Charge Form Factor and Sum Rules of Electromagnetic Response Functions in ^{12}C

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An *ab initio* calculation of the ^{12}C elastic form factor and sum rules of longitudinal and transverse response functions measured in inclusive (e, e') scattering are reported, based on realistic nuclear potentials and electromagnetic currents. The longitudinal elastic form factor and sum rule are found to be in satisfactory agreement with available experimental data. A direct comparison between theory and experiment is difficult for the transverse sum rule. However, it is shown that the calculated transverse sum rule has large contributions from two-body currents, indicating that these mechanisms lead to a significant enhancement of the quasielastic transverse response. This fact may have implications for the anomaly observed in recent neutrino quasielastic charge-changing scattering experiments on ^{12}C.

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The current picture of the nucleus as a system of protons and neutrons interacting among themselves via two- and three-body forces and with external electroweak probes via one- and two-body currents—a dynamical framework we will refer to below as the standard nuclear physics approach (SNPA)—has been shown to reproduce satisfactorily a variety of empirical properties of light nuclei with mass number $A \leq 12$, including energy spectra [1–7]; static properties [1,3,4,8,9] of low-lying states, such as charge radii and magnetic and quadrupole moments; and longitudinal electron scattering [10,11]. However, it has yet to be established conclusively whether such a picture quantitatively and successfully accounts for the observed electroweak structure and response of these systems, at least those with $A > 4$, in a wide range of energy and momentum transfers.

Recent neutrino quasielastic charge-changing scattering data on ^{12}C [12] show an excess, at relatively low energy, of measured cross section relative to theoretical calculations. However, those calculations use rather crude models of nuclear structure—Fermi gas [12,13] or local density approximations [14] of the nuclear matter spectral function—as well as simplistic treatments of the reaction mechanism, and hence do not include the potentially important two-nucleon effects that would be present in the standard approach.

We report an exact quantum Monte Carlo (QMC) calculation of the elastic form factor and sum rules associated with the longitudinal and transverse response functions measured in inclusive electron scattering experiments on ^{12}C. In addition to providing the first step toward a comprehensive study, within the SNPA, of the quasielastic electroweak response functions of light nuclei, the comparison of these results with the data provides an important verification of the SNPA.

The sum rules are defined as [15]

$$S_\alpha(q) = C_\alpha \int_{\omega_{th}^+}^\infty d\omega \frac{R_\alpha(q, \omega)}{G_E^{p2}(Q^2)}, \qquad (1)$$

where $R_\alpha(q, \omega)$ is the longitudinal $(\alpha = L)$ or transverse $(\alpha = T)$ response function, q and ω are the momentum and energy transfers, ω_{th} is the energy transfer corresponding to the inelastic threshold (the first excited-state energy is at 4.44 MeV relative to the ground state in ^{12}C), $G_E^p(Q^2)$ is the proton electric form factor evaluated at four-momentum transfer $Q^2 = q^2 - \omega^2$, and the C_α's are appropriate normalization factors, given by

$$C_L = \frac{1}{Z}, \qquad C_T = \frac{2}{(Z\mu_p^2 + N\mu_n^2)} \frac{m^2}{q^2}. \qquad (2)$$

Here, m is the nucleon mass, and Z (N) and μ_p (μ_n) are the proton (neutron) number and magnetic moment, respectively. These factors have been introduced so that $S_\alpha(q \to \infty) \simeq 1$ under the approximation that the nuclear charge and current operators originate solely from the charge and spin magnetization of individual protons and neutrons and that relativistic corrections to these one-body operators—such as the Darwin-Foldy and spin-orbit terms in the charge operator—are ignored.

It is well known [16] that the sum rules above can be expressed as ground-state expectation values of the type

$$S_\alpha(q) = C_\alpha[\langle 0|O_\alpha^\dagger(\mathbf{q})O_\alpha(\mathbf{q})|0\rangle - |\langle 0; \mathbf{q}|O_\alpha(\mathbf{q})|0\rangle|^2], \quad (3)$$

