Axial Symmetry Breaking in Self-Induced Flavor Conversion of Supernova Neutrino Fluxes

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(Received 31 May 2013; published 28 August 2013)

Neutrino-neutrino refraction causes self-induced flavor conversion in dense neutrino fluxes. For the first time, we include the azimuth angle of neutrino propagation as an explicit variable and find a new generic multi-azimuth-angle instability which, for simple spectra, occurs in the normal neutrino mass hierarchy. Matter suppression of this instability in supernovae requires larger densities than the traditional bimodal case. The new instability shows explicitly that solutions of the equations for collective flavor oscillations need not inherit the symmetries of initial or boundary conditions. This change of paradigm requires reconsideration of numerous results in this field.

DOI: 10.1103/PhysRevLett.111.091101 PACS numbers: 97.60.Bw, 14.60.Pq

Introduction.—Flavor oscillations depend strongly on matter because the weak-interaction potential $\sqrt{2}G_{\mathrm{F}}n_e$ can far exceed the oscillation energy $\omega = \Delta m^2/2E$ [1,2]. A matter gradient can cause complete flavor conversion [Mikheev-Smirnov-Wolfenstein (MSW) effect], notably for neutrinos streaming from a supernova (SN) core [3–5]. In addition, the large neutrino flux itself causes strong neutrino-neutrino refraction [6] and can lead to self-induced flavor conversion [7–10]. This effect is very different from MSW conversion because the flavor content of the overall ensemble remains fixed. Instead, flavor is exchanged between different momentum modes and can lead to interesting spectral features [9–15]. Self-induced flavor conversion can become large because of self-amplification within the interacting neutrino system, which in turn requires instabilities (collective run-away solutions) in flavor space [16–19].

Our main point is that run-away solutions need not inherit the symmetries of initial or boundary conditions. For self-induced flavor conversion in SNe, global spherical symmetry was always assumed and, therefore, axial symmetry in every direction. However, our linearized stability analysis shows that local axial symmetry can be broken by a multi-azimuth-angle (MAA) instability. For simple spectra it arises in the normal hierarchy (NH) of neutrino masses, whereas the traditional bimodal instability [8,20,21] occurs in the inverted hierarchy (IH).

Core-collapse SNe show large convective overturns, the standing accretion shock instability, or simply rotation. However, our new effect is not caused by the concomitant asymmetries of neutrino emission, but by the intrinsic flavor instability of an axially symmetric neutrino flux, an effect which does not strongly depend on the exact azimuth distribution of emission.

In the early universe, one can integrate out the factor $1 - \mathbf{v}\cdot\mathbf{v}'$ from the current-current neutrino interaction and then finds the bimodal instability [20]. However, for equal neutrino and anti-neutrino densities it was found that allowing angle modes to evolve independently enables run-away solutions in both hierarchies [22]. In this early study it was not recognized that such multi-angle instabilities are far more general.

Two-flavor neutrino-neutrino refraction can be written in the form of the spin-pairing Hamiltonian that appears in many areas of physics [23]. When all flavor spins interact with each other with the same strength, this Hamiltonian has as many invariants as variables and thus is integrable, explaining the N-mode coherent solutions [23–26]. After including the factor $1 - \mathbf{v}\cdot\mathbf{v}'$, these simple properties are probably lost. It would be interesting to study this multiangle spin-pairing Hamiltonian to develop a deeper mathematical understanding of our system.

Our more modest goal here is to prove explicitly the existence of the MAA instability in the simplest SN setting and how it is affected by matter, allowing for a first understanding of the MAA effect.

Equations of motion.—We describe neutrinos by 3×3 flavor matrices $\varrho(t, \mathbf{r}, E, \mathbf{v})$, where the diagonal elements are occupation numbers for ν_e, ν_μ, and ν_τ while the off diagonal elements encode correlations caused by flavor oscillations. We use negative E to denote $\bar\nu$ in which case ϱ includes a minus sign: the diagonal elements are negative $\bar\nu$ occupation numbers. (One needs 6×6 matrices to include ν–$\bar\nu$ correlations that could arise from novel lepton-number violating interactions [27,28] or from Majorana spin-flavor oscillations [29,30].)

