Experimental Generation of Quantum Discord via Noisy Processes

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Quantum systems in mixed states can be unentangled and yet still nonclassically correlated. These correlations can be quantified by the quantum discord and might provide a resource for quantum information processing tasks. By precisely controlling the interaction of two ionic qubits with their environment, we investigate the capability of noise to generate discord. Firstly, we show that noise acting on only one quantum system can generate discord between two. States generated in this way are restricted in terms of the rank of their correlation matrix. Secondly, we show that classically correlated noise processes are capable of generating a much broader range of discordant states with correlation matrices of any rank. Our results show that noise processes prevalent in many physical systems can automatically generate nonclassical correlations and highlight fundamental differences between discord and entanglement.

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Over the last decade, improvements in experimental control over quantum systems have enabled scientists to generate nonclassical states of light and matter. Entanglement between quantum systems [1] has been at the forefront of this research, largely due to the role that it plays in Bell inequalities [2] and as a resource for certain information processing tasks [3]. However, entanglement is not the only kind of nonclassical correlation that can exist between systems. Discord is a measure of these other correlations in bipartite systems, which becomes distinguishable from entanglement for mixed states [4–7]. There are mixed states with discord yet no entanglement, and these are the focus of the present work.

In recent years, we have seen a significant research effort towards understanding discord, focusing largely on its characterization and application, see Ref. [7] for a review. There is evidence that, like entanglement, discord can be viewed as a resource for quantum information processing tasks [6–13], see also Refs. [14–16]. Recently, quantum states of light have been generated which belong to the regime where there is discord but no entanglement [9,13,17,18]. Following extension of discord to continuous variable systems [19,20], experiments have investigated Gaussian discord dynamics under various decoherence channels [18].

In this Letter, we experimentally address the question of which physical processes are required to generate discord between quantum systems: a mode of questioning that has been very successful in understanding the structure of entanglement [21]. Although we use trapped ions as a test bed, our results are relevant to any many-body quantum system, such as atoms, photons, or superconducting junctions. The Letter is organized as follows: after briefly reviewing the discord, we demonstrate how operations on one qubit can generate discord between two qubits. Secondly, we review the correlation rank as a way to assess the nature of correlations in quantum states and then show how discordant states with any rank can be generated via noisy processes.

Two quantum systems A and B have discord when considering measurements on system A if and only if their state cannot be written in the form $\rho_{AB} = \sum_i p_i |\psi_i^A\rangle\langle\psi_i^A| \otimes \rho_i^B$, where $\langle\psi_i^A|\psi_j^A\rangle = \delta_{ij}$, ρ_i^B are density matrices of qubit B and p_i are probabilities. For a state ρ_{AB}, a von Neumann (VN) measurement [3] of A with eigenvectors $\Pi_i = |\psi_i^A\rangle\langle\psi_i^A|$ will leave the total state unchanged, i.e., $\sum_i \Pi_i \rho_{AB} \Pi_i^\dagger = \rho_{AB}$. The discord D of a bipartite system is quantified as the difference between two definitions of the mutual information, I and J, i.e., $D = I - J$. I captures the total correlations by the difference in entropy of systems when taken individually and when taken together $I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$, where S is the VN entropy [3] and $\rho_A(\rho_B)$ is the reduced density matrix of system A (B). J captures the classical correlations and can be interpreted as information gain about one subsystem as a result of a measurement on the other. $J(\rho_{AB}) = S(\rho_B) - S(\rho_{B|A})$, where $S(\rho_{B|A})$ is the entropy of B after a measurement of A (with unknown result) and is maximized over all VN measurements of A, see Ref. [7]. Discord can be asymmetric with respect to exchange of the two systems since it quantifies the extent to which measurements on one system affect the total system for an independent observer. For the discord that considers measurements on system A (B), we will use the label D_A (D_B).

Consider the separable two-qubit mixed state

$$\rho_1 = \frac{1}{2}(|+\rangle_A\langle+| \otimes |+\rangle_B\langle+| + |-\rangle_A\langle-| \otimes |-\rangle_B\langle-|), \quad (1)$$

where $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. Although manifestly correlated, state ρ_1 is fully classical since it is diagonal in the local orthogonal $|\pm\rangle \otimes |\pm\rangle$ basis. The discord is symmetric, $D_A = D_B = 0$; a VN measurement $\{\Pi_\pm = |\pm\rangle\langle\pm|\}$ of either qubit leaves ρ_1 unchanged. Surprisingly, discord can be generated by applying an amplitude damping process that acts only on one of the qubits in state ρ_1 [22,23].

