Multifractality at Anderson Transitions with Coulomb Interaction

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(Received 13 May 2013; published 5 August 2013)

We explore mesoscopic fluctuations and correlations of the local density of states (LDOS) near localization transition in a disordered interacting electronic system. It is shown that the LDOS multifractality survives in the presence of the Coulomb interaction. We calculate the spectrum of multifractal dimensions in $2+\epsilon$ spatial dimensions and show that it differs from that in the absence of interaction. The multifractal character of fluctuations and correlations of the LDOS can be studied experimentally by scanning tunneling microscopy of two-dimensional and three-dimensional disordered structures.

DOI: 10.1103/PhysRevLett.111.066601 PACS numbers: 72.15.Rn, 71.30.+h, 73.23.−b

Fifty-five years after its discovery [1], Anderson localization remains an actively developing field [2]. One of the central directions of the current research is the physics of Anderson localization transitions [3]. These include both metal-insulator transitions and quantum Hall plateau transitions (and, more generally, transitions between different phases of topological insulators). Such transitions have been experimentally observed and studied in a variety of semiconductor structures [4]. Recent discoveries of graphene [5] and time-reversal-invariant topological insulator materials [6] have further broadened the arena for their experimental exploration. In addition to electronic systems, there is a number of further experimental realizations of Anderson localization, including localization of light [7], cold atoms [8], ultrasound [9], and optically driven atomic systems [10].

Anderson transitions are quantum phase transitions and are characterized by critical scaling of various physical observables. A particularly remarkable property of Anderson transitions is the multifractality of critical wave functions which implies their very strong fluctuations. Specifically, at the critical point, the wave-function moments or equivalently, the averaged participation ratios $\langle P_q\rangle = \langle \int d^d r |\psi(\mathbf{r})|^{2q}\rangle$ show anomalous multifractal scaling with respect to the system size L,

$$L^d\langle|\psi(\mathbf{r})|^{2q}\rangle \propto L^{-\tau_q}, \qquad \tau_q = d(q-1)+\Delta_q, \tag{1}$$

where d is the spatial dimension, $\langle\ldots\rangle$ denotes the averaging over disorder, and Δ_q are anomalous multifractal exponents distinguishing the critical point from a conventional metallic phase, where $\Delta_q \equiv 0$. We refer the reader to Ref. [3] for an overview of this research area. Very recently, a complete classification of observables characterizing critical wave functions [that includes multifractal moments (1) as a "tip of the iceberg"] was developed [11].

The above results on multifractality have been obtained for noninteracting disordered systems. In the case of broken spin invariance, they remain valid in the presence of short-range (e.g., screened by external gate) electron-electron interaction which, in this case, is irrelevant in the renormalization-group (RG) sense [12]. On the other hand, the long-range ($1/r$) Coulomb interaction is RG relevant and may have a strong impact on the localization properties of the system (see Refs. [16,17] for reviews). In particular, it induces a metal-insulator transition in (otherwise localized) two-dimensional (2D) systems with preserved spin and time-reversal invariances [18]. Further, the Coulomb interaction induces a strong suppression of the local density of states (LDOS) $\rho(E)$ near zero energy E (counted from the chemical potential) [19,20]. The LDOS can be measured in a tunneling experiment, and this phenomenon is known as the zero-bias anomaly (ZBA). Specifically, in a 2D weakly disordered system the disorder-averaged LDOS behaves as [16,17,21–25]

$$\langle\rho(E)\rangle \propto \exp\left\{-\frac{1}{4\pi g}\log^2|E|\right\}, \tag{2}$$

where g is the dimensionless (measured in units e^2/h) conductivity. The physics of 2D disordered systems is closely related to the behavior at Anderson transition, since $d=2$ is a logarithmic (lower critical) dimension. The unconventional behavior (2) with squared logarithm in the exponential (rather than with a simple logarithm that would yield a power law, as normally expected for critical behavior) is related to the fact that the LDOS is affected by gauge-type phase fluctuations that yield a suppression of the Debye-Waller type. For the Anderson transition in $d = 2+\epsilon$ dimensions (with $\epsilon \ll 1$ allowing a parametric control of the theory) in systems with broken time reversal and/or spin symmetries, one of the logarithmic factors in

Eq. (2) transforms into a factor $\sim 1/\epsilon$ [16,17]. Since the critical conductance g_* is of order $1/\epsilon$ as well, this yields

$$\langle \rho(E) \rangle \propto |E|^\beta, \qquad \beta = O(1), \tag{3}$$

with the precise value of the critical exponent β depending on the symmetry class. Specifically, up to corrections of order ϵ, one finds $\beta \simeq 1/2$, $1/[4(1-\ln 2)]$, and 1, for the problems with magnetic impurities, magnetic field, and spin-orbit scattering, respectively [16,17]. In view of a combination of disorder and interaction physics, such metal-insulator transitions are often called Mott-Anderson (or Anderson-Mott) transitions. We remind that in the absence of interaction, the LDOS is uncritical, $\beta = 0$, in conventional (Wigner-Dyson) symmetry classes.