In the absence of collisions, neutrino propagation is described by the Liouville equation [31,32]

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{r}})\varrho = -i[\mathsf{H}, \varrho], \tag{1}$$

where ϱ and H are functions of t, $\mathbf{r}$, E, and $\mathbf{v}$. (Except for ϱ, we use capital sans-serif letters to denote matrices in flavor space.) The Hamiltonian matrix is

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_{\mathrm{F}}\left[\mathsf{N}_\ell + \int_{-\infty}^{+\infty} dE' \int \frac{d\mathbf{v}'}{(2\pi)^3}\varrho'(1 - \mathbf{v}\cdot\mathbf{v}')\right],$$

where M^2 is the matrix of mass-squares, causing vacuum oscillations. The matrix of charged-lepton densities, N_ℓ, includes the background matter effect. The $d\mathbf{v}'$ integral is over the unit sphere and ϱ' depends on t, $\mathbf{r}$, E', and $\mathbf{v}'$.

In general, this is an untractable 7-dimensional problem. As a simplification we assume stationarity and drop the time dependence. We also assume spherically symmetric emission, but no longer enforce local axial symmetry of the solution. We still assume that variations in the transverse direction are small so that ϱ depends only on r, E, and $\mathbf{v}$. In other words, we study neutrino propagation only in the neighborhood of a chosen location and do not worry about the global solution.

We consider neutrinos that stream freely after emission at some fiducial inner boundary R ("neutrino sphere"). If we describe neutrinos by their local $\mathbf{v}$, the zenith range of occupied modes depends on radius. To avoid this effect we use instead the emission angle ϑ_R to label the modes. The variable $u = \sin^2\vartheta_R$ is even more convenient because blackbody-like isotropic emission at R corresponds to a uniform distribution on $0 \leq u \leq 1$. The radial velocity of a mode u at radius r is $v_{r,u} = (1 - uR^2/r^2)^{1/2}$ and the transverse velocity is $\beta_{r,u} = u^{1/2}R/r$.

To study quantities that evolve only as a consequence of flavor oscillations, we introduce flux matrices [33] by

$$\frac{\mathsf{F}(r, E, u, \varphi)}{4\pi r^2}\frac{dEdud\varphi}{v(u, r)} = \varrho(r, \mathbf{p})\frac{d^3\mathbf{p}}{(2\pi)^3}, \qquad (2)$$

where φ is the azimuth angle of $\mathbf{v}$. The Liouville equation finally becomes $\partial_r \mathsf{F} = -i[\mathsf{H}, \mathsf{F}]$, the vacuum and matter terms receive a factor v^{-1}, and the ν–ν part is

$$\mathsf{H}_{\nu\nu} = \sqrt{2}G_\mathrm{F}\int d\Gamma'\mathsf{F}'\frac{1 - vv' - \boldsymbol{\beta}\cdot\boldsymbol{\beta}'}{vv'}, \qquad (3)$$

where $\int d\Gamma' = \int_{-\infty}^{+\infty} dE' \int_0^1 du' \int_0^{2\pi} d\varphi'$. In addition, we find $\boldsymbol{\beta}\cdot\boldsymbol{\beta}' = \sqrt{uu'}(R^2/r^2)\cos(\varphi - \varphi')$. Enforcing axial symmetry would remove the $\boldsymbol{\beta}\cdot\boldsymbol{\beta}'$ term, and this is what was done in the previous literature.

Two flavors.—Henceforth, we consider only two flavors e and $x = \mu$ or τ and describe energy modes by $\omega = \Delta m^2/2E$. We write the 2×2 flux matrices in the form

$$\mathsf{F} = \frac{\mathrm{Tr}\mathsf{F}}{2} + \frac{F_e^R - F_x^R}{2}\begin{pmatrix} s & S \\ S^* & -s \end{pmatrix}, \qquad (4)$$

where $F_{e,x}^R(\omega, u, \varphi)$ are the flavor fluxes at the inner boundary radius R. All other quantities depend on r, ω, u, and φ. The flux summed over all flavors, $\mathrm{Tr}\,\mathsf{F}$, is conserved and can be ignored in commutators. The ν_e survival probability, $\frac{1}{2}(1 + s)$, is given in terms of what we call the swap factor $-1 \leq s \leq 1$. The off diagonal element S is complex and $s^2 + |S|^2 = 1$.