Amplitude damping of a single qubit can be described by the quantum map $\epsilon'_{ad}(\rho) = E_0\rho E_0^\dagger + E_1\rho E_1^\dagger$ with Kraus operators $E_0 = |0\rangle\langle 0| + \sqrt{1-p}|1\rangle\langle 1|$ and $E_1 = \sqrt{p}|0\rangle\langle 1|$. For $p = 0$ ($p = 1$), the qubit undergoes zero (complete) amplitude damping. Consider now the case where after preparation of ρ_1 qubit B undergoes this process; i.e., it interacts with a dissipative Markovian bath that causes the excited state $|1\rangle$ to decay to the ground state $|0\rangle$. Throughout the damping process, the state is of the form $\epsilon_{ad}(\rho_1) = 1/2(|+\rangle_A\langle+| \otimes \tau_{+B} + |-\rangle_A\langle-| \otimes \tau_{-B})$, where $\tau_{\pm B} = \epsilon'_{ad}(|\pm\rangle\langle\pm|)$ are generic density matrices representing the state of qubit B. The effect of the damping process is to reduce the distinguishability of $\tau_{\pm B}$, which are initially orthogonal. Subsequently, qubit A becomes correlated with nonorthogonal states of qubit B. For $0 < p < 1$, there is no VN measurement of B after which the original state $\epsilon_{ad}(\rho_1)$ is recovered and therefore the discord $D_B > 0$.

Experiments are carried out using two $^{40}Ca^+$ ions in a linear Paul trap. A qubit is encoded in an $S_{1/2}$ ground and a $D_{5/2}$ metastable state. For details on the generation of the initial classically correlated state ρ_1 and implementation of the amplitude damping channel, see the Supplemental Material [24]. Two-qubit state tomography is performed for a range of amplitude damping probabilities p. Maximum likelihood reconstruction [25] is employed. The set of zero discord two-qubit states is of zero measure in the total set [26], and therefore any white noise (e.g., measurement projection noise) is likely to result in the reconstruction of a discordant state. For numerical simulations of this effect, see the Supplemental Material [24]. All quantities are derived from the reconstructed density matrix, including discord via a numerical optimization [5,17].

Amplitude damping results are summarized in Fig. 1. Figure 1(a) shows that statistically significant amounts of discord D_B are generated, while D_A remains small and constant to within error. All states contain less than 0.001 tangle (a measure of entanglement [27]). Figure 1(b) presents the dynamics of the states $\tau_{\pm B}$ of qubit B on a cross section of the Bloch sphere. One sees that the distinguishability is reduced by the damping process, and

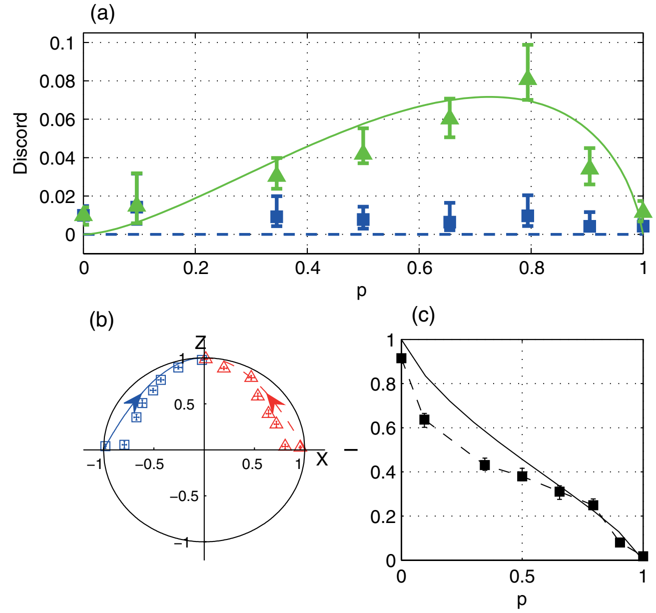

FIG. 1 (color online). Amplitude damping results $\epsilon_{ad}(\rho_1)$. (a) Discord D_A (ideal dashed blue line, data squares) and D_B (ideal green line, data triangles) as a function of amplitude damping probability p. (b) Ideal trajectories $\tau_{+B} = \epsilon'_{ad}(|+\rangle \times \langle+|)$ (red dashed line) and $\tau_{-B} = \epsilon'_{ad}(|-\rangle\langle-|)$ (blue solid line) on the X-Z plane of the Bloch sphere. Experimental results (open shapes) derived from the reduced state of qubit B in Eq. (1) after projecting qubit A, in the reconstructed density matrix, into either $|+\rangle$ (triangles) or $|-\rangle$ (squares). (c) Total correlations, $I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$. Ideal (solid line) and measured (solid squares).

consequently, for all $0 < p < 1$, qubit A becomes correlated with nonorthogonal states of qubit B.