where $O_\alpha(\mathbf{q})$ is either the charge $\rho(\mathbf{q})$ ($\alpha = L$) or transverse current $\mathbf{j}_\perp(\mathbf{q})$ ($\alpha = T$) operator divided by $G_E^p(Q^2)$, $|0; \mathbf{q}\rangle$ denotes the ground state of the nucleus recoiling with total momentum $\mathbf{q}$, and averages over the spin projections have been suppressed because ^{12}C has $J^\pi = 0^+$. The $S_\alpha(q)$ as defined in Eq. (1) only includes the inelastic contribution to $R_\alpha(q, \omega)$; i.e., the elastic contribution represented by the second term on the right-hand side of Eq. (3) has been removed. It is proportional to the longitudinal (F_L) or transverse (F_T) elastic form factors. For ^{12}C, F_T vanishes, while $F_L(q)$ (to be discussed below) is given by $F_L(q) = G_E^p(Q_{el}^2)\langle 0; \mathbf{q}|O_L(\mathbf{q})|0\rangle/Z$, with the four-momentum transfer $Q_{el}^2 = q^2 - \omega_{el}^2$ and ω_{el} corresponding to elastic scattering $\omega_{el} = \sqrt{q^2 + m_A^2} - m_A$ (m_A is the ^{12}C mass).

The sum rules $S_\alpha(q)$ provide a useful tool for studying integral properties of the response of the nucleus to an external electromagnetic probe, and their calculation does not require any knowledge of the complicated structure of the nuclear excitation spectrum. Unfortunately, direct comparison between the calculated and experimentally extracted sum rules cannot be made unambiguously for two reasons. First, the experimental determination of S_α requires measuring the associated R_α in the whole energy-transfer region, from threshold up to ∞. Inclusive electron scattering experiments only allow access to the spacelike region of the four-momentum transfer ($\omega < q$). While the response in the timelike region ($\omega > q$) could, in principle, be measured via e^+e^- annihilation, no such experiments have been carried out to date. Therefore, for a meaningful comparison between theory and experiment, one needs to estimate the strength outside the region covered by the experiment. We will return to this issue below. For the moment, it suffices to say that in the past, this has been accomplished in the case of $S_L(q)$ either by extrapolating the data [17] or, in the few-nucleon systems, by parametrizing the high-energy tail and using energy-weighted sum rules to constrain it [18,19].

The second reason that direct comparison of theoretical and "experimental" sum rules is difficult lies in the inherent inadequacy of the current SNPA to account for explicit pion production mechanisms. The latter mostly affect the transverse response and make its Δ-peak region outside the range of applicability of this approach. However, the one- and two-body charge and current operators adopted in the present work should provide a realistic and quantitative description of both longitudinal and transverse response in the quasielastic region, where nucleon and (virtual) pion degrees of freedom are expected to be dominant. At low and intermediate momentum transfers ($q \lesssim 400$ MeV/c), the quasielastic and Δ peak are well separated, and it is therefore reasonable to study sum rules of the transverse response.

The ground-state wave function of ^{12}C is obtained from a Green's function Monte Carlo (GFMC) solution of the Schrödinger equation including the Argonne v_{18}

(AV18) two-nucleon (NN) [20] and Illinois-7 (IL7) three-nucleon (NNN) [2] potentials. The AV18 consists of a long-range component induced by one-pion exchange and intermediate-to-short-range components modeled phenomenologically and fits the NN scattering database for energies up to $E_{lab} = 350$ MeV with a χ^2 per datum close to one. The IL7 includes a central (albeit isospin dependent) short-range repulsive term and two-and three-pion-exchange mechanisms involving excitation of intermediate Δ resonances. Its strength is determined by four parameters which are fixed by a best fit to the energies of 17 low-lying states of nuclei in the mass range $A \leq 10$, obtained in combination with the AV18 NN potential. As already noted, the AV18 + IL7 Hamiltonian reproduces well the spectra of nuclei with $A \leq 10$ [2]—in particular, the attraction provided by the Illinois NNN potentials in isospin 3/2 triplets is crucial for the p-shell nuclei—and the p-wave resonances with $J^\pi = (3/2)^-$ and $(1/2)^-$ in low-energy neutron scattering off ^{4}He [21].

The ^{12}C ground-state wave function is evolved in imaginary time by GFMC calculations from a variational Monte Carlo (VMC) wave function that contains both explicit alpha clustering and the five possible $J^\pi = 0^+$ p-shell states [1,22]. These are multiplied by two- and three-body noncentral correlations [22]. Our ground-state energy and rms charge radius are $-93.3(4)$ MeV and 2.46(2) fm, respectively, in good agreement with the experimental values of -92.16 MeV and (2.471 ± 0.005) fm [23].

