Kinetic Theory of Plasma Sheaths Surrounding Electron-Emitting Surfaces

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A one-dimensional kinetic theory of sheaths surrounding planar, electron-emitting surfaces is presented which accounts for plasma electrons lost to the surface and the temperature of the emitted electrons. It is shown that ratio of plasma electron temperature to emitted electron temperature significantly affects the sheath potential when the plasma electron temperature is within an order of magnitude of the emitted electron temperature. The sheath potential goes to zero as the plasma electron temperature equals the emitted electron temperature, which can occur in the afterglow of an rf plasma and some low-temperature plasma sources. These results were validated by particle in cell simulations. The theory was tested by making measurements of the sheath surrounding a thermionically emitting cathode in the afterglow of an rf plasma. The measured sheath potential shrunk to zero as the plasma electron temperature cooled to the emitted electron temperature, as predicted by the theory.

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In laboratory plasmas, electron emission from surfaces can influence many aspects of the discharge. Surfaces can emit electrons due to secondary electron emission (SEE), affecting the operation of plasma devices such as Hall thrusters [1]. In tokamak divertors, secondary electron emission modifies the sheath [2], which, if incorrectly estimated, can lead to unexpected particle fluxes which can severely damage surfaces. Emissive probes make use of thermionic electron emission to measure the plasma potential, and the details of the emissive sheath are critical to making accurate measurements [3]. Although often the emitted electron temperature ($T_{\rm ee}$) is much smaller than the plasma electron temperature ($T_{\rm ep}$), in some cases such as the afterglow of a radio frequency (rf) plasma or very low-temperature plasma sources, the $T_{\rm ep}$ can approach the $T_{\rm ee}$. Additionally, SEE can produce electrons with energies comparable to plasma electrons. In this Letter, we show that a kinetic treatment of the electrons lost to the wall and the energy distribution function of the electrons emitted from the surface is necessary to accurately capture emissive sheath behavior when $T_{\rm ep} \sim T_{\rm ee}$.

The first fluid theory of a collisionless, emissive sheath surrounding a floating, electron-emitting surface (also called a cathode in this work) was developed by Hobbs and Wesson [4]. Their model was of a one-dimensional, planar, electron-emitting surface facing a plasma with cold ions and Maxwellian plasma electrons with temperature $T_{\rm ep}$. As the emission level increases, the sheath potential shrinks due to space-charge effects, and with sufficient emission, the sheath potential (the potential difference between the sheath edge and the surface) saturates at $1.02T_{\rm ep}/e$. If the electron emission is further increased, a virtual cathode forms, which is a potential minimum in the sheath structure. This analysis of emissive sheaths was generalized for nonfloating surfaces [5] and for non-Maxwellian plasma electron velocity distribution functions [6].

When the emissive sheath reaches the space-charge-limited (SCL) condition, it functions like a double layer on the surface of the emitter [7,8]. Double layers can separate two plasmas of different temperatures and typically include at least three of four possible particle species: trapped ions, trapped electrons, free ions, and free electrons [9]. For electron-emitting surfaces, the plasma provides the trapped electrons and free ions, as in a typical sheath, while the surface acts as the second plasma, providing free electrons that are typically colder than the plasma electrons. It has been noted that the electric field in a double layer depends on the electron temperatures [10], but this Letter provides a rigorous analysis of the potential as a function of temperature ratio.

By removing some of the assumptions of fluid theory, kinetic theory can offer a more accurate solution to the sheath problem. Sizonenko considered the effects of secondary electron emission on sheaths in the context of tokamak divertors, accounting for the plasma electrons lost to the surface and the energy distribution of the emitted electrons [11]. His solution gives a sheath potential of $0.95T_{\rm ep}/e$ but did not include the effects of the plasma to the emitted electron temperature ratio on the sheath potential. A kinetic theory of emissive sheaths was proposed by Schwager and verified with particle in cell (PIC) simulations [12]. She demonstrated that small ion masses and high ion temperatures reduce the emissive sheath potential but did not investigate the effects of the emitted electron temperature. Those results have been extended to a bi-Maxwellian plasma [13]. It has also been noted that a higher emitted electron temperature can increase the potential between the virtual cathode minimum and the emissive surface [14].