Our results do not imply that the total correlations between two systems can be increased via operations on only one system. Figure 1(c) shows that the total correlations captured by the mutual information I continually decrease during the damping channel. The correct interpretation is that the process converts some of the preexisting classical correlations quantified by J into quantum correlations ($I - J$). Indeed, if there are no classical correlations present in the initial state, then no discord can be generated by operating on only one system [28]. Furthermore, only a very restricted class of discordant states can be created by such operations: a set of measure zero in the total set [28]. That there are fundamentally different kinds of discordant two-qubit states, in terms of how they can be generated, raises the question of whether there is another way of quantifying the correlations in these systems.

An alternative view on quantum correlations is presented in Ref. [28], where in addition to discord, the rank R of the correlation matrix is considered. Although operations on one system can convert classical to quantum correlations, they cannot increase the correlations in terms of R, which is therefore considered as an additional quantity of interest. R is calculated in the following way. First, the correlation matrix (M) is constructed by writing ρ in a basis of local Hermitian operators (e.g., the Pauli spin operators). R is the number of nonzero singular values of M, which is equivalent to the minimal number of bipartite product operators needed to represent a quantum state. Originally, R was introduced as a witness for discord [29]. A state with $R = 1$ is completely uncorrelated. If $R > d_A$, where d_A is the dimension of the smallest system, then the discord is nonzero. For a system of two qubits, $d_A = d_B = 2$. The lowest rank of a two-qubit system containing discord is 2, but in general, an $R = 2$ state may or may not contain discord. The maximum is $R = 4$ representing a highly correlated state. In Ref. [8], it is shown that R determines the extent to which a state can act as a resource in quantum state transmission.

Figure 2 presents a selection of reconstructed states and their singular values from the amplitude damping experiment. The results are largely consistent with an invariant value $R = 2$. Small nonzero singular values consistent with a higher rank are analyzed in the Supplemental Material [24]. Numerical simulations show that these small values, the largest of which is $0.06 + (0.02/-0.02)$, are consistent with the effects of measurement projection noise. The difficulty of verifying that singular values are strictly zero clearly makes the witness criteria for discord experimentally challenging.

How then can these strongly correlated high-rank discordant states be generated? We now show that classically correlated noise processes which operate on both qubits are sufficient to generate states of all ranks. Consider qubits interacting with an environment that causes them to suffer identical single-qubit rotations around some axis by an amount that is not known and fluctuates from experiment to experiment. Complete dephasing under this noise can be modeled by a quantum map $\epsilon_{cd}^{\vec{n}} = (1/2\pi) \times \int_0^{2\pi} K_{\vec{n}}(\theta)\rho K_{\vec{n}}^{\dagger}(\theta)$ with operators $K_{\vec{n}}(\theta) = R_{\vec{n}}(\theta) \otimes R_{\vec{n}}(\theta)$ and $R_{\vec{n}}(\theta) = e^{-i\theta\vec{n}\,\vec{\sigma}/2}$ is a single-qubit rotation of angle θ around a normalized axis vector $\vec{n}$. The integral over θ generates a dephasing effect between eigenstates of the rotation operator. This type of noise occurs in any experimental situation with fluctuating external fields that couple equally to the qubits and, as we will show, can lead to the automatic generation of high-rank correlations between quantum systems.

