Internal Bremsstrahlung Signature of Real Scalar Dark Matter and Consistency with Thermal Relic Density

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A gamma-ray excess from the Galactic center consistent with line emission around 130 GeV was recently found in the Fermi-LAT data. Although the Fermi-LAT Collaboration has not confirmed its significance, such a signal would be a clear signature of dark matter annihilation. Until now, there have been many attempts to explain the excess by dark matter. However, these efforts tend to give too-small cross sections into photons if consistency with the correct thermal relic density of dark matter is required. In this Letter, we consider a simple Yukawa interaction that can be compatible with both aspects and show which parameters are favored.

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Observations of gamma rays, cosmic-ray positrons, antiprotons, and neutrinos are being performed to look for dark matter (DM) signatures. In particular, Fermi-LAT public data have recently been examined in detail, and a gamma-ray excess around 130 GeV from the region of the Galactic center has been claimed [1,2]. The Fermi-LAT Collaboration also found the excess at 135 GeV independently [3]; however, they found a much lower significance in the reprocessed data set [4]. Many authors have provided models of this excess by monochromatic gamma rays from DM annihilation or decay; see, for example, Refs. [5–23]. If the source of the gamma-ray excess is DM annihilation, the required cross section into two photons is $\sigma v_{\gamma\gamma} = 1.27 \times 10^{-27}\ \mathrm{cm}^3/\mathrm{s}$ for an Einasto DM density distribution; this value can change for a different DM profile [2]. The process of DM annihilation into two photons is loop suppressed because DM does not have electric charge. The loop-suppression factor is naively expected to be $\alpha_{\mathrm{em}}^2/(4\pi)^2 \sim 10^{-7}$ compared with the annihilation cross section $\sigma v_{\mathrm{th}} \sim 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$, where α_{em} is the electromagnetic fine structure constant. This value of σv_{th} is needed to achieve the correct relic density of DM. Thus, it seems difficult to be consistent with the thermal relic density of DM unless some enhancement of the cross section is introduced [24]. In other words, if we assume DM is thermally produced, the gamma-ray production cross section is fixed to a value that is too small to explain the excess around 130 GeV.

Another possibility is the explanation via internal bremsstrahlung (IB) of Majorana DM [1]. The possibility of explaining the 130 GeV excess with IB has been explored in Refs. [1,25–28]. This process is the radiative correction for the final state charged particles and the intermediate particle. The IB process generates a linelike energy spectrum. The suppression factor compared with σv_{th} is roughly $\alpha_{\mathrm{em}}/\pi \sim 10^{-3}$, which is larger than the monochromatic photon case. Thus, the IB process has better prospects than the monochromatic $\gamma\gamma$ process from this point of view. However, even with the IB process, it seems difficult to be compatible with the thermal relic density of DM. For standard p-wave annihilating neutralino DM, the IB signal is still a factor of a few below the nominally required rate for the observed density.

In this Letter, we consider IB for real scalar DM interacting with a fermionic mediator and a light fermion. As we discuss below, the annihilation cross section into a light fermion-antifermion pair is expanded with the relative velocity of DM, with a suppressed constant term. As a result, a higher order term of the cross section can be dominant in the early Universe, and the cross section into gamma rays becomes relatively large at present times, thus reconciling the relic density value and the interpretation of the gamma-ray excess by DM annihilation.

We consider a real scalar DM particle χ which has the following Yukawa interaction with the electromagnetically charged fermion f and the fermionic mediator ψ:

$$\mathcal{L} = y_L \chi \bar{\psi} P_L f + \mathrm{H.c.}, \tag{1}$$

where the fermion f is typically a light lepton or a quark. The annihilation cross section into $f\bar{f}$ is expanded as $\sigma v_{f\bar{f}} = a + bv^2 + cv^4 + \mathcal{O}(v^6)$ with the DM relative velocity v, and it is calculated under the approximation of $m_f \ll m_\chi$ as

$$\sigma v_{f\bar{f}} = \frac{y_L^4}{4\pi m_\chi^2}\frac{m_f^2}{m_\chi^2}\frac{1}{(1+\mu)^2} - \frac{y_L^4}{6\pi m_\chi^2}\frac{m_f^2}{m_\chi^2}\frac{1+2\mu}{(1+\mu)^4}v^2 + \frac{y_L^4}{60\pi m_\chi^2}\frac{1}{(1+\mu)^4}v^4 + \mathcal{O}(v^6), \tag{2}$$