Realistic models for the electromagnetic charge and current operators include one- and two-body terms (see Ref. [24] for a recent overview). The former follow from a nonrelativistic expansion of the single-nucleon four-current, in which corrections proportional to $1/m^2$ are retained. Leading two-body terms are derived from the static part of the NN potential (the AV18 in the present case), which is assumed to be due to exchanges of effective pseudoscalar (π-like) and vector (ρ-like) mesons. The corresponding charge and current operators are constructed from nonrelativistic reductions of Feynman amplitudes with the π-like and ρ-like effective propagators projected out of the central, spin-spin, and tensor components of the NN potential. They contain no free parameters, and their short-range behavior is consistent with that of the potential. In particular, the longitudinal part of these two-body currents satisfies, by construction, current conservation with the (static part of the) NN potential. Additional contributions—purely transverse and hence unconstrained by current conservation—come from the $M1$ excitation of Δ resonances treated perturbatively in the intermediate state (for the current) and from the $\rho\pi\gamma$ transition mechanism (for the charge and current). For these, the values of the various coupling constants are taken from experiment [24]. As documented in Refs. [15,25,26], these charge and current operators reproduce quite well a variety of few-nucleon electromagnetic observables, ranging from elastic

form factors to low-energy radiative capture cross sections to the quasielastic response in inclusive (e, e') scattering at intermediate energies.

The spin-orbit and convection terms in $O_L(\mathbf{q})$ and $O_T(\mathbf{q})$ require gradients of both the bra and ket in Eq. (3); however, we cannot compute gradients of the evolved GFMC wave function. Therefore we compute these terms for only the VMC wave function and add them perturbatively to the GFMC results. They are generally quite small, although the convection term is significant for small q; see Fig. 3 below.

The calculations were made on Argonne's IBM Blue Gene/Q (Mira). Our GFMC program uses the Asynchronous Dynamic Load Balancing (ADLB) library [27] to achieve parallelization to more than 250 000 Message Passing Interface (MPI) processes with 80% efficiency to calculate the energy. The calculations of operators presented here require much more memory than just the energy evaluation, and we typically used four MPI processes on each 16 Gbyte node. We achieve good OpenMP scaling in each process: using 16 threads (the most possible) instead of only 4 reduces the time per configuration per q value from about 12 to 6 min. For each Monte Carlo configuration, we averaged over 12 directions of $\hat{\mathbf{q}}$ in Eq. (3); these were in four groups of three orthogonal directions obtained by implementing the method of uniformly distributed random rotations on a unit sphere [28]. The 12 calculations for each of 21 magnitudes of q (252 independent calculations) were distributed to different MPI processes by ADLB, with an efficiency above 95% on more than 32 000 MPI processes.
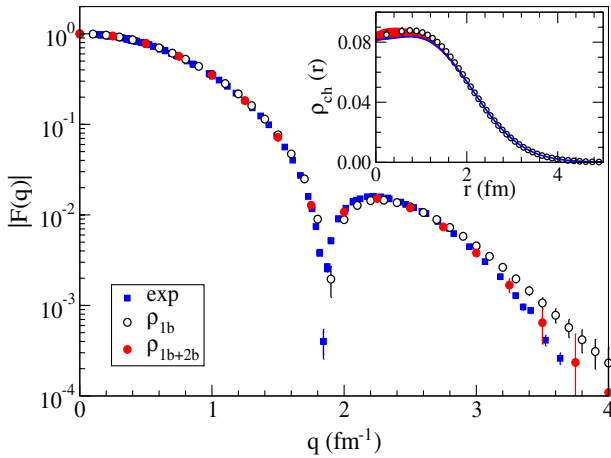
The calculated longitudinal elastic form factor (F_L) of ^{12}C is compared to experimental data in Fig. 1. These data are from an unpublished compilation by Sick [23,29] and are well reproduced by theory over the whole range of momentum transfers. The results labeled one body (1b) include, in addition to the proton, the neutron contribution as well as the Darwin-Foldy and spin-orbit relativistic corrections to the single-nucleon charge operator, while those labeled two body (2b) also contain the contributions due to the π-like, ρ-like, and $\rho\pi\gamma$ (two-body) charge operators. These two-body contributions are negligible at low q and become appreciable only for $q > 3$ fm^{-1}, where they interfere destructively with the one-body contributions, bringing theory into closer agreement with experiment. The Simon [30], Galster [31], and Höhler [32] parametrizations are used for the proton electric, neutron electric, and proton and neutron magnetic form factors, respectively.