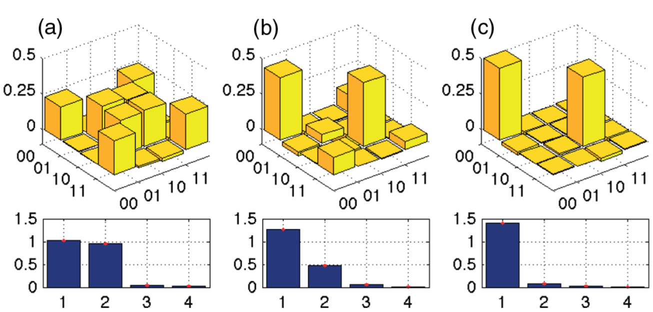

FIG. 2 (color online). Amplitude damping results. Real values of measured density matrices for damping probabilities (p) (a) 0.00, (b) 0.79, and (c) 1.00. Imaginary components are $\leq$ 0.03. Below, bar charts show the singular values of the corresponding correlation matrix. The number of nonzero values gives the rank. The fidelities with target states are all greater than 0.98.

We investigate the effects of correlated noise due to ambient magnetic (B) field noise. The transition frequency of each qubit is determined by the local B field, which is largely identical for both ions due to their small separation ($\approx 5\ \mu$m). B-field noise causes correlated qubit rotations in the Z basis, i.e., $\vec{n} \cdot \vec{\sigma} = \sigma_z$, and θ is proportional to the B field. Over the repeated experiments required to estimate expectation values, B-field fluctuations smear out the phases between eigenstates of the σ_z operators. Although these effects are always present, typical experiments require orders of magnitude less time than that required for complete dephasing. Here, we intentionally expose various quantum states for long times to investigate the phenomenon in detail. As previously shown [30], when applied to certain entangled Greenberger-Horne-Zeilinger states, this noise process can be extremely detrimental, resulting in a fully classical $R = 2$ state. Here we show the converse—the process can also generate quantum correlations.

The dephasing channel is applied to the classically correlated $R = 2$ state ρ_1 by introducing a delay between initialization of the qubits into this state and performing tomography. A delay of 10 ms is sufficient to achieve almost complete dephasing. Experimentally reconstructed density matrices and singular values of ρ_1 and $\epsilon_{cd}^{\vec{n}}(\rho_1)$ are shown in Figs. 2(a) and 3(a), respectively. The $|00\rangle\langle 11|$ coherence element of the density matrix is almost completely lost [31], while the $|01\rangle\langle 10|$ coherence is largely unaffected since it is in a decoherence-free subspace [32]. The discord increases from $0.010 + (0.014/-0.005)$ to $0.19 + (0.03/-0.03)$, and the singular values are consistent with an increase from an $R = 2$ to an $R = 3$ state.

In the Supplemental Material [24], we present new theoretical results on the conditions under which correlated noise can change R. The final rank can be obtained with the aid of a simple geometrical picture expressing the relationship between the rotation axis $\vec{n}$ of the correlated dephasing and two normalized vectors $\vec{v}$, $\vec{w} \in \mathbb{R}^3$, which provide all the necessary information about $R = 2$ states. Specifically, $R = 2$ states where the reduced state of each qubit is completely mixed (e.g., state ρ_1) can be written as $\rho = 1/4(\mathbb{I} \otimes \mathbb{I} + d\vec{v} \cdot \vec{\sigma} \otimes \vec{w} \cdot \vec{\sigma})$. The final rank depends on the overlap between $\vec{n}$ and the vectors $\vec{v}$ and $\vec{w}$, respectively. In ρ_1, the qubits are correlated in the X direction, i.e., $\vec{v} = \vec{w} = \vec{e}_x$, and the dephasing rotations are conducted around the orthogonal Z direction, $\vec{n} = \vec{e}_z$. In this case, an $R = 3$ state is generated since $\vec{n}\,\vec{v} = \vec{n}\,\vec{w} = 0$. However, if $\vec{n}$ is neither equal nor orthogonal to $\vec{v}$