We introduce the dimensionless spectrum $g(\omega, u, \varphi)$, representing $F_e^R - F_x^R$. It is negative for antineutrinos where $\omega < 0$, and normalized to the $\bar\nu$ flux, i.e., $\int_{-\infty}^0 d\omega \int_0^1 du \int_0^{2\pi} d\varphi g(\omega, u, \varphi) = -1$. The ν–$\bar\nu$ asymmetry is $\epsilon = \int d\Gamma g$ where $\int d\Gamma = \int_{-\infty}^{+\infty} d\omega \int_0^1 du \int_0^{2\pi} d\varphi$.

Refractive effects are provided by the r-dependent parameters [17]

$$\lambda = \sqrt{2}G_\mathrm{F}[n_e(r) - n_{\bar e}(r)]\frac{R^2}{2r^2},$$
$$\mu = \frac{\sqrt{2}G_\mathrm{F}[F_{\bar\nu_e}(R) - F_{\bar\nu_x}(R)]}{4\pi r^2}\frac{R^2}{2r^2}. \qquad (5)$$

In analogy to g, we normalize the effective ν–ν interaction energy μ to the $\bar\nu_e$–$\bar\nu_x$ flux difference at R. The factor $R^2/2r^2$ highlights that only the multiangle impact of refraction is relevant [17].

So finally the stability analysis uses the spectrum $g(\omega, u, \varphi)$, the effective ν–ν interaction energy $\mu \propto r^{-4}$, and the total matter effect parametrized by $\bar\lambda = \lambda + \epsilon\mu$. For $\Delta m^2 > 0$, our equations correspond to IH, whereas NH can be implemented with $\Delta m^2 \to -\Delta m^2$ or equivalently via $\omega \to -\omega$ in the vacuum term of H.

Linearized stability analysis.—At high density, neutrinos are produced in flavor eigenstates and propagate as such until the initially small off diagonal elements of F grow large. This can happen by an MSW resonance, which in SNe typically occurs at much larger distances than self-induced conversions. In the latter case, which we study here, the sudden growth is caused by an exponential runaway solution. We assume that no such instability occurs out to $r \gg R$, so we use the large-distance approximation where the transverse neutrino velocity is small. To linear order in S, we have $s = 1$ and find

$$i\partial_r S = (\omega + u\bar\lambda)S - \mu\int d\Gamma'[u + u' - 2\sqrt{uu'}\cos(\varphi - \varphi')]g'S'. \qquad (6)$$

We write solutions as $S(r, \omega, u, \varphi) = Q_\Omega(\omega, u, \varphi)e^{-i\Omega r}$ with complex eigenfrequency $\Omega = \gamma + i\kappa$ and eigenvector $Q_\Omega(\omega, u, \varphi)$, which satisfy the eigenvalue equation

$$(\omega + u\bar\lambda - \Omega)Q_\Omega = \mu\int d\Gamma'[u + u' - 2\sqrt{uu'}\cos(\varphi - \varphi')]g'Q'_\Omega. \qquad (7)$$

The rhs has the form $a + bu + \sqrt{u}(c\cos\varphi + d\sin\varphi)$ with complex numbers a, b, c, and d, so the eigenvectors are

$$Q_\Omega = \frac{a + bu + \sqrt{u}(c\cos\varphi + d\sin\varphi)}{\omega + u\bar\lambda - \Omega}. \qquad (8)$$

After inserting Eq. (8) into (7), self-consistency requires

$$\begin{pmatrix} I_1-1 & I_2 & I^{c}_{3/2} & I^{s}_{3/2} \\ I_0 & I_1-1 & I^{c}_{1/2} & I^{s}_{1/2} \\ -2I^{c}_{1/2} & -2I^{c}_{3/2} & -2I^{cc}_{1}-1 & -2I^{sc}_{1} \\ -2I^{s}_{1/2} & -2I^{s}_{3/2} & -2I^{sc}_{1} & -2I^{ss}_{1}-1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}=0, \tag{9}$$

where

$$I^{c(s)}_{n}=\mu\int dud\omega d\varphi\frac{u^{n}g(\omega,u,\varphi)}{\omega+u\bar{\lambda}-\Omega}\cos\varphi(\sin\varphi). \tag{10}$$

Nontrivial solutions exist if the determinant of the matrix vanishes. The mass hierarchy IH $\to$ NH is changed by $\omega\to-\omega$ in the denominator of Eq. (10).