where the Yukawa coupling y_L is assumed to be real and the parameter μ is the ratio of masses defined as $\mu \equiv m_\psi^2/m_\chi^2 > 1$. The first and second terms of Eq. (2), which are called the s wave and p wave, respectively, agree with the Appendix of Ref. [29]. In addition, the d-wave term which is proportional to v^4 is easily found to be the leading term in the limit of $m_f \to 0$. The s wave and p wave are suppressed by the factor m_f^2/m_χ^2; thus, the d wave becomes the dominant contribution to the cross section in the early Universe when μ is not large enough. Conversely, the s wave becomes dominant today even if the mass of particle f is as low as the electron mass. The nonrelativistic thermally averaged cross section $\langle \sigma v_{f\bar{f}} \rangle$ is given by substituting $\langle v^2 \rangle \to 6T/m_\chi$ and $\langle v^4 \rangle \to 60T^2/m_\chi^2$, where T is the temperature of the Universe. This replacement coincides with Refs. [30,31]. The thermally averaged cross section is important to estimate the relic density of DM. The typical value of the temperature that sets the correct relic density is roughly $m_\chi/T \approx 20$–25.

Next, we consider the radiative correction for the above two-body process, that is, $\chi\chi \to f\bar{f}\gamma$, shown in Fig. 1. This process is the IB of the real scalar DM, and the emitted photon can be a gamma-ray signal, which is comparable with the Fermi-LAT excess. The amplitude for the total IB is separated to two pieces of final state radiation (FSR) and virtual internal bremsstrahlung (VIB). One cannot generally treat the VIB diagram separately in a gauge invariant manner, and it is only the sum of all three diagrams in Fig. 1 that is gauge invariant. Here, we define the FSR amplitude as the leading term of the differential cross section in Eq. (4), and the VIB one as the amplitude removing the chiral suppression in the s wave to the annihilation cross section. These definitions of FSR and VIB makes the following discussion clear; however, we note that they are different definitions from Ref. [32]. Thus, the differential cross section is expressed as

$$\frac{d\sigma v_{f\bar{f}\gamma}}{dx} = \frac{d\sigma v_{f\bar{f}\gamma}^{\rm FSR}}{dx} + \frac{d\sigma v_{f\bar{f}\gamma}^{\rm VIB}}{dx}, \tag{3}$$

with $x = E_\gamma/m_\chi$, where the interference term between the FSR and VIB amplitudes is neglected here.

The first term in Eq. (3) of FSR can be written in the model-independent way:

$$\frac{d\sigma v_{f\bar{f}\gamma}^{\rm FSR}}{dx} \approx \sigma v_{f\bar{f}} \frac{Q^2\alpha_{\rm em}}{\pi} \frac{(1-x)^2+1}{x} \log\left(\frac{4m_\chi^2(1-x)}{m_f^2}\right), \tag{4}$$

where Q stands for the electromagnetic charge of ψ and f. A similar result is obtained for a bosonic final state, but the x dependence is different [33]. The FSR differential cross section is proportional to the two-body cross section $\sigma v_{f\bar{f}}$. This implies that if $m_f \ll m_\chi$, the FSR gives a very small contribution and it can be negligible at present times. The energy spectrum of FSR is broad and it is not suitable to explain the gamma-ray excess. If the FSR contribution is not suppressed, the energy spectrum of the first term in Eq. (3) invariably becomes greater than the second term.