We are thus facing the following important question: does multifractality survive in the presence of the Coulomb interaction between electrons? The goal of this Letter is to answer this question. Specifically, we will show that on top of the ZBA suppression of the average $\langle \rho(E) \rangle$, the LDOS of a strongly interacting critical system does show multifractal fluctuations and correlations. We will also calculate the corresponding spectrum of anomalous dimensions in $2+\epsilon$ spatial dimensions up to the two-loop order and demonstrate that it differs from that of a noninteracting system.

Note that the question addressed in this Letter is of direct experimental relevance. In particular, a recent work [26] performed scanning tunneling microscopy (STM) of a magnetic semiconductor $Ga_{1-x}Mn_xAs$ near metal-insulator transition and detected LDOS fluctuations and correlations of mutlifractal character. Strong fluctuations of the LDOS in a strongly disordered 3D system (presumably, on the insulating side of the transition) have been also observed in Ref. [27]. Further, recent works on STM of 2D semiconductor systems and graphene [28] demonstrated the feasibility to explore fluctuations and correlations of the LDOS also near the quantum Hall transitions. Finally, strong spatial fluctuations of the LDOS have been also detected near the superconductor-insulator transition in disordered films [29] that is known to have much in common with the metal-insulator transition.

We turn now to the presentation of our results. The LDOS is formally defined as an imaginary part of the single-particle Green function, $\rho(E, \boldsymbol{r}) = (-1/\pi)\,\mathrm{Im}G(E; \boldsymbol{r}, \boldsymbol{r})$. We find that near the Anderson localization transition, the moments of LDOS normalized to its average show multifractal scaling,

$$\langle [\rho(E, \boldsymbol{r})]^q \rangle / \langle \rho(E) \rangle^q \sim (\mathcal{L}/l)^{-\Delta_q}, \tag{4}$$

where l denotes a microscopic length scale of the order of elastic scattering mean free path and $\mathcal{L} = \min\{\xi, L_\phi, L\}$ is the shorter of the three lengths: the localization (correlation) length ξ, the dephasing length L_ϕ, and the system size L [30]. The correlation length diverges at the transition point in a power-law fashion, $\xi \sim |g - g_*|^{-\nu}$, with an exponent ν. Further, the dephasing length (controlled by inelastic scattering processes) diverges at zero energy (we remind that all energies are counted from the chemical potential), $L_\phi \sim |E|^{-1/z}$, with a dynamical exponent z.

The power-law scaling (4) of the normalized LDOS moments is governed by a set of exponents Δ_q. These exponents control also spatial power-law correlations of the LDOS at scales $R < \mathcal{L}$. (At large distances, $R \gg \mathcal{L}$, the LDOS becomes essentially uncorrelated.) In particular, the correlation function of two LDOS at different points shows at $l < R < \mathcal{L}$ the following scaling:

$$\langle \rho(E, \boldsymbol{r}) \rho(E, \boldsymbol{r} + \boldsymbol{R}) \rangle / \langle \rho(E) \rangle^2 \sim \left(\frac{\mathcal{L}}{R} \right)^\eta, \tag{5}$$

where $\eta = -\Delta_2$. Correlations between the LDOS at different energies have analogous scaling properties,

$$\frac{\langle \rho(E, \boldsymbol{r}) \rho(E + \omega, \boldsymbol{r} + \boldsymbol{R}) \rangle}{\langle \rho(E) \rangle \langle \rho(E + \omega) \rangle} \sim \left(\frac{L_\omega}{R} \right)^\eta, \qquad R < L_\omega, \tag{6}$$

where $L_\omega \sim \omega^{-1/z}$ and it is assumed that $L_\omega < \mathcal{L}$.