Axial symmetry of neutrino emission.—As a next step, we simplify to $g(\omega,u,\varphi)\to g(\omega,u)/2\pi$. Now only the φ integrals with $\sin^2\varphi$ and $\cos^2\varphi$ survive and yield $I^{cc}_{1}=I^{ss}_{1}=\frac{1}{2}I_1$, leaving us with

$$\begin{pmatrix} I_1-1 & I_2 & 0 & 0 \\ I_0 & I_1-1 & 0 & 0 \\ 0 & 0 & -(I_1+1) & 0 \\ 0 & 0 & 0 & -(I_1+1) \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}=0. \tag{11}$$

This system has nontrivial solutions if

$$(I_1-1)^2=I_0I_2 \quad \text{or} \quad I_1=-1, \tag{12}$$

where the integral expressions are the same as in the previous azimuthally symmetric case [17].

The first equation corresponds to nontrivial solutions for a and b and yields the instabilities found in previous works. In IH this is the well-known bimodal solution, present even for the single-angle case of only 1 zenith mode. In NH the bimodal solution does not exist and multiangle effects are necessary for any run-away solution. For a nontrivial distribution of zenith angles, the first equation leads to a solution [17] which we now denote the multizenith angle (MZA) instability.

The second equation allows for nonzero c and d, providing solutions with nontrivial φ dependence, unstable only in NH. The previous solutions remain unaffected by MAA, whereas in NH new solutions appear.

These cases become more explicit if we ignore matter ($\bar{\lambda}=0$) and assume the spectrum factorizes as $g(\omega,u)\to g(\omega)h(u)$. With $I=\mu\int d\omega g(\omega)/(\omega-\Omega)$, Eq. (12) is

$$I^{-1}=q_j=\begin{cases}\langle u\rangle+\langle u^2\rangle^{1/2} & \text{for } j=\text{bimodal},\\ \langle u\rangle-\langle u^2\rangle^{1/2} & \text{for } j=\text{MZA},\\ -\langle u\rangle & \text{for } j=\text{MAA}.\end{cases} \tag{13}$$

Note that q_j is positive in the first case, and negative in the second and third. In IH, the first case is the only one providing an instability (bimodal) and exists for any u distribution. In NH, the first case is always stable, while the second case yields the MZA solution. It does not exist for single angle where $\langle u\rangle=\langle u^2\rangle^{1/2}$. The third case exists for any u distribution. For simple (single-crossed) spectra, it provides the new MAA solution only in NH.

We illustrate these findings with a simple example and consider single neutrino energy ($\omega=\pm\omega_0$), i.e., the spectrum $g(\omega)=-\delta(\omega+\omega_0)+(1+\epsilon)\delta(\omega-\omega_0)$. Equation (13) is now equivalent to the quadratic equation

$$\frac{\omega_0^2-\Omega^2}{2\omega_0+\epsilon(\omega_0+\Omega)}=\mu q_j\equiv\mu_j, \tag{14}$$

where $j=$ bimodal, MZA or MAA. The solutions are

$$\Omega_j=\frac{1}{2}\left(-\epsilon\mu_j\pm\sqrt{(2\omega_0-\epsilon\mu_j)^2-8\omega_0\mu_j}\right). \tag{15}$$

Exponentially growing solutions ($\kappa=\mathrm{Im}\Omega>0$) can only exist when $\omega_0\mu_j>0$. Inverted hierarchy corresponds to $\omega_0>0$ and only the first case has $q_j>0$, providing the bimodal instability. Normal hierarchy corresponds to $\omega_0<0$ so that the second and third cases provide the MZA and MAA instabilities.

The system is unstable for μ_j between the limits $2\omega_0/(\sqrt{1+\epsilon}\pm1)^2$. The maximum growth rate obtains for $\mu_j=2\omega_0(2+\epsilon)/\epsilon^2$ and is $\kappa_{\max}=2|\omega_0|\sqrt{1+\epsilon}/\epsilon$. Therefore, a typical growth rate is a few times the vacuum oscillation frequency. For $\epsilon=1/2$ we find $\kappa_{\max}=2\sqrt{6}|\omega_0|\approx4.90|\omega_0|$.

Azimuth distribution.—According to the expression for the eigenfunction Q_Ω in Eq. (8), the off diagonal elements of the ϱ matrices develop an exponentially growing "dipole term" $c\cos(\varphi)+d\sin(\varphi)$, which represents an ellipse in the complex plane. Its orientation and ellipticity is chosen by some initial disturbance. If neutrino emission is not axially symmetric, it provides a macroscopic seed, but otherwise the situation is largely the same.