In Figs. 2 and 3, we show by the open squares the experimental sum rules $S_L(q)$ and $S_T(q)$ obtained by integrating up to ω_{max} the longitudinal and transverse response functions (divided by the square of G_E^p) extracted from world data on inclusive (e, e') scattering off ^{12}C [17]. For $q = 1.53$, 1.94, and 2.90 fm^{-1}, ω_{max} in the longitudinal (transverse) case corresponds to, respectively, 140, 210, and 345 (140, 180, 285) MeV. We also show by the solid squares the experimental sum rules obtained by estimating the contribution of strength in the region $\omega > \omega_{max}$. This estimate $\Delta S_\alpha(q)$ is made by assuming that for $\omega > \omega_{max}$, i.e., well beyond the quasielastic peak, the (longitudinal or

FIG. 1 (color online). The longitudinal elastic form factor of ^{12}C obtained from the AV18 + IL7 Hamiltonian with one-body only (empty circles) and one- plus two-body (solid circles) terms in the charge operator is compared to experimental data (solid squares). Also displayed are the statistical errors of the QMC calculation. The inset shows the calculated charge density in coordinate space with one-body (empty circles) and (one + two)-body (red band) terms compared with an analysis of the experimental data (solid line) [36].
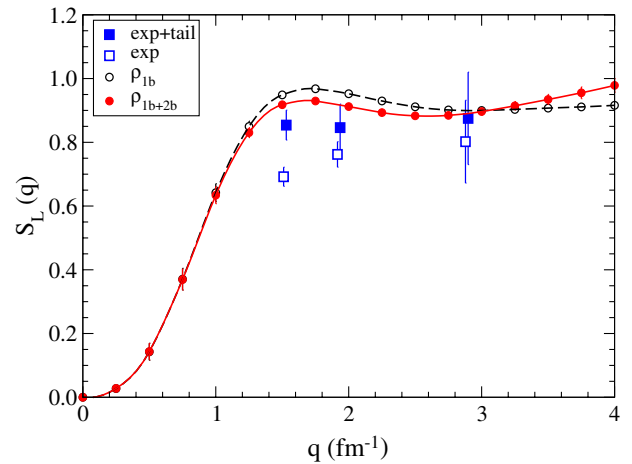
FIG. 2 (color online). The longitudinal sum rule of ^{12}C obtained from the AV18 + IL7 Hamiltonian with one-body only (empty circles, dashed line) and one- plus two-body (solid circles, solid line) terms in the charge operator is compared to experimental data without (empty squares), labeled exp, and with (solid squares), labeled exp+tail, the tail contribution; see the text. Also displayed are the statistical errors of the QMC calculation.
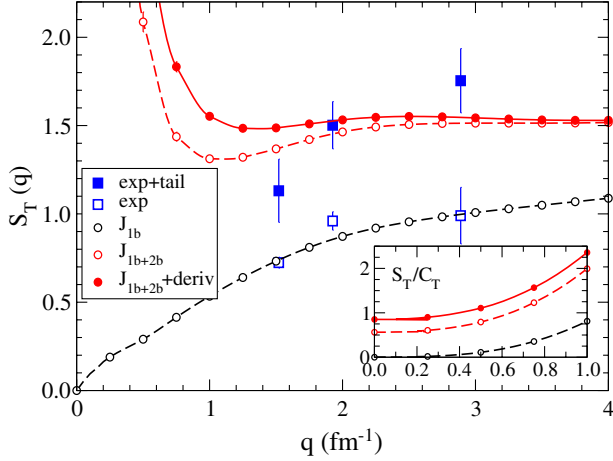
FIG. 3 (color online). Same as in Fig. 2, but for the transverse sum rule. The open symbols do not contain derivative terms, while a VMC evaluation of the derivative terms is included for the solid red dots. The inset shows $S_T(q)/C_T$ in the small q region.