To derive the above results, we use the nonlinear σ model (NLσM) field-theoretical approach to interacting disordered systems [16,17]. To keep the analysis parametrically under control, we consider the Anderson transition in $d = 2 + \epsilon$ dimensions, where the critical conductance g_* is large (i.e., the corresponding σ model coupling $t_* = 1/\pi g_*$ is weak). This allows us to obtain an ϵ expansion for critical exponents. In the spirit of the usual ideology of critical phenomena, it is expected that the scaling results (4)–(6) are of general validity and hold also at strong-coupling critical points of Coulomb-interacting disordered systems, such as metal-insulator transitions in 3D or quantum Hall plateau transitions.

We consider a system of disordered fermions with Coulomb interaction in the absence of time reversal and spin rotational symmetries, which corresponds to the symmetry class "MI(LR)" in the terminology of Ref. [17]. The RG analysis of the Anderson metal-insulator transition in $d = 2 + \epsilon$ dimensions for this symmetry class was developed up to two-loop order in Refs. [31,32]. Renormalization of the dimensionless conductance g is governed by the following β function [32]:

$$-\frac{dt}{d \ln y} = \beta(t) = \epsilon t - 2t^2 - 4At^3 + O(t^4), \tag{7}$$

where y is the running RG length scale, $t = 1/\pi g$, and $A \approx 1.64$. The condition $\beta(t_*) = 0$ determines the position of the critical point: $t_* = (\epsilon/2)(1 - A\epsilon) + O(\epsilon^3)$ (and thus the critical conductance $g_* = 1/\pi t_*$). Further, the localization length exponent is determined by the derivative of the β function at the fixed point, $\nu = -1/\beta'(t_*) = 1/\epsilon - A + O(\epsilon)$. The dynamical exponent connecting the energy and length scaling at criticality is also known up to the two-loop order: $z = 2 + \epsilon/2 + (2A - \pi^2/6 - 3)\epsilon^2/4 + O(\epsilon^3)$ [31].

To determine the scaling of LDOS moments, we translate the corresponding correlation functions into the NLσM language. We use the Matsubara version of the interacting σ model, and introduce replicas in order to perform the disorder averaging. A detailed two-loop RG analysis (see the Supplemental Material [33]) demonstrates that the scaling behavior of moments of the normalized LDOS $[\rho(E,\boldsymbol{r})/\langle\rho(E)\rangle]^q$ is governed by the anomalous dimensions

$$\zeta_q(t) = \frac{q(1-q)t}{2}\left[1 + \left(2 - \frac{\pi^2}{6}\right)t\right] + O(t^3). \quad (8)$$

This proves the anomalous scaling (4) and determines the multifractal exponents at the critical point:

$$\Delta_q = \zeta_q(t_*) = \frac{q(1-q)\epsilon}{4}\left[1 + \left(1 - A - \frac{\pi^2}{12}\right)\epsilon\right] + O(\epsilon^3). \quad (9)$$

An extension of this analysis onto correlation functions of the LDOS at different spatial points and/or energies yields Eqs. (5) and (6) and their generalizations onto higher correlation functions [33].

To illustrate the origin of the obtained fluctuations and correlations of the LDOS, we show in Fig. 1 representative diagrams for the correlation function $\langle\rho(E,\boldsymbol{r})\times\rho(E+\omega,\boldsymbol{r}+\boldsymbol{R})\rangle$. Each LDOS is given by a fermionic loop dressed by interaction lines. Averaging each loop over disorder generates diffusive vertex corrections and yields the ZBA. On the other hand, diffusons connecting the loops lead to multifractal correlations. The RG effectively sums up the leading contributions of the diagrams with multiple interaction lines and intra- and interloop diffusons inserted in all possible ways.

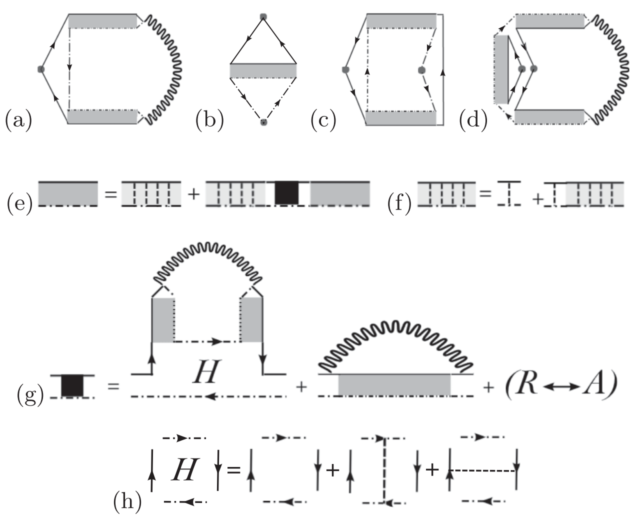

FIG. 1. Representative diagrams for the average LDOS (a) and to the correlation function of two LDOS $\langle\rho(E,\boldsymbol{r})\times\rho(E+\omega,\boldsymbol{r}+\boldsymbol{R})\rangle$ (b)–(d). The retarded (advanced) single-particle Green function is denoted by solid (dashed-dotted) line. The wavy solid line denotes the dynamically screened Coulomb interaction. The shaded rectangular is a representation for the diffuson (e) with self-energy due to interaction shown in (g). The shaded rectangular with dashed lines stands for a bare diffuson (f). The impurity line is denoted by the dashed line. The white rectangle with symbol "H" stands for the Hikami box shown in (h).

It is instructive to compare our findings with the known results for the Anderson transition in $d = 2 + \epsilon$ dimensions in the absence of interactions. In the case of noninteracting disordered fermions without time reversal symmetry (the Wigner-Dyson unitary class A), the β function, the critical point, and the localization length exponent are known up to the five-loop order [34]

$$-\frac{dt}{d\ln y} = \beta^{(0)}(t) = \epsilon t - \frac{1}{2}t^3 - \frac{3}{8}t^5 + O(t^6), \quad (10)$$

$t_* = (2\epsilon)^{1/2}(1 - 3\epsilon/4) + O(\epsilon^{5/2})$, and $\nu = 1/2\epsilon - 3/4 + O(\epsilon)$. The anomalous dimensions of operators which determine the scaling behavior of the LDOS moments have been computed at the four-loop level [35] with the result

$$\zeta_q^{(0)}(t) = \frac{q(1-q)t}{2}\left(1 + \frac{3t^2}{8} + \frac{3\zeta(3)}{16}q(q-1)t^3\right) + O(t^5), \quad (11)$$

where $\zeta(3) \approx 1.2$ stands for the Riemann zeta function. This leads to the following expression for the corresponding multifractal exponents:

$$\Delta_q^{(0)} = q(1-q)\left(\frac{\epsilon}{2}\right)^{1/2} - \frac{3\zeta(3)}{32}q^2(q-1)^2\epsilon^2 + O(\epsilon^{5/2}). \quad (12)$$

Comparing Eqs. (7) and (10), one sees that the Coulomb interaction changes the β function and, consequently, the fixed point and critical exponents. Thus, Anderson transitions with and without the Coulomb interaction belong to different universality classes.

While we have shown that multifractality of the LDOS persists in the presence of the Coulomb interaction; the values of the multifractal dimensions, Eq. (9), are essentially different from their noninteracting counterparts (12). This happens because of a difference in the corresponding scaling functions [cf. Eqs. (8) and (11)] and because of different values of critical resistance t_*. We mention that in both cases in the two-loop approximation, the spectrum of anomalous dimensions Δ_q [and thus the so-called singularity spectrum $f(\alpha)$ that is obtained by the Legendre transformation (see, e.g., Ref. [3])] is parabolic, $\Delta_q \simeq \gamma q(1-q)$. It is expected, however, that a higher-loop contribution will break the exact parabolicity in the Coulomb case, in analogy with what happens (in the four-loop order) in the noninteracting model.

For small ϵ, when the values of the exponents are parametrically controlled, the Coulomb interaction

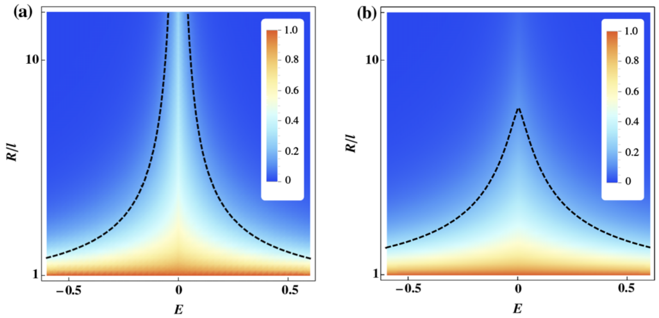

FIG. 2 (color online). Schematic color-code plot of the autocorrelation function $\langle[\rho(E,\boldsymbol{r}) - \langle\rho(E)\rangle][\rho(E,\boldsymbol{r}+\boldsymbol{R}) - \langle\rho(E)\rangle]\rangle/\langle\rho(E)\rangle^2$ for the system (a) at the critical point, $g = g_*$, and (b) slightly on the metallic side, $(g - g_*)/g_* = 0.2$. The energy is measured in units of elastic scattering rate which sets the ultraviolet cutoff of the NLσM theory. The dashed line in (a) and (b) corresponds to $R/l \sim (|E|^{2/z} + |g/g_* - 1|^{2\nu})^{-1/2}$.

considerably reduces the numerical values of the anomalous exponent, i.e., it weakens multifractality. As an example, for $\epsilon = 1/8$ we get $\gamma = 0.25$ in the absence and $\gamma = 0.03$ in the presence of interaction. In the physically most interesting case of dimensionality $d = 3$, i.e., $\epsilon = 1$, we can only use the one-loop term as an estimate. This yields in the noninteracting case $\gamma = 0.7$ and $\eta = -\Delta_2 = 1.4$, in fairly good agreement with numerical results. An analogous estimate based on our results for the interacting system yields $\gamma = 0.25$ and $\eta = -\Delta_2 = 0.5$. Note that at $\epsilon = 1$, the second-loop term in Eq. (9) is numerically of the same order (by absolute value) as the one-loop term; thus, this estimate is expected to be quite rough.

To visualize the critical LDOS correlations near a metal-insulator transition, we present in Fig. 2 a color-code plot of the autocorrelation function $\langle[\rho(E,\boldsymbol{r}) - \langle\rho(E)\rangle] \times [\rho(E,\boldsymbol{r}+\boldsymbol{R}) - \langle\rho(E)\rangle]\rangle/\langle\rho(E)\rangle^2$ [cf. Eq. (5)]. This presentation is analogous to Figs. 4A and 4B of the experimental paper [26]. For this plot, we have chosen the following values of the critical exponents: $\nu = 1$, $z = 1.5$, $\eta = 0.5$, which are theoretical estimates obtained by taking $\epsilon = 1$ in the one-loop results for the case of the Coulomb interaction. The left panel [Fig. 2(a)] corresponds to the case when the system is exactly at the transition point, $g = g_*$. We see the long-range multifractal correlation at low energies; since $\nu > 1/z$ (as is also the case for experimental estimates of the corresponding exponents at 3D metal-insulator transitions and at quantum Hall transitions), the range of correlation $\mathcal{L}$ is controlled by the dephasing length L_ϕ. In the right panel [Fig. 2(b)], the system is slightly off the transition; i.e., $g - g_*$ is now nonzero. In this case, $\mathcal{L}$ is governed by the correlation (localization) length ξ in a certain window around zero energy and by L_ϕ outside this window. All essential features of Fig. 2 compare well with Fig. 4 of Ref. [26].

To summarize, we have shown that the multifractal fluctuations and correlations of the LDOS persist in the presence of the Coulomb interaction but the spectrum of multifractal exponents is modified. By using the NLσM approach, we have calculated the multifractality spectrum of an interacting system without time reversal and spin symmetries up to the two-loop order in $2 + \epsilon$ dimensions. Our results are in an overall agreement with the experimental data of Ref. [26].

We hope that our work will motivate further experimental studies of multifractality of interacting electrons near metal-insulator and quantum Hall transitions. On the theoretical side, our Letter paves a way to a systematic investigation of multifractality at interacting critical points of localization transitions. There exists by now a vast knowledge on properties of multifractality in the absence of interaction, including, in particular, systems of different symmetry classes and different dimensionalities, symmetries of mutlifractal spectra, termination and freezing, implications of conformal symmetry, connection to entanglement entropy, and manifestation of multifractality in various observables [3,11,36–38]. In the presence of the Coulomb interaction, the corresponding physics remains to be explored. In addition to metal-insulator transitions and transitions between different phases of topological insulators, we envision a possibility to extend this analysis also to superconductor-insulator transitions.

While we were preparing this Letter for publication, a preprint appeared [39] where an analogous problem was addressed numerically within a self-consistent Hartree-Fock approximation.

We thank A. Yazdani, A. W. W. Ludwig, B. I. Shklovskii, and M. A. Skvortsov for discussions. The work was supported by DFG-RFBR in the framework of SPP 1285, BMBF, Russian President Grant No. MK-4337.2013.02, Dynasty Foundation, RFBR Grant No. 12-02-00579, Russian Ministry of Education and Science under Contract No. 8385, and by RAS Programs “Quantum mesoscopic and disordered systems,” “Quantum Physics of Condensed Matter,” and “Fundamentals of nanotechnology and nanomaterials.”

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