The second term in Eq. (3) represents the VIB contribution [34,35]. This process is well known for enhancing the s-wave component in such chirally suppressed models. The differential cross section of the VIB process for Majorana DM has been calculated in Refs. [32,36–38]. Similarly, it is calculated for real scalar DM by following Ref. [32] as

$$\frac{d\sigma v_{f\bar{f}\gamma}^{\rm VIB}}{dx} = \frac{Q^2\alpha_{\rm em}y_L^4}{4\pi^2 m_\chi^2}(1-x)\left[\frac{2x}{(\mu+1)(\mu+1-2x)} - \frac{x}{(\mu+1-x)^2} - \frac{(\mu+1)(\mu+1-2x)}{2(\mu+1-x)^3}\log\left(\frac{\mu+1}{\mu+1-2x}\right)\right], \tag{5}$$

and the total cross section is obtained by integrating Eq. (5) in the range of $0 \le x \lesssim 1$ as follows:

$$\sigma v_{f\bar{f}\gamma}^{\rm VIB} = \frac{Q^2\alpha_{\rm em}y_L^4}{8\pi^2 m_\chi^2}\left\{(\mu+1)\left[\frac{\pi^2}{6} - \log^2\left(\frac{\mu+1}{2\mu}\right) - 2{\rm Li}_2\left(\frac{\mu+1}{2\mu}\right)\right] + \frac{4\mu+3}{\mu+1} + \frac{4\mu^2-3\mu-1}{2\mu}\log\left(\frac{\mu-1}{\mu+1}\right)\right\}, \tag{6}$$

where ${\rm Li}_2(z)$ is the dilogarithm function defined by ${\rm Li}_2(z) = -\int_0^1 \log(1-zt)/t dt$. The above VIB cross section for real scalar DM is a factor of 8 times larger than that for Majorana DM. Note that the continuum gamma-ray spectrum due to hadronization should be added in Eq. (3) when the final state particles are tauons or light quarks.

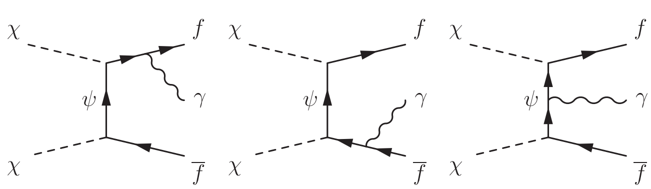

FIG. 1. Internal bremsstrahlung processes of (real) scalar DM.

The above discussion is valid when the other interactions are sufficiently suppressed. Here, we add the interaction with the right-handed component of the fermion f, and we estimate how much hierarchy is necessary among the interactions in order for the above scheme to work. If the interaction with the left- and right-handed fermions is

$$\mathcal{L} = \chi\bar{\psi}(y_L P_L + y_R P_R)f + {\rm H.c.}, \tag{7}$$

the s-wave and p-wave components are not suppressed as in Eq. (2), and they are given by

$$\sigma v_{f\bar{f}} = \frac{y_L^2 y_R^2}{\pi m_\chi^2}\frac{\mu}{(1+\mu)^2} - \frac{y_L^2 y_R^2}{3\pi m_\chi^2}\frac{\mu+3\mu^2}{(1+\mu)^4}v^2 + \mathcal{O}(v^4). \tag{8}$$

This formula coincides with the Appendix of Ref. [29]. We can estimate the required condition among the parameters to validate the above discussion. The s-wave component should be suppressed enough, compared with the d wave, leading to the condition

$$\left(\frac{y_R}{y_L}\right)^2 \lesssim \frac{v^4}{60\mu(1+\mu)^2}. \tag{9}$$

Therefore, $y_R/y_L \lesssim 0.02$ is required when $\mu \sim 1$ and $v^2 \sim 0.3$. For example, even if y_R does not exist at tree level, y_R is induced at one-loop level from the left-handed Yukawa coupling y_L. When $\mu \sim 1$, the right-handed Yukawa coupling y_R is then

$$y_R \approx -\frac{y_L^3}{2(4\pi)^2}\mu\frac{m_f}{m_\chi}. \tag{10}$$

This is sufficiently small compared with y_L because of the factor m_f/m_χ, and the requirement Eq. (9) is satisfied.

We comment about other DM models, such as complex scalar and fermionic DM. In the case of complex scalar DM, the s-wave component of the two-body cross section is suppressed by the factor m_f^2/m_χ^2, just like real scalar DM, but the p-wave component remains. Thus, this framework discussed above does not work. For Majorana DM, the nonsuppressed p-wave term is also present, and we cannot reconcile the DM relic abundance and the explanation of the gamma-ray excess by DM annihilation. For the Dirac DM case, the s-wave term exists for the two-body process, and the FSR process is always larger than the VIB process. Thus, we cannot obtain the linelike gamma-ray spectrum from VIB because the VIB signal is swamped by the broad FSR spectrum.

