Bottom-Quark Forward-Backward Asymmetry in the Standard Model and Beyond

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We computed the bottom-quark forward-backward asymmetry at the Tevatron in the standard model (SM) and for several new physics scenarios. Near the Z pole, the SM bottom asymmetry is dominated by tree level exchanges of electroweak gauge bosons. While above the Z pole, next-to-leading order QCD dominates the SM asymmetry as was the case with the top-quark forward-backward asymmetry. Light new physics, $M_{\rm NP} \lesssim 150$ GeV, can cause significant deviations from the SM prediction for the bottom asymmetry. The bottom asymmetry can be used to distinguish between competing new physics (NP) explanations of the top asymmetry based on how the NP interferes with s-channel gluon and Z exchange.

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Introduction.—Measurements [1–3] of the forward-backward asymmetry in top-quark pair production ($A_{\rm FB}^{t\bar{t}}$) by the CDF and D0 collaborations at the Tevatron have attracted a lot of attention recently. At high invariant mass, the CDF measurement $A_{\rm FB}^{t\bar{t}}(M_{t\bar{t}} \gtrsim 450\ {\rm GeV}) = 0.295 \pm 0.058({\rm stat}) \pm 0.031({\rm syst})$ is approximately 3σ away from the standard model (SM) prediction, 0.100 ± 0.030 [3]. In addition, CDF observes that $A_{\rm FB}^{t\bar{t}}$ has an approximately linear dependence on both the invariant mass and the magnitude of the rapidity difference ($|\Delta y_{t\bar{t}}|$) of the $t\bar{t}$ pair with slopes that are more than 2σ away from the SM prediction.

Soon after CDF reported evidence for a mass-dependent $t\bar{t}$ asymmetry, it was realized [4–6] that measuring the forward-backward asymmetry in bottom quark production ($A_{\rm FB}^{b\bar{b}}$) may provide insight into the source of the $t\bar{t}$ asymmetry. Any new physics (NP) explanation of $A_{\rm FB}^{t\bar{t}}$ involving left- (right-)handed quarks that respects $SU(2)_L$ (custodial) symmetry will in general also create an asymmetry in $b\bar{b}$ production. The CDF collaboration is in the process of measuring the $b\bar{b}$ forward-backward asymmetry, and has stated [7] how it is binning the data and how sensitive it expects to be to a potential signal. However, $A_{\rm FB}^{b\bar{b}}$ will likely be more difficult to measure than $A_{\rm FB}^{t\bar{t}}$. Among the reasons for this are that gluon fusion, which does not produce an asymmetry, is responsible for $\gtrsim 90\%$ of bottom quark production at the Tevatron. In addition, the $b\bar{b}$ asymmetry is measured by selecting dijet events containing a soft muon and relating the charge of the muon to the charge of the b that produced it [7]. This is potentially problematic because $B - \bar{B}$ mixing and cascade decays will partially wash out the correlation between the charge of what is detected and the charge of bottom quark that produced it [8].

In this Letter, we computed the bottom-quark forward-backward asymmetry at the Tevatron in the SM and for several NP scenarios. It is necessary to know the SM prediction in order to determine whether or not any NP can possibly be present. Since a small asymmetry is expected in the SM, $A_{\rm FB}$ provides an excellent window to observe NP. An interesting difference between the bottom and top quark asymmetries is that the Z pole is in the signal region for the $b\bar{b}$ asymmetry. This leads to tree level exchanges of electroweak gauge bosons dominating the SM contribution to $A_{\rm FB}$ near the Z pole, as well as the opportunity for there to be significant interference effects between NP and tree level Z exchange.