This kinetic theory of planar emissive sheaths answers the question of what is the sheath potential of a collisionless plasma sheath adjacent to a floating surface that emits electrons such that the emission is just SCL, making the electric field at the surface zero. It was assumed that there are no instabilities. Normalized values were used: $\Phi = -(e\phi/T_{\rm ep})$ and $\mathcal{E}_0 = (E_0/T_{\rm ep})$, where ϕ is the potential referenced to the sheath edge ($\Phi = 0$ at the sheath edge), E_0 is the ion energy at the sheath edge, and $\mathcal{E}_0$ is that energy normalized to the plasma electron temperature. The normalized potential of the emissive surface is Φ_w.

The plasma electrons were assumed to be Maxwellian with temperature $T_{\rm ep}$. After they enter the sheath, most are reflected back out, but some are energetic enough to reach the surface, where they are lost with no reflection. The electrons lost to the wall modify the plasma electron density in the sheath, which can be calculated by integrating over a Maxwellian velocity distribution function that is missing the tail where $v > \sqrt{(2T_{\rm ep}/m_e)(\Phi_w - \Phi)}$:

$$\frac{n_{\rm ep}(\Phi)}{n_{\rm ep}(0)} = \exp(-\Phi)\left(\frac{1+\mathrm{erf}(\sqrt{\Phi_w - \Phi})}{1+\mathrm{erf}(\sqrt{\Phi_w})}\right). \quad (1)$$

The emitted electrons were assumed to have a half-Maxwellian distribution with temperature $T_{\rm ee}$, which is the distribution for thermionic emission where the $T_{\rm ee}$ is equal to the temperature of the surface. This distribution was chosen because it is the easiest to treat analytically, but the analysis can be generalized to SEE. The plasma electron temperature to emitted electron temperature ratio is $\Theta \equiv T_{\rm ep}/T_{\rm ee}$. By integrating over the half-Maxwellian distribution, the emitted electron density is

$$\frac{n_{\rm ee}(\Phi)}{n_{\rm ee}(0)} = \frac{\exp[\Theta(\Phi_w - \Phi)]\mathrm{erfc}[\sqrt{\Theta(\Phi_w - \Phi)}]}{\exp(\Theta\Phi_w)\mathrm{erfc}(\sqrt{\Theta\Phi_w})}. \quad (2)$$

Enforcing charge neutrality at the sheath edge and flux balance such that the net current through the sheath is zero and the emitting surface is electrically floating, the densities $n_{\rm ep}(0)$ and $n_{\rm ee}(0)$ can be reexpressed in terms of Φ_w and the ion density at the sheath-presheath boundary. The presheath is a quasineutral region of weak electric field that accelerates the ions.

The ions were assumed to be cold ($T_i = 0$) and reach the sheath-presheath boundary with a velocity consistent with Bohm's criterion. The ions are, therefore, described by the fluid equations which have been often used:

$$\frac{n_i(\Phi)}{n_0} = \sqrt{\frac{1}{1+\frac{\Phi}{\mathcal{E}_0}}}. \quad (3)$$

The boundary condition at the sheath-presheath boundary is that the potential is zero by definition and the electric field is zero. By integrating Poisson's equation twice over the potential, the differential equation can be reduced to an integral equation:

$$\int_0^{\Phi_w}[n_{\rm ep}(\Phi) + n_{\rm ee}(\Phi) - n_i(\Phi)]d\Phi = 0. \quad (4)$$

The ion flux was assumed to be small compared to the emitted electron flux (terms on the order of $\sqrt{m_e/m_i}$ were neglected). Integrating the densities over the potential yields an equation which is a function of only Φ_w and $\mathcal{E}_0$, given some Θ. Bohm's criterion generalized to account for the emitted electrons

$$\left.\left(\frac{dn_i}{d\Phi} - \frac{dn_{\rm ep}}{d\Phi} - \frac{dn_{\rm ee}}{d\Phi}\right)\right|_{\Phi=0} \geq 0 \quad (5)$$

was assumed to be marginally fulfilled and can be solved to find the expression for $\mathcal{E}_0$ as a function of Φ_w [15]. These two equations can be solved to calculate the sheath potential Φ_w and the ion energy at the sheath edge $\mathcal{E}_0$ for a given value of Θ.