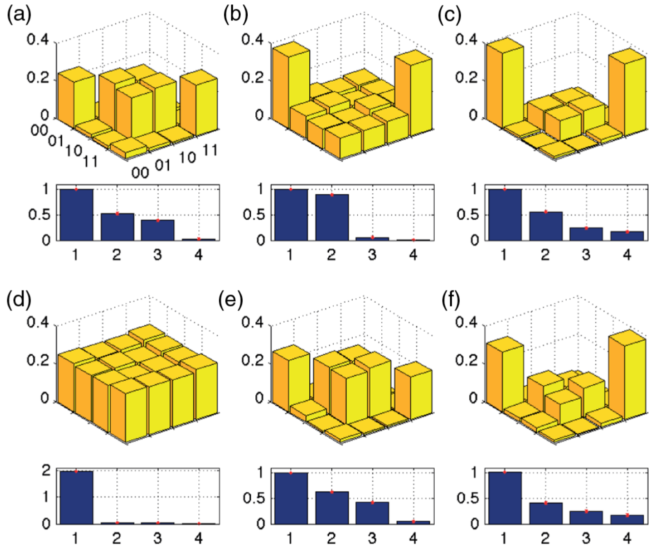

FIG. 3 (color online). Correlated dephasing results. Real values of experimentally reconstructed density matrices and corresponding singular values of the correlation matrix (underneath) for target states: (a) $\epsilon_{cd}^{\vec{n}}(\rho_1)$, (b) ρ_2, (c) $\epsilon_{cd}^{\vec{n}}(\rho_2)$, (d) $|\psi\rangle = |+,+\rangle$, where $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, (e) $\epsilon_{cd}^{\vec{n}}(|\psi\rangle\langle\psi|)$, (f) $\epsilon_{cd}^{\vec{n}}[K_{\vec{m}}(\pi/2)\epsilon_{cd}^{\vec{n}}(|\psi\rangle\langle\psi|)K_{\vec{m}}(\pi/2)^\dagger]$, for $\vec{m}\cdot\vec{\sigma} = \sigma_y$ and $\vec{n}\cdot\vec{\sigma} = \sigma_z$. The discord D_B is (a) $0.19 + (0.03/-0.03)$, (b) $0.01+(0.01/-0.01)$, (c) $0.19+(0.02/-0.03)$, (d) $0.012+(0.004/-0.005)$, (e) $0.23+(0.02/-0.02)$, (f) $0.12+(0.03/-0.04)$ The tangles are all less than 0.003, and the discord D_A is equal to D_B to within error in each case. Imaginary components of density matrices are all ≤ 0.003.

and $\vec{w}$, then an $R = 4$ state can be generated. We demonstrate this by preparing the new initial state $\rho_2 = K_{\hat{n}}(\pi/8)\rho_1 K_{\hat{n}}(\pi/8)^\dagger$ for $\vec{n}\cdot\vec{\sigma} = \sigma_y$ by applying a 729 nm laser pulse $K_{\hat{n}}(\pi/8)$ to ρ_1 and allowing B-field dephasing. Figures 3(b) and 3(c) present the results: the significantly increased third and fourth singular values are consistent with the conversion from an $R = 2$ to an $R = 4$ state.

In all cases considered so far, classical correlations have been present in the initial state. We also find that completely uncorrelated states ($R = 1$, $\rho = \rho_A \otimes \rho_B$) can be converted to high-rank states via correlated noise (see Supplemental Material [24]). The conditions for this can be described in terms of the reduced Bloch vectors of the two qubits $\vec{r}^A$ and $\vec{r}^B$. If either $\vec{n} = \vec{r}^A$ or $\vec{n} = \vec{r}^B$, then R is unchanged by correlated noise. In any other case, an $R = 3$ state is generated. Figures 3(d) and 3(e) demonstrate the conversion from an $R = 1$ to an $R = 3$ state. Figure 3(f) shows that even conversion from $R = 1$ to $R = 4$ is possible by combining dephasing with simple unitary operations. Specifically, as detailed in Fig. 3, we allow an $R = 1$ state to fully decohere via correlated B-field noise into an $R = 3$ state, apply single-qubit rotations to both qubits, and then allow the state to fully decohere in the same way again. Sequences of such unitary operations are common tools and regularly employed in quantum information processing tasks.

In conclusion, we have shown that in stark contrast to entanglement, discord can be generated by common noise processes. Specifically, two quantum systems can develop discord via operations on just one of them. Not all discordant states can be made this way, and the correlation rank provides a way to distinguish between fundamentally different kinds of discordant states. We have shown that noise processes generated by classically fluctuating fields are sufficient to generate discordant states with any rank, even starting with completely uncorrelated states. Our results should be relevant to a wide range of experimental systems since the noise processes considered are commonly present and can automatically lead to the generation of nonclassical correlations in many-body quantum systems. The current work can be extended to more ions (see Supplemental Material [24]). Finally, we note that all the generated states of nonzero discord lead to nonzero fidelities for the remote state preparation protocol [13], even those which have been created by local noise. On the other hand, states of $R > 2$, which can be created by correlated dephasing, enable the implementation of the quantum information transmission protocol [8].

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