transverse) response in a nucleus like ^{12}C (R_α^A) is proportional to that in the deuteron (R_α^d), which can be exactly calculated [24]. In particular, R_α^d has been calculated using AV18, but very similar results are obtained by using next-to-next-to-next-to-leading order [33] instead. Thus, we set $R_\alpha^A(q, \omega > \omega_{max}) = \lambda(q)R_\alpha^d(q, \omega)$ and determine $\lambda(q)$ by matching the experimental ^{12}C response to the calculated deuteron one. In practice, $\Delta S_\alpha(q)$ follows from

$$\Delta S_\alpha(q) = \lambda(q)C_\alpha^A\left[\frac{S_\alpha^d(q)}{C_\alpha^d} - \int_{\omega_{th}^+}^{\omega_{max}}\frac{R_\alpha^d(q, \omega)}{G_E^{p2}(Q^2)}\right], \quad (4)$$

where the C_α^A and C_α^d are the normalization factors associated with the nucleus and deuteron, respectively, and $S_\alpha^d(q)$ is the deuteron sum rule. It is worthwhile emphasizing that, for the transverse case, this estimate is particularly uncertain for the reasons explained earlier. In particular, the data on R_T at $q = 1.94$ and 2.90 fm − 1 [17] suggest that at $\omega \sim \omega_{max}$, there might be already significant strength that has leaked in from the Δ-peak region.

The scaling assumption above assumes that the high-energy part of the response is dominated by two-nucleon physics and that the most important contribution is from deuteronlike np pairs. The high-energy response can be obtained from the Fourier transform of the short-time response $(2\pi)S_\alpha(q, \omega) = \int dt\langle0|O_\alpha^\dagger(\mathbf{q}) \times \exp(-iHt)O_\alpha(\mathbf{q})|0\rangle$, or equivalently from the small imaginary time dependence of the propagator. At short times, the full propagator is governed by the product of pair propagators (assuming three-nucleon interactions are weak), and hence we expect the scaling with deuteronlike pairs.

The sum rules computed with the AV18 + IL7 Hamiltonian and one-body only or one- and two-body terms in the charge (S_L) and current (S_T) operators are shown, respectively, by the dashed and solid lines in Figs. 2 and 3. In the small q limit, $S_L(q)$ vanishes quadratically, while the divergent behavior in $S_T(q)$ is due to the $1/q^2$ present in the normalization factor C_T. In this limit, $O_T(\mathbf{q} = 0) = i[H, \sum_i \mathbf{r}_i P_i]$ [25,26], where H is the Hamiltonian and P_i is the proton projector, and therefore $S_T(q)/C_T$ is finite; indeed, the associated strength is due to collective excitations of electric-dipole type in the nucleus. In the large q limit, the one-body sum rules differ from one because of relativistic corrections in $O_L(\mathbf{q})$, primarily the Darwin-Foldy term which gives a contribution $-\eta/(1 + \eta)$ to $S_L^{1b}(q)$, where $\eta \simeq q^2/(4m^2)$, and because of the convection term in $O_T(\mathbf{q})$, which gives a contribution $\simeq (4/3)C_T T_p/m$ to $S_T^{1b}(q)$, where T_p is the proton kinetic energy in the nucleus.

The calculated $S_L(q)$ is in satisfactory agreement with the experimental values, including tail contributions, and no significant quenching of longitudinal strength is observed. This is achieved by dividing the experimental $R_L(q, \omega)$ by the (square of the) free proton electric form factor. In contrast to S_L, the transverse sum rule has large two-body contributions—at $q = 2.5$ fm^{-1} these increase S_T^{1b} by about 50%. Studies of Euclidean transverse response functions in the few-nucleon systems within the same SNPA adopted here [15] suggest that a significant portion of this excess transverse strength is in the quasi-elastic region. Clearly, a direct QMC calculation of the ^{12}C response functions is needed to resolve this issue conclusively. It will also be interesting to see the extent to which these considerations—in particular, the major role played by two-body currents—will remain valid in the weak sector probed in neutrino scattering, and possibly provide an explanation for the observed ^{12}C anomaly mentioned in the introduction. There are indications from random phase approximation calculations including $2p$-$2h$ states that this may indeed be the case [34,35].

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