In this sense, our main point is that the linearized system supports run-away solutions where the exponentially growing off diagonal ϱ elements depend on φ even if the diagonal elements, represented by $g(\omega,u,\varphi)$, do not depend on φ because of axially symmetric emission.

If we represent the φ dependence by N discrete angles φ_i with $i=1,\ldots,N$, the corresponding distributions $\delta(\varphi-\varphi_i)$ can be expanded in terms of functions $\cos(n\varphi)$ and $\sin(n\varphi)$. One can then show that the linearity of the eigenfunctions Q_Ω in $\cos(\varphi)$ and $\sin(\varphi)$ implies that no new instabilities arise in the discretized system. No spurious instabilities appear, in contrast to discrete zenith angles [19], where the eigenfunctions depend on u in nonlinear ways.

Impact of matter.—If there is only one zenith angle, matter has no impact on κ because $\bar{\lambda}u$ in the resonance denominator simply shifts the real part of Ω. In general, $\bar{\lambda}u$ is different for every zenith angle trajectory, along which neutrinos acquire different matter-induced phases. If $\bar{\lambda}$ is

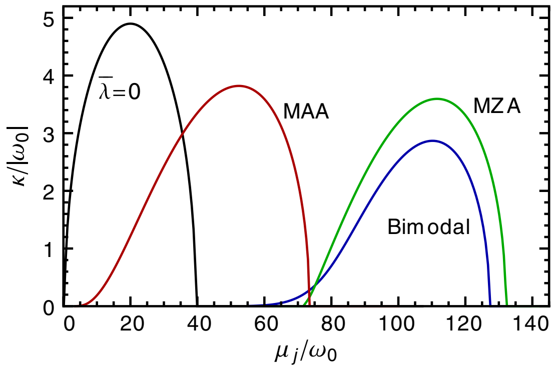

FIG. 1 (color online). Growth rate κ for blackbody-like zenith distribution, single energy $\pm\omega_0$, and $\epsilon = 1/2$. Black line: All cases for $\bar{\lambda} = \lambda + \epsilon\mu = 0$ (no matter effect). Other lines: Indicated unstable cases for $\bar{\lambda} = 300|\omega_0|$.

large, the unstable region shifts to larger μ-values [17] as shown in Fig. 1 for all three cases. We have used blackbody-like zenith distribution (uniform on $0 \le u \le 1$) where $q_{\text{bimodal}} = 1/2 + 1/\sqrt{3} \approx 1.077$, $q_{\text{MZA}} = 1/2 - 1/\sqrt{3} \approx -0.077$ and $q_{\text{MAA}} = -1/2$. On the horizontal axis in Fig. 1, we use μ_j as a variable, so the physical μ range is very different for the 3 cases.

Numerically it appears that for large $\bar{\lambda}$, the instability occurs for $\alpha_j\mu \sim \bar{\lambda}$, where α_j is a coefficient different for each case. It also appears that for the bimodal and MZA cases, actually $\alpha_j \sim |q_j|$ and we roughly have $\bar{\lambda} \sim |\mu_j|$. Note that $\bar{\lambda} = \epsilon\mu + \lambda \sim |q_{\text{MZA}}|\mu = 0.077\mu$, so that, for reasonable values of ϵ, the matter density λ would have to be negative—the MZA instability is self-suppressed by the unavoidable effect of neutrinos themselves, and plays no role in a realistic SN situation. On the other hand, we find the new MAA instability the least sensitive to matter effects, as the instability region shifts only for much larger interaction strength ($\alpha_{\text{MAA}} \sim 6|q_{\text{MAA}}|$).

Schematic SN example.—During the SN accretion phase, the matter effect can be so large as to suppress collective flavor conversions [18,33,34]. In Fig. 2 we juxtapose the instability regions for the IH bimodal and the new NH MAA instabilities for a simplified SN model. We use single energy and blackbody-like emission at the neutrino sphere, ignoring the halo flux [35]. We choose physical parameters R, $\mu(R)$, and ϵ that mimic the more realistic $15M_\odot$ accretion-phase model used in our previous study [18,35]. We show the region where $\kappa r > 1$, i.e., where the growth rate is deemed "dangerous." We also show $\lambda(r)$, where the shock wave is seen at 70 km.