The sheath potential as a function of Θ is graphed in Fig. 1. The dashed lines indicate the solutions as $\Theta \rightarrow \infty$. The fluid theory result for cold emitted electrons ($\Theta \rightarrow \infty$)

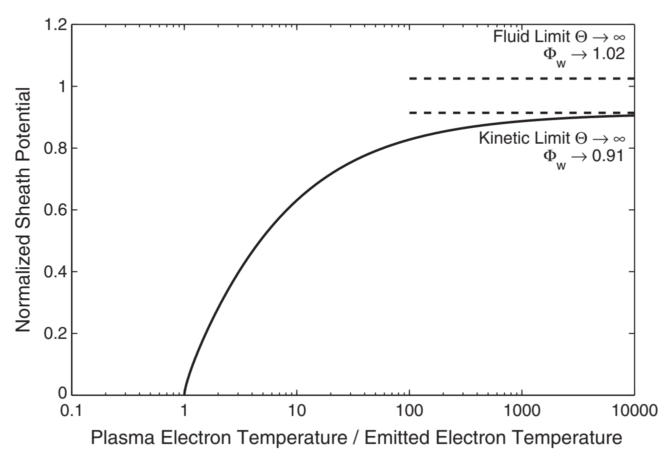

FIG. 1. The normalized sheath potential as a function of the plasma electron temperature to emitted electron temperature ratio (Θ).

is $\Phi_w = 1.02$, the result first derived by Hobbs and Wesson [4]. For the kinetic theory when $T_{ee} \ll T_{ep}$, the sheath potential drops to $\Phi_w = 0.91$, only a 10% reduction from the fluid theory result, but the difference between the two theories becomes greater as the two temperatures become comparable. A typical thermionically heated emissive probe emits electrons with a temperature of ~0.2 eV while T_{ep} can be as low as 1 eV or less in some low-temperature laboratory experiments. For these parameters, $\Theta = 5$ and $\Phi_w = 0.51$. Including the effects of the plasma electrons lost to the wall and nonzero T_{ee} yields a sheath potential half that is predicted by the widely used fluid theory.

As $\Theta \to 1$, the sheath potential goes to zero. This is expected because at $\Theta = 1$, the electrons lost to the wall would be replaced by electrons emitted from the wall at the same temperature. For the electrons, it would be as if the surface was not there, and it is a result that has been observed in PIC simulations [16]. Only by considering both the plasma electrons lost to the surface and the temperature of the emitted electron can this result be obtained.

The predictions of the planar kinetic theory were compared to one-dimensional planar electrostatic direct implicit particle in cell (EDIPIC) code simulations [1,16]. The ions were singly ionized argon, T_{ep} was 1 eV, and the ion temperature was 0.025 eV (room temperature). There were no collisions in this simulation, so as to be consistent with the collisionless kinetic theory. At the plasma boundary, a constant flux of $7.12 \times 10^{17}\ \mathrm{m^{-2}\,s^{-1}}$ electron-ion pairs were injected into the system, and any electrons escaping to the plasma boundary were reinjected with a Maxwellian distribution. The electric field at this boundary was set to be zero. The cathode boundary was electrically floating, the electric potential was fixed at zero, and electrons were emitted with a half-Maxwellian distribution and a flux of $3.7 \times 10^{19}\ \mathrm{m^{-2}\,s^{-1}}$, which was determined to be sufficient to allow the SCL sheath to form. Any additional current in excess of that which forms the SCL sheath is reflected back to the cathode boundary by the virtual cathode. Simulations were run for emitted electron temperatures of 0.2, 0.1, 0.05, 0.02, and 0.01 eV. The simulated system length was 5 mm, chosen because larger lengths were subject to ion acoustic instabilities, but those instabilities are beyond the scope of this Letter.