We numerically analyze the consistency between the thermal relic abundance of DM and the gamma-ray excess. In the following calculation, fermion f is taken to be the electron. The thermal relic density of DM is obtained by solving the Boltzmann equation [30,31,39]

$$\frac{z}{Y_{\rm eq}}\frac{dY}{dz} = -\frac{\Gamma}{H}\left(\frac{Y^2}{Y_{\rm eq}^2}-1\right), \tag{11}$$

where Y is defined as the DM number density n_χ divided by the entropy density of the Universe, and $Y_{\rm eq}$ is the value of Y in thermal equilibrium. The reaction rate Γ is defined as $\Gamma \equiv \langle\sigma v\rangle n_\chi^{\rm eq}$ with the number density in thermal equilibrium, H is the Hubble parameter, and z is a dimensionless parameter defined by $z = m_\chi/T$. The total cross section implies $\langle\sigma v\rangle = \langle\sigma v_{f\bar{f}}\rangle + \langle\sigma v_{f\bar{f}\gamma}\rangle$. In general, we need approximately $\langle\sigma v\rangle \sim 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$ in order to get the correct relic abundance of DM, which is $\Omega_\chi h^2 = 0.120$ observed by Planck [40].

We have only three parameters: m_χ, μ, and y_L. We solve the Boltzmann equation numerically with an implicit method. The contours of Yukawa coupling y_L which satisfy the observed DM relic density are depicted in Fig. 2 in the m_χ-μ plane. We can see from the figure that a larger

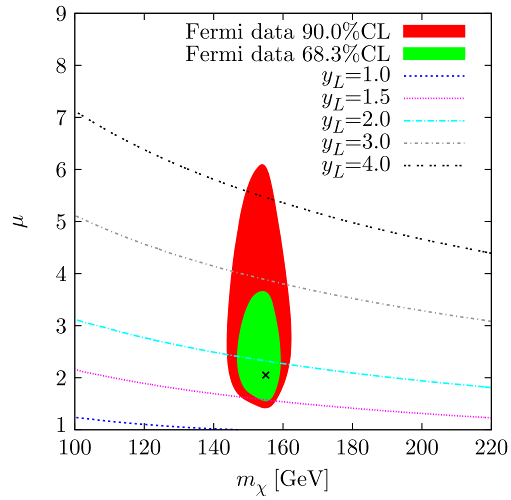

FIG. 2 (color online). The contours satisfying the DM relic density and the favored region to fit to the gamma-ray excess in the m_χ-μ plane. The fermion f is assumed to be an electron here.

Yukawa coupling is required for larger μ. Coannihilation between the DM χ and the mediator ψ begins to be effective in the region of $\mu \lesssim 1.2$. For example, the process $\chi\psi \to fH$ can occur with the interaction of Eq. (1) and the standard model (SM) Yukawa couplings, where H is the SM Higgs boson. However, it would be small for the light SM charged particles. If the other interactions lead to effective coannihilation, this should be taken into account, as it may affect the numerical analysis.

The gamma-ray flux coming from DM annihilation for the target region $\Delta\Omega$ is given by

$$\frac{d\Phi_\gamma^{\rm DM}}{dE_\gamma} = \frac{r_\odot}{8\pi}\frac{\rho_\odot^2}{m_\chi^2}\bar{J}\langle\sigma v_\gamma\rangle\frac{dN_\gamma}{dE_\gamma}, \tag{12}$$

where $r_\odot = 8.5$ kpc is the distance of Earth from the Galactic center, and $\rho_\odot = 0.4\ \mathrm{cm}^3/\mathrm{s}$ is the local DM density [41]. The parameter $\bar{J}$ is defined as

$$\bar{J} \equiv \frac{1}{\Delta\Omega}\int db d\ell \cos b \int_{\text{line of sight}} \frac{ds}{r_\odot}\left(\frac{\rho(r,b,\ell)}{\rho_\odot}\right)^2, \tag{13}$$