The matter profile never intersects the bimodal instability region; i.e., this instability is suppressed everywhere in this specific SN example. On the other hand, $\lambda(r)$ intersects the MAA instability region just outside the shock wave. This simplified case illustrates that the MAA instability can arise in SN models where the bimodal instability is suppressed. It also shows that the "danger spots" are in very different places, although it remains to be seen if this finding is generic.

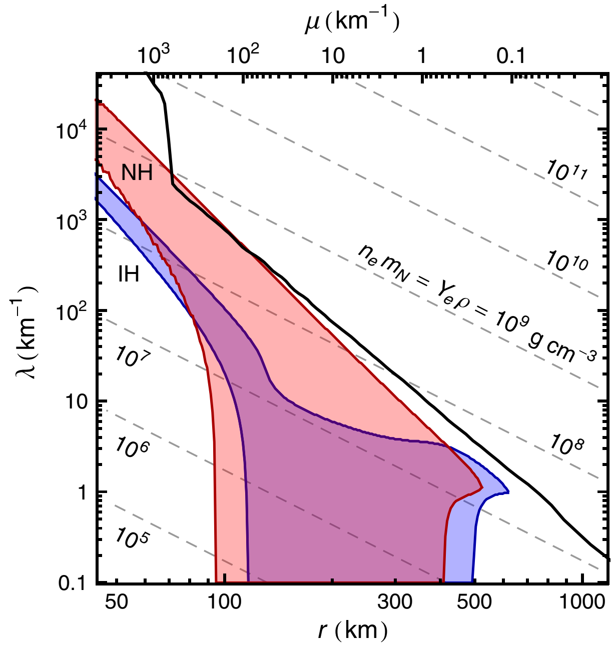

FIG. 2 (color online). Region where $\kappa r > 1$ for IH (blue) and NH (red), depending on radius r and multiangle matter potential λ for our simplified SN model. Thick black line: SN density profile. Thin dashed lines: Contours of constant electron density, where Y_e is the electron abundance per baryon. (The IH case corresponds to Fig. 4 of Ref. [18], except for the simplified spectrum used here.)

Conclusions.—All previous studies of self-induced neutrino conversion in SNe or the early universe were based on the false premise that solutions of the equations of motion would inherit the symmetries of the initial or boundary conditions. We have shown that azimuth-angle instabilities are a generic phenomenon of collective neutrino oscillations. Every single case in the previous literature with enforced axial symmetry may have missed the dominant effect.

We have linearized the equations of motion around the initial state of neutrinos in flavor eigenstates. The system then shows either the bimodal or the MAA instability, but not both. (For more complicated spectra that would lead to multiple spectral splits [14], the bimodal instability occurs for positive spectral crossings, the MAA instability for negative ones.) However, evolved bimodal solutions, where the off diagonal ϱ entries are not small, may still become φ-unstable, and the other way round.

Both instabilities can be suppressed by matter, but the required density is larger for MAA. Therefore, it is not necessarily clear if collective flavor conversions are generically suppressed during the SN accretion phase, an important question for possible neutrino mass hierarchy determination [5]. For those cases where suppression is not effective, dedicated numerical studies are needed.

More fundamentally, one also needs to question the validity of other common symmetry assumptions. For example, we have assumed a stationary solution inherited from stationary neutrino emission. Doubts may be motivated, in particular, by the role of the small backward flux caused by residual neutrino scattering that causes significant refraction [35,36]. Even without worrying about the backward flux, it has never been proven that a stationary boundary condition implies a stationary solution for a dense interacting neutrino stream. In the early universe, homogeneous initial conditions need not guarantee homogeneous solutions. It remains to be seen if the interacting neutrino system can spontaneously break translation symmetry in space or time.

We thank B. Dasgupta, I. Tamborra, and H.-T. Janka for comments on the manuscript. This work was partly supported by the Deutsche Forschungsgemeinschaft under Grant No. EXC-153 (Cluster of Excellence ''Origin and Structure of the universe'') and by the European Union under Grant No. PITN-GA-2011-289442 (FP7 Initial Training Network ''Invisibles''). D.S. acknowledges support by the Fundação para a Ciência e Tecnologia (Portugal) under Grant No. SFRH/BD/66264/2009.

Note added in proof.—Motivated by the preprint version of our paper, a numerical study has appeared that confirms the existence and importance of the MAA instability [37]. Moreover, two of us have devised a simple toy example of two counter-propagating beams that shows a flavor instability in both neutrino mass hierarchies and explains the physical nature of the MAA instability [38].

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