As in Schwager's simulations [12], a source sheath accelerated ions to fulfill Bohm's criterion. The potential profile generated by EDIPIC when $\Theta = 100$ is shown in the inset of Fig. 2, and the source sheath is the potential drop near the plasma boundary. In the longer system length simulation that reached a steady state, it was observed that the source sheath did not accelerate the ions to marginally fulfill Bohm's criterion ($E_0 = (1/2)T_{ep}$), but rather the ions reached an energy of $0.71T_{ep}$ (a velocity of $1.42c_s$, where c_s is the ion sound speed) at the sheath edge. The source sheath allows the ions to become supersonic in order to

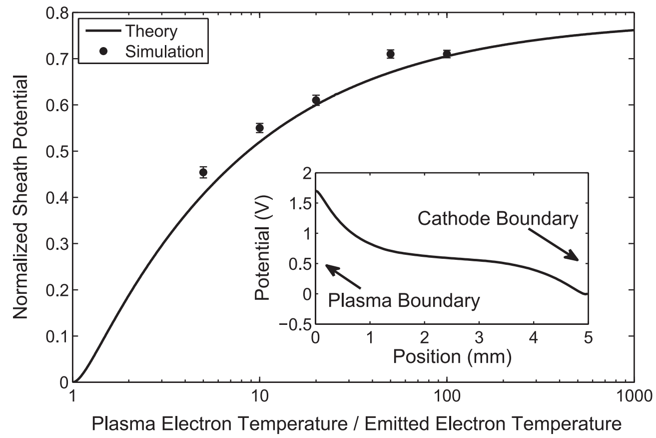

FIG. 2. A comparison of the emissive sheath as predicted by the kinetic theory (solid line) to that simulated using EDIPIC (points). The inset shows the potential profile generated by EDIPIC when $\Theta = 100$.

fulfill the boundary conditions and quasineutrality. While the kinetic theory presented above was formulated for the marginal solution to Bohm's criterion, it can easily be rewritten for supersonic ion flow at the sheath edge. A modification to account for the supersonic flow was made in order to make a meaningful comparison between the PIC simulations and the kinetic theory.

The potential at the sheath edge was taken to be the potential at which the ion energy was 0.71 eV. The emissive sheath potential was the difference between the potential at the sheath edge and that at the minimum of the virtual cathode. The emissive sheath potential as predicted by the kinetic theory and calculated from the PIC simulation results is shown in Fig. 2, showing very good agreement and validating the predictions of the theory.

The experiments to test the kinetic theory of the emissive sheath were performed in a modified version of the gaseous electronics conference reference cell [17]. The vacuum chamber was a cylinder 22.3 cm high and 25.1 cm in diameter. The working gas was helium, and the neutral pressure was 25 mTorr. The ion mass is unimportant to the emissive sheath, and experiments with helium were expected to match the simulations with argon. 30 W of 10 MHz rf power was capacitively coupled into the plasma. The rf signal was pulsed at a rate of 60 Hz, turning the power off for 2.5 ms. A floating planar barium doped tungsten cathode (a HeatWave Labs, Inc., TB-198), 2.54 cm in diameter, was heated so it thermionically emitted electrons. The Debye length was small compared to the cathode size. The emissive sheath around the cathode was SCL for the entire afterglow.

The semilog plot of the Langmuir probe (LP) current-voltage (I-V) trace was used to measure T_{ep}, and the emissive probe (EP) technique of the inflection point in the limit of zero emission was used to measure the plasma potential (ϕ_p) [18]. The slow-sweep probe method was used to obtain temporally resolved I-V traces for both measurements. The probe was biased at a constant potential, and current was measured as a function of time. That procedure was performed for a range of bias voltages and by transposing the data, and probe current versus bias voltage at various times were obtained [19]. This method allowed the inflection point technique, the most accurate EP technique for measuring ϕ_p [20], to be used to make time resolved measurements for the first time.

Measurements of T_{ep} were crucial for testing the kinetic emissive sheath theory, but late in the afterglow ($> 250\ \mu$s), the plasma was so diffuse that the large noise to signal ratio made it impossible to extract meaningful T_{ep} data from the LP. To approximate T_{ep} for the entire afterglow, the floating potential of a nonemitting surface can be used: $T_{ep} \approx e(\phi_p - \phi_{f,EP})/\ln(\mu/2\pi)^{1/2} = e(\phi_p - \phi_{f,EP})/3.5$, where $\phi_{f,EP}$ is the floating potential of the EP when cold and μ is the ion to electron mass ratio [21]. The T_{ep} approximation was validated by the LP technique early in the afterglow where T_{ep} could still be extracted. The approximation measurements were within the error bars of the Langmuir probe measurements, suggesting that this approximation provides a good measure of T_{ep} in this plasma. Later in the afterglow, as the electrons cool, Maxwellianization of the electrons occurs because electron-electron collisions are proportional to $T_{ep}^{-3/2}$. Therefore, the formula should be valid throughout the afterglow.