where b and ℓ are the galactic latitude and longitude of the target region. The integral variable s is related with the distance from the Galactic center r as $r(s,b,\ell) = \sqrt{r_\odot^2 + s^2 - 2r_\odot s \cos b \cos\ell}$. Note that the energy dependence arises only from the energy spectrum dN_γ/dE_γ, and $\langle\sigma v_\gamma\rangle dN_\gamma/dE_\gamma$ simply corresponds to Eq. (3) at the present situation. We use the generalized Navarro-Frenk-White profile [42], which is written as

$$\rho(r) = \frac{\rho_s}{(r/r_s)^\alpha(1+r/r_s)^{3-\alpha}}. \tag{14}$$

It corresponds to the normal Navarro-Frenk-White profile if $\alpha = 1$. The parameter ρ_s is the normalization factor in order to fix to $\rho(r_\odot) = 0.4\ \mathrm{GeV/cm}^3$. The parameters r_s

and α are taken as $r_s = 20$ kpc and $\alpha = 1.15$. This DM profile, as well as the parameter values, are the same as those in Ref. [43] used to fit the excess around 130 GeV in the Fermi-LAT data. We focus on the region of Reg4 in Refs. [2,43] to compare the gamma-ray flux from DM annihilation and the claimed gamma-ray excess. The background of the gamma-ray flux is evaluated by the fitting function [2] in the unit of $\mathrm{GeV}^{-1}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$

$$\frac{d\Phi_\gamma^B}{dE_\gamma} = 2.4 \times 10^{-5} E_\gamma^{-2.55}. \tag{15}$$

We find the best fit point in the parameter space of m_χ, μ, and y_L to give the gamma-ray excess. The 53 data points counting from the upper energy are taken from Ref. [43] and used for a chi-square analysis. Simultaneously, the constraint of the DM relic density is also imposed. As a result of the analysis, we get the best fit point of $m_\chi = 155$ GeV, $\mu = 2.05$, and $y_L = 1.82$ with $\chi^2_{\rm min} = 65.57$ (51 degree of freedom). From the values, the cross section is calculated as $\langle \sigma v_{f\bar{f}\gamma} \rangle = 4.72 \times 10^{-27}$ cm^3/s, which is comparable with 6.2×10^{-27} cm^3/s obtained in Ref. [1], while the parameter setting is slightly different. The favored m_χ-μ region to fit to the gamma-ray excess is shown in Fig. 2, where the Yukawa coupling y_L is fixed by the constraint of the thermal relic density of DM at each point. From the figure, we can see that the DM abundance and the gamma-ray excess coming from DM annihilation are consistent with each other. The favored region in large μ would be slightly changed if the monochromatic photon induced by the box diagrams is taken into account in the model [44]. The fitting of the gamma-ray excess with the evaluated values is depicted in Fig. 3.

The Yukawa interaction considered here contributes to the anomalous magnetic moment of fermion f; thus, it may constrain the strength of the interaction. The anomalous magnetic moment of f is calculated from the Yukawa interaction Eq. (1) as [29]

$$\delta a_f = \frac{y_L^2}{(4\pi)^2} \frac{m_f^2}{m_\chi^2} \frac{2 + 3\mu - 6\mu^2 + \mu^3 + 6\mu \log\mu}{6(1-\mu)^4}. \tag{16}$$

The current experimental bound [45,46] for the electron anomalous magnetic moment is given as $\delta a_e \equiv a_e(\mathrm{SM}) - a_e(\mathrm{exp}) = 1.06 \times 10^{-12}$ [47], while the value at the fitting point is 2 orders of magnitude lower: $\delta a_e \approx 9.4 \times 10^{-15}$. For the muon, the experimental bound is $\delta a_\mu = 25.5 \times 10^{-10}$ [48,49] and our value is 4.0×10^{-10}, 1 order of magnitude below the bound. However, for example, if we have simultaneous Yukawa couplings to both the electron and muon, the Yukawa coupling is extremely constrained by charged lepton flavor violating processes like $\mu \to e\gamma$ unless destructive interference occurs.