Standard model calculation.—The definition of the forward-backward asymmetry in heavy quark production we use is

$$A_{\rm FB} = \frac{\sigma(\Delta y > 0) - \sigma(\Delta y < 0)}{\sigma(\Delta y > 0) + \sigma(\Delta y < 0)}. \quad (1)$$

Here, Δy is the difference in the rapidity of the quark and antiquark, $\Delta y \equiv y_Q - y_{\bar{Q}}$, and is invariant under boosts along the collision axis. A frame dependent asymmetry may also be defined using y_Q instead of Δy as the discriminating observable. Leading order (LO) QCD is completely symmetric with respect to Δy, and thus, does not generate an asymmetry. Starting with next-to-leading order (NLO) QCD, contributions to the asymmetry as an expansion in powers of α_s can be written schematically as

$$A_{\rm FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \cdots}{\alpha_s^2 D_0 + \alpha^2 \tilde{D}_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \cdots}$$
$$= \alpha_s \frac{N_1}{D_0} + \frac{\alpha^2}{\alpha_s^2}\frac{\tilde{N}_0}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \cdots. \quad (2)$$

Analytic formulae for the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha)$ terms of $A_{\rm FB}$ are given in [9,10]. These results are based on analogous calculations [11,12] for the $e^- e^+ \to \gamma^\star \to \mu^- \mu^+$ asymmetry. Prior results on the QCD asymmetry also exist [13–15]. The $\mathcal{O}(\alpha^2/\alpha_s^2)$ term for $A_{\rm FB}^{t\bar{t}}$ was computed in [16]. Electroweak (EW) Sudakov corrections are shown in [17] to increase the $\mathcal{O}(\alpha_s)$ contribution to the inclusive $A_{\rm FB}^{b\bar{b}}$ by a factor of 1.07. While the N_1 and D_1 terms in Eq. (2) are known completely and have been studied [18–27] in depth, N_2 is only partially known [28–30]. Since it would be inconsistent to include the $N_1 D_1/D_0$ term in our

TABLE I. The $\mathcal{O}(\alpha^2/\alpha_s^2)$ and $\mathcal{O}(\alpha_s)$ contributions to $A_{FB}^{b\bar{b}}$ in various bins.

Bin	$\mathcal{O}(\alpha^2/\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$A_{FB}^{b\bar{b}}$ [%]
$35 \leq M_{b\bar{b}}/\text{GeV} < 75$	0.	$0.179^{+0.014}_{-0.011}$	$0.18 \pm 0.05^{+0.01}_{-0.01}$
$75 \leq M_{b\bar{b}}/\text{GeV} < 95$	$2.167^{+0.661}_{-0.550}$	$0.676^{+0.032}_{-0.026}$	$2.84 \pm 0.20^{+0.69}_{-0.58}$
$95 \leq M_{b\bar{b}}/\text{GeV} < 130$	$0.554^{+0.178}_{-0.147}$	$1.241^{+0.058}_{-0.048}$	$1.79 \pm 0.37^{+0.24}_{-0.20}$
$130 \leq M_{b\bar{b}}/\text{GeV}$	$0.150^{+0.046}_{-0.039}$	$3.369^{+0.237}_{-0.199}$	$3.52 \pm 1.01^{+0.28}_{-0.24}$
$0.0 \leq \lvert\Delta y_{b\bar{b}}\rvert < 0.5$	$0.023^{+0.005}_{-0.005}$	$0.032^{+0.002}_{-0.001}$	$0.06 \pm 0.01^{+0.01}_{-0.01}$
$0.5 \leq \lvert\Delta y_{b\bar{b}}\rvert < 1.0$	$0.082^{+0.020}_{-0.017}$	$0.166^{+0.012}_{-0.010}$	$0.25 \pm 0.05^{+0.03}_{-0.03}$
$1.0 \leq \lvert\Delta y_{b\bar{b}}\rvert \leq 2.0$	$0.133^{+0.034}_{-0.029}$	$0.382^{+0.031}_{-0.024}$	$0.51 \pm 0.11^{+0.07}_{-0.05}$
Inclusive	$0.074^{+0.018}_{-0.015}$	$0.226^{+0.021}_{-0.016}$	$0.30 \pm 0.07^{+0.04}_{-0.03}$

calculation without the N_2 term, we drop the $\mathcal{O}(\alpha_s^2)$ contribution to A_{FB}. To account for this neglect of higher order terms, we assign an uncertainty to our calculation of 30% of the $\mathcal{O}(\alpha_s)$ contribution, originating from $\alpha_s D_1 \approx 0.3 D_0$.