The cathode was heated to 900 °C ($T_{ee} = 0.1$ eV), and ϕ_p, T_{ep} and the cathode floating potential were measured as a function of time throughout the afterglow. The potential difference between the plasma and the floating surface was graphed versus the Θ in Fig. 3, in the same way as Fig. 1, with the inset showing the decay of T_{ep} versus time in the afterglow. The first feature to notice is that as $T_{ep} \to T_{ee}$, the floating potential of the planar heated electrode approached ϕ_p. This result supports that prediction made by the kinetic theory (see Fig. 1). The data show that when $T_{ep} = T_{ee}$, the floating potential is greater than ϕ_p, which is caused by the barrier potential to limit excess electron emission. As Θ increases, so does the sheath potential to a maximum of $\sim 2T_{ep}/e$. The measured experimental values included not only the potential drop of the sheath but also the presheath, which depended on collisionality and geometry, among other things. The potential drop across a presheath of a weakly collisional plasma such as this one is typically $\sim T_{ep}/e$ [22], which would make the maximum emissive sheath potential $\sim T_{ep}/e$, as expected. One difference between Figs. 1 and 3 is the scale of the horizontal axes: the emissive sheath potential is not greatly affected by Θ until that ratio is below about 3, while the kinetic theory predicts that significant change in the emissive sheath potential occurs for $\Theta < 100$. The instability observed in the PIC simulations, but not considered in the kinetic theory, is likely responsible for this difference.

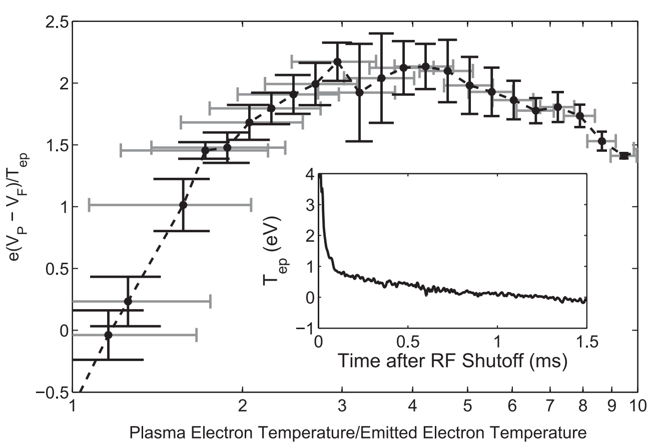

FIG. 3. Measurements of the potential difference between the plasma potential in the bulk (ϕ_p) and the floating potential of the heated electrode (ϕ_f) normalized to the plasma electron temperature (T_{ep}) versus the plasma to the emitted electron temperature ratio (Θ). The dashed line connects the data points and is not a fit.

Two sources of uncertainty were significant. First, noise in the measurement of ϕ_p propagated uncertainty to the T_{ep} approximation. Second, leakage currents in measuring the Langmuir probe floating potential were a source of error. Even small leakage currents in electric circuitry of the probe could be a source of error in measurements of the floating potential and, as a result, in the determination of T_{ep}.

In conclusion, using a one-dimensional kinetic model of the planar, collisionless, emissive sheath, it was shown that the sheath potential goes to zero as the plasma electron temperature approaches the emitted electron temperature, a result not captured by previous theories. Only by considering both the effect of the plasma electrons lost to the surface and the emitted electron temperature can this phenomenon be accurately described. Results from one-dimensional EDIPIC simulations of an emissive sheath agreed well with the theory. The kinetic theory of emissive sheaths was examined experimentally in the afterglow of a capacitively coupled rf plasma. The slow-sweep probe method was used in conjunction with the inflection point in the limit of zero emission EP technique to obtain temporally resolved EP I-V traces for the first time. The theory, simulations, and experiments all demonstrate that the emissive sheath potential goes to zero as the plasma electron temperature approaches the emitted electron temperature. Differences between theory and experiment for intermediate values of T_{ep}/T_{ee} indicate that instabilities can play an important role in determining the emissive sheath potential.

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