We have discussed scalar DM having Yukawa interaction with the left-handed light fermion f and the mediator ψ. The annihilation cross section of the DM into $f\bar{f}$ is highly suppressed since the s wave and p wave are proportional to the ratio of masses m_f^2/m_χ^2. As a result, the d wave can be dominant in the early Universe, and the DM relic abundance is obtained by the d-wave cross section. Simultaneously, the VIB component of the radiative correction for the process that is $\chi\chi \to f\bar{f}\gamma$ has an s wave and it gives the linelike gamma-ray signal. The recently observed gamma-ray excess is well explained without inconsistency with the thermal relic density of DM. We have three parameters of m_χ, μ, and y_L and obtained the best fit point $m_\chi = 155$ GeV, $\mu = 2.05$, and $y_L = 1.82$ by fitting to the gamma-ray excess with the constraint of the DM thermal relic density.

The framework discussed here works when the other interactions of DM are small enough. The lightest right-handed sneutrino DM in supersymmetric extended models would be a realistic candidate since the chargino plays a role in the mediator ψ. To do that, the neutrino Yukawa interaction should be large. Therefore, supersymmetric radiative seesaw models such as Refs. [50,51] would be promising concrete models to implement this scheme because the neutrino Yukawa coupling can be order one and tiny neutrino masses are derived without contradiction. In addition, inverse seesaw models with supersymmetry would also be good candidates for the framework [52].

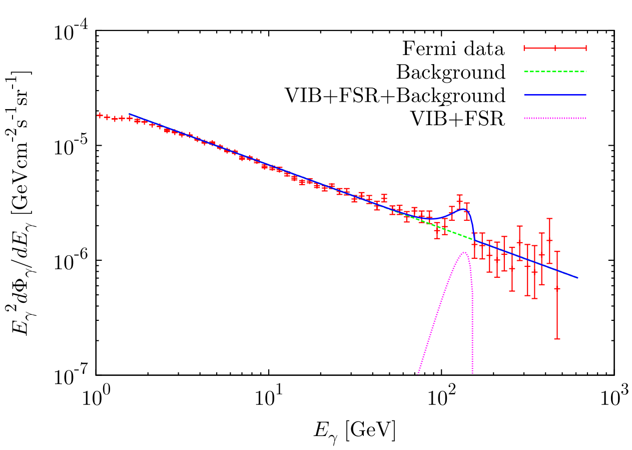

FIG. 3 (color online). Fitting to the gamma-ray excess via the VIB process. We use the best fit parameters found here. The data are taken from Ref. [43]. Note that the energy dispersion of the Fermi instrument is included; it is approximately 10% at $E_\gamma = 100$ GeV. This may alter the fit region.

