derived from the defining parameters of the theory.

Similar considerations can be inspired by the contributions of type $p_{x,n}^2 - P_x^2/N^2$ neglected for the last equality in Eq. (8) under the assumption of quasi-rigid motion. My Eq. (8) also shows that for "deformed-Heisenberg quantum spacetimes" one could have an amplification of the quantum-spacetime effects when the body is not quasi-rigid and the context is such that terms of type $p_{x,n}^2 - P_x^2/N^2$ are large, as it happens in particular for bodies at very high temperatures. This is not the case of the macroscopic bodies considered in the phenomenological proposals of Refs. [13,14], but could inspire some new phenomenological proposals. In pursuing such opportunities one should take into account that the properties of the center of mass of bodies in such extreme regimes would still be different from the ones of the constituents. For appropriately large departures from quasi-rigid motion in deformed-Heisenberg quantum spacetimes the Planck-scale properties of the center of mass of a macroscopic body could actually be stronger than those of the constituents.

It is also possible that for some models of quantum spacetime, the starting points of my analysis, constituted by Eqs. (1) and (2), are inapplicable even when the center-of-mass degrees of freedom cleanly decouple from internal degrees of freedom. For one of the cases here considered, the one of Lie-algebra noncommutative spacetime, this is already established in the literature, although it does not affect my analysis. Indeed, in Lie-algebra spacetimes, the law of composition of momenta is expected to be deformed but the law of composition of spacetime coordinates is undeformed, as first shown in Ref. [18]. Interestingly, the derivation of my main result for Lie-algebra spacetimes, Eq. (4), requires exclusively Eq. (1), so it is not affected by this issue. Requirement (2) is crucial for my main result concerning "deformed-Heisenberg noncommutativity," Eq. (8), but the available literature on those quantum spacetimes does not advocate any deformation of composition laws (see, e.g., Refs. [26,27]). Similarly, the available literature on "Moyal noncommutativity," for which my main result is Eq. (6), does not advocate [22,23] any modification of Eqs. (1) and (2). So, the analysis I here reported is not challenged by any available results on composition laws in quantum spacetimes. However, this issue must be monitored since the understanding of known quantum-spacetime models is still in progress. Moreover, new models might at some point be proposed with deformed composition laws such that my argument would not then be applicable to them.

While my main focus here was on phenomenological prospects, in closing I should also emphasize some severe technical challenges that, according to my analysis, must be faced in theory work on the quantum-spacetime idea. A first challenge comes from the fact that my analysis shows that macroscopic bodies have quantum-spacetime properties different from those of their constituents, but it gives no indication of which constituents are those "fundamental enough" to be affected by the full strength of Planck-scale effects. Think, for example, of molecules: my analysis suggests that molecules are affected more weakly by quantum-spacetime effects than the atoms within them, but should the Planck-length magnitude of quantum-spacetime effects be assumed for atoms or for protons and neutrons within the nuclei of atoms? or for quarks?

And, a second challenge would need to be faced even assuming this first challenge is eventually addressed in a given quantum-spacetime picture, so that actually the picture predicts the magnitude of quantum-spacetime effects for, say, protons and also predicts how much weaker than for protons the effects are for, say, Cs atoms. We would clearly need a completely new type of theory of mechanics, in which the spacetime properties of different particles are different. We should renounce to one of the key aspects of simplicity that survived previous evolutions of our formulation of the laws of physics: the general-relativistic description of spacetime, just like the special-relativistic one and the Newtonian one, is indeed such that the implications of spacetime for particle properties are independent of compositeness and are therefore the same for protons and large atoms.

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