Our calculation was done by convolving the analytic formula of [10,16] with MSTW 2008 NLO PDFs [35] using the deterministic numeric integration algorithm Cuhre from the CUBA library [36]. α_s is set by the MSTW2008 best-fit value, $\alpha_s(M_Z) = 0.120$. We fixed $\mu_R = \mu_F = M_Z$ and $n_{lf} = 4$. The other numeric values employed in this analysis were: $m_b = 4.7$ GeV, $M_Z = 91.1876$ GeV, $\Gamma_Z = 2.4952$ GeV, $\alpha(M_Z) = 1/128.93$, and $\sin^2\theta_W = 0.231$.

To mimic CDF's analysis [7], we required the $b\bar{b}$ pair in our calculation to have a maximum acollinearity of $\delta = \pi - 2.8$ radians. The phase space that is available to the gluon in the $b\bar{b}g$ final state is discussed in [37]. Additional cuts, $|y_{b,\bar{b}}| \leq 1$ and $p_{\perp b,\bar{b}} \geq 15$ GeV, were made. We found the $\mathcal{O}(\alpha)$ corrections decrease the contribution of $\mathcal{O}(\alpha_s)$ to $A_{FB}^{b\bar{b}}$ by 3–11%, depending on the bin. However, we neglect this $\mathcal{O}(\alpha)$ contribution as it is mostly canceled by the increase in $A_{FB}^{b\bar{b}}$ due to electroweak Sudakov effects [17], and the sum of the two effects is small compared to the uncertainty in the total contribution. The flavor excitation process, $qg \to qb\bar{b}$, as well as t-channel W exchange were also neglected as they are numerically small [10,16].

Our results for the $\mathcal{O}(\alpha^2/\alpha_s^2)$ and $\mathcal{O}(\alpha_s)$ contributions to binned $A_{FB}^{b\bar{b}}$ are shown in Table I. In the second and third columns, the uncertainty is due to varying $\mu_R = \mu_F$ from $M_Z/2$ to $2M_Z$. In the fourth column, the first uncertainty is due to neglect of higher-order terms, and the second is the combined scale uncertainty. The uncertainty in the $\mathcal{O}(\alpha^2/\alpha_s^2)$ contribution to $A_{FB}^{b\bar{b}}$ is larger than the $\mathcal{O}(\alpha_s)$ term because the extra power of α_s makes it more sensitive to the choice of scales and PDFs.

Based on CDF's expected sensitivities [7] and assuming the standard model (and the measurements follow a Gaussian distribution), CDF should be able to exclude $A_{FB}^{b\bar{b}}(75 \leq M_{b\bar{b}}/\text{GeV} < 95) = 0$ at the 2.2σ confidence level (C.L.). Although the central value for the asymmetry in the ≥ 130 GeV invariant mass bin is slightly larger than the 75–95 GeV bin, CDF should only be able to exclude $A_{FB}^{b\bar{b}}(130 \leq M_{b\bar{b}}/\text{GeV}) = 0$ at the 1.2σ C.L. The likelihood of excluding zero asymmetry in the 95–130 GeV invariant mass bin is comparable to the likelihood in the ≥ 130 GeV bin. In the SM, all the other (mass or rapidity) bins should be consistent with zero at the 1σ level based on experimental uncertainties.

LO event generators can predict the $\mathcal{O}(\alpha^2/\alpha_s^2)$ contribution to the asymmetry. MADGRAPH 5.1.5.5 [38] with CTEQ6L1 PDFs [39] gives $A_{FB}^{b\bar{b}}(75 \leq M_{b\bar{b}}/\text{GeV} < 95) = (2.26 \pm 0.32(\text{stat})^{+0.24}_{-0.74}(\text{scale}))\%$, in good agreement with our calculation.

It has been suggested [5,6] that measuring the charm-quark forward-backward asymmetry at the Tevatron ($A_{FB}^{c\bar{c}}$) and the bottom-quark charge asymmetry at the LHC ($A_C^{b\bar{b}}$) may also provide insight into the origin of the $A_{FB}^{t\bar{t}}$ anomaly. We computed SM asymmetries of a few percent in suitably chosen kinematic regions for both $A_{FB}^{c\bar{c}}$ and $A_C^{b\bar{b}}$. While the central values for these asymmetries are comparable to those of $A_{FB}^{b\bar{b}}$, it is unlikely that these asymmetries will be observed any time soon in the absence of NP. For $A_{FB}^{c\bar{c}}$, c tagging is less efficient than b tagging. For $A_C^{b\bar{b}}$, the kinematic regions where the asymmetry becomes a few percent have small production cross sections and will require the LHC to run for at least a year at 14 TeV to collect enough data for the SM asymmetry to be statistically distinguishable from zero. Furthermore, the EW contribution to the cross section in these kinematic regions is negligible, and no Z-resonance effects are expected.

New physics scenarios.—Many new physics models have been proposed [40–50] as explanations of the anomalously [51] large $t\bar{t}$ forward-backward asymmetry. For the stringent constraints that these models must overcome, see [53–62]. Prospects for discovery at the LHC are discussed in [41,42,47,49,50,53–55,58–62], among others. Predictions for $A_{FB}^{b\bar{b}}$ in the context of various NP scenarios

TABLE II. The gauge and flavor representations for the models under consideration. T_Q^a and T_L^b are generators of $SU(3)_{Q_L}$ and $SU(2)_L$, respectively.

Case	SM	G_F	Relevant Interaction	Reference
G'	$(8, 1)_0$	$(1, 1, 1)$	$g_a(\bar{U}_R \not{G}' U_R + \bar{D}_R \not{G}' D_R - \bar{Q}_L \not{G}' Q_L)$	[42,44]
ϕ	$(1, 2)_{1/2}$	$(3, 1, \bar{3})$	$\lambda(\phi^0 \bar{t}_L V_{tb} u_R + \phi^- \bar{b}_L u_R) + \text{H.c.}$	[45]
V	$(1, 3)_0$	$(1, 1, 8)$	$\eta V_\mu^{a,b}(\bar{Q}_L^{\alpha i}\gamma^\mu (T_Q^a)_\alpha^\beta (T_L^b)_i^j Q_{L\beta j})$	[46,47]

have already been made in [6,8,50,53,63,64]. We expanded on these works by taking into account the resonance effects of the Z and limiting ourselves to the energy regime accessible at the Tevatron. In particular, we are interested in seeing if the NP contribution to $A_{\rm FB}^{b\bar{b}}$ can be large enough to be distinguishable from the SM predictions we computed above based on the expected sensitivities given in [7]. Any NP in the bottom sector must not spoil the agreement between the SM and precise measurements of flavor changing decays and meson mixing observables such as $\mathrm{Br}(b \to s + \gamma)$ and $B - \bar{B}$ mixing. These and other constraints, such as same-sign top production, are more easily satisfied in flavor symmetric models in which the NP particles form complete representations of the quark global flavor symmetry group, $G_F = SU(3)_{U_R} \times SU(3)_{D_R} \times SU(3)_{Q_L}$. Furthermore, the flavor symmetry guarantees a definite relationship between $A_{\rm FB}^{t\bar{t}}$ and $A_{\rm FB}^{b\bar{b}}$. We consider three different models, a light, broad axigluon (G'), a scalar weak doublet (ϕ), and an $SU(3)_{Q_L}$ octet of electroweak triplet (EWT) vectors (V); see Table II.

It is convenient to split the contributions to the forward-backward asymmetry into two terms

$$A_{\rm FB} = A_{\rm FB}^{\rm I} + A_{\rm FB}^{\rm II}. \quad (3)$$

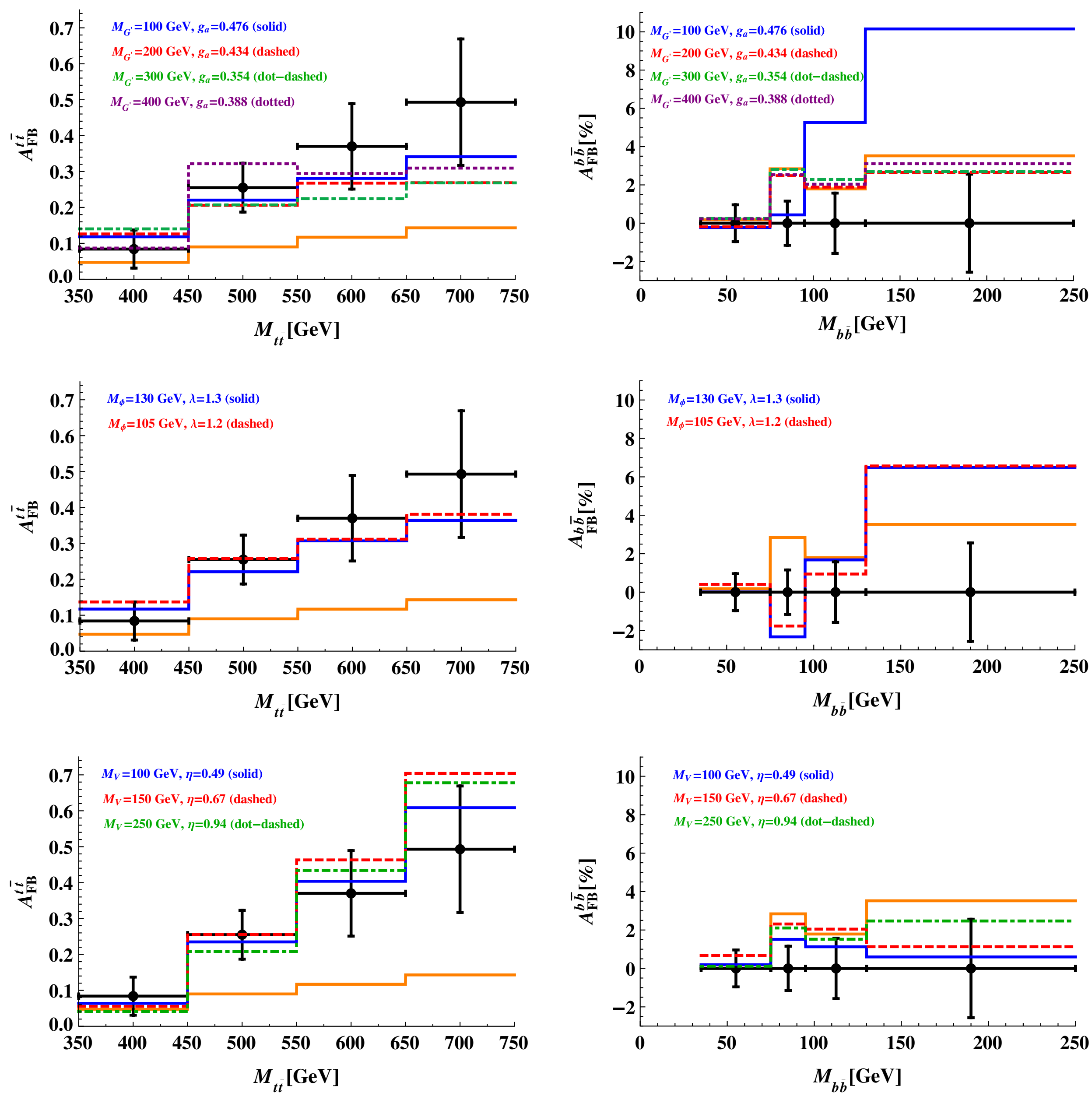

FIG. 1 (color online). Predictions for the binned $A_{\rm FB}^{t\bar{t}}$ (left) and $A_{\rm FB}^{b\bar{b}}$ (right) from the axigluon (top), scalar weak doublet (middle), and flavor octet vector (bottom) models. SM predictions are in orange. In black are CDF's measurements [3] and expected sensitivities [7] for $A_{\rm FB}^{t\bar{t}}$ and $A_{\rm FB}^{b\bar{b}}$, respectively.

$A_{\rm FB}^{\rm I}$ contains the $\mathcal{O}(\alpha_s)$ contribution to $A_{\rm FB}^{b\bar{b}}$ and can be obtained from Table I. The $\mathcal{O}(\alpha)$ contribution to the asymmetry could also be included in $A_{\rm FB}^{\rm I}$, but we neglect it in what follows. On the other hand, $A_{\rm FB}^{\rm II}$ contains the SM $\mathcal{O}(\alpha^2/\alpha_s^2)$ contribution to the asymmetry as well as contributions from NP. This includes both pure NP contributions and interference between NP and tree level s-channel gluon and Z exchange. We calculated $A_{\rm FB}^{\rm II}$ using FEYNRULES 1.6.1 [65] to implement the NP models in MADGRAPH 5.1.5.5 [38] including electroweak processes (QED = 2). For $A_{\rm FB}^{t\bar{t}}$, 10^5 events were generated for a given set of parameters using the CTEQ6L1 [39] PDFs with the renormalization and factorization scales set to m_t. For $A_{\rm FB}^{b\bar{b}}$, 10^5 events were generated for each mass bin for a given set of parameters with $\mu_R = \mu_F = M_Z$. As was the case for the SM analysis, a cut was placed on the rapidity of the bottom quarks, $|y_{b,\bar{b}}| \leq 1$.

Predictions for the binned $t\bar{t}$ and $b\bar{b}$ asymmetries from the NP models are shown in the left and right columns of Fig. 1, respectively. Overflow is included in the rightmost bins. The widths of the axigluon and the EWT vectors were chosen to be 10% of their masses. For the scalars, the natural width to quarks was used. Axigluon benchmark points were taken from Table I of [54]. Benchmark points for the ϕ and V models were chosen based on adding approximately 10% to the inclusive $t\bar{t}$ asymmetry, having a roughly linear dependence of $A_{\rm FB}^{t\bar{t}}$ on $M_{t\bar{t}}$ and adding (or subtracting) less than 1 pb from the $t\bar{t}$ production cross section at the Tevatron.

We have given three classes of models that can accommodate $A_{\rm FB}^{t\bar{t}}$ and produce a $A_{\rm FB}^{b\bar{b}}$ that is distinguishable from the SM prediction. However, this is not generally the case. For example, a flavor octet, EW singlet model ($V_\mu^b (T_L^b)_i^j \to V_\mu \delta_i^j$ in Table II), can accommodate $A_{\rm FB}^{t\bar{t}}$ without causing any significant deviations from the SM predictions because it only produces $b\bar{b}$ from $d\bar{d}$ initial states, whereas the other models involve $u\bar{u}$ initial states. While all three models considered can interfere with gluon exchange, ϕ and V can also interfere with the Z, which dominates the NP contribution to $A_{\rm FB}^{b\bar{b}}$ in the Z pole bin for these models.

In addition to the $A_{\rm FB}^{t\bar{t}}$ anomaly, there is the longstanding puzzle of the $b\bar{b}$ forward-backward asymmetry at LEP1, $A_{\rm FB}^{(0,b)}$, which is 2.4σ below the SM value [66]. Furthermore, the ratio of the partial width $Z \to b\bar{b}$ to the inclusive hadronic width, R_b, is 2.3σ above the SM prediction [67]. Assuming only the bottom quark's coupling to the Z is modified, the value of δg_{Rb} which provides the best-fit to the EWPD collected at LEP is 0.016 [68], which is more than 20% of the LO SM coupling. See [48,49] for attempts to simultaneously explain $A_{\rm FB}^{t\bar{t}}$ and $A_{\rm FB}^{(0,b)}$. In models where the NP couples to quarks in a flavor universal way, the loop correction that gives the best-fit value for δg_{Rb} will give an analogous correction to $\delta g_{Ru,d}$, which is much larger than allowed by atomic parity violation experiments [57]. The tree level $V - Z$ mixing of [69] is not a viable explanation either for the same reason. Axigluon models give $\delta g_{Rb} = \delta g_