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[1] T. Bringmann, X. Huang, A. Ibarra, S. Vogl, and C. Weniger, J. Cosmol. Astropart. Phys. 07 (2012) 054.
[2] C. Weniger, J. Cosmol. Astropart. Phys. 08 (2012) 007.
[3] E. Bloom *et al.* (on behalf of the Fermi-LAT Collaboration), arXiv:1303.2733.
[4] M. Ackermann, Fermi-LAT Collaboration, arXiv:1305.5597.
[5] E. Tempel, A. Hektor, and M. Raidal, J. Cosmol. Astropart. Phys. 09 (2012) 032; 11 (2012) A01.
[6] J. M. Cline, Phys. Rev. D **86**, 015016 (2012).
[7] K.-Y. Choi and O. Seto, Phys. Rev. D **86**, 043515 (2012); **86**, 089904 (2012).
[8] B. Kyae and J.-C. Park, Phys. Lett. B **718**, 1425 (2013).
[9] M. R. Buckley and D. Hooper, Phys. Rev. D **86**, 043524 (2012).
[10] D. Das, U. Ellwanger, and P. Mitropoulos, J. Cosmol. Astropart. Phys. 08 (2012) 003.
[11] Z. Kang, T. Li, J. Li, and Y. Liu, arXiv:1206.2863.
[12] W. Buchmuller and M. Garny, J. Cosmol. Astropart. Phys. 08 (2012) 035.
[13] J.-C. Park and S. C. Park, Phys. Lett. B **718**, 1401 (2013).
[14] J. M. Cline, G. D. Moore, and A. R. Frey, Phys. Rev. D **86**, 115013 (2012).
[15] L. Wang and X.-F. Han, Phys. Rev. D **87**, 015015 (2013).
[16] K. Schmidt-Hoberg, F. Staub, and M. W. Winkler, J. High Energy Phys. 01 (2013) 124.
[17] Y. Farzan and A. R. Akbarieh, arXiv:1211.4685.
[18] G. Chalons, M. J. Dolan, and C. McCabe, J. Cosmol. Astropart. Phys. 02 (2013) 016.
[19] M. Asano, T. Bringmann, G. Sigl, and M. Vollmann, Phys. Rev. D **87**, 103509 (2013).
[20] C. B. Jackson, G. Servant, G. Shaughnessy, T. M. P. Tait, and M. Taoso, arXiv:1302.1802.
[21] J. Kumar and P. Sandick, Phys. Rev. D **87**, 123534 (2013).
[22] A. Ibarra, H. M. Lee, S. Lóp. Gehler, W.-I. Park, and M. Pato, J. Cosmol. Astropart. Phys. 05 (2013) 016.
[23] K.-Y. Choi, H. M. Lee, and O. Seto, Phys. Rev. D **87**, 123541 (2013).
[24] S. Tulin, H.-B. Yu, and K. M. Zurek, Phys. Rev. D **87**, 036011 (2013).
[25] T. Bringmann and C. Weniger, Phys. Dark Univ. **1**, 194 (2012).
[26] L. Bergstrom, Phys. Rev. D **86**, 103514 (2012).
[27] B. Shakya, arXiv:1209.2427.
[28] M. Garny, A. Ibarra, M. Pato, and S. Vogl, arXiv:1306.6342.
[29] C. Boehm and P. Fayet, Nucl. Phys. **B683**, 219 (2004).
[30] M. Srednicki, R. Watkins, and K. A. Olive, Nucl. Phys. **B310**, 693 (1988).
[31] P. Gondolo and G. Gelmini, Nucl. Phys. **B360**, 145 (1991).
[32] P. Ciafaloni, M. Cirelli, D. Comelli, A. De Simone, A. Riotto, and A. Urbano, J. Cosmol. Astropart. Phys. 06 (2011) 018.
[33] A. Birkedal, K. T. Matchev, M. Perelstein, and A. Spray, arXiv:hep-ph/0507194.
[34] L. Bergstrom, Phys. Lett. B **225**, 372 (1989).
[35] R. Flores, K. A. Olive, and S. Rudaz, Phys. Lett. B **232**, 377 (1989).
[36] T. Bringmann, L. Bergstrom, and J. Edsjo, J. High Energy Phys. 01 (2008) 049.
[37] L. Bergstrom, T. Bringmann, and J. Edsjo, Phys. Rev. D **78**, 103520 (2008).
[38] M. Asano, T. Bringmann, and C. Weniger, Phys. Lett. B **709**, 128 (2012).
[39] E. W. Kolb and M. S. Tuner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1989).
[40] P. A. R. Ade *et al.* (Planck Collaboration), arXiv:1303.5076.
[41] M. Cirelli, G. Corcella, A. Hektor, G. Hutsi, M. Kadastik, P. Panci, M. Raidal, F. Sala, and A. Strumia, J. Cosmol. Astropart. Phys. 03 (2011) 051; 10 (2012) E01.
[42] J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. **490**, 493 (1997).
[43] C. Weniger, AIP Conf. Proc. **1505**, 470 (2012).
[44] F. Giacchino, L. Lopez-Honorez, and M. H. G. Tytgat, arXiv:1307.6480.
[45] D. Hanneke, S. Fogwell, and G. Gabrielse, Phys. Rev. Lett. **100**, 120801 (2008).
[46] D. Hanneke, S. F. Hoogerheide, and G. Gabrielse, Phys. Rev. A **83**, 052122 (2011).
[47] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. Lett. **109**, 111807 (2012).
[48] G. W. Bennett *et al.* (Muon $g-2$ Collaboration), Phys. Rev. D **73**, 072003 (2006).
[49] P. von Weitershausen, M. Schafer, H. Stockinger-Kim, and D. Stockinger, Phys. Rev. D **81**, 093004 (2010).
[50] D. Suematsu and T. Toma, Nucl. Phys. **B847**, 567 (2011).
[51] H. Fukuoka, D. Suematsu, and T. Toma, J. Cosmol. Astropart. Phys. 07 (2011) 001.
[52] S. Khalil, H. Okada, and T. Toma, J. High Energy Phys. 07 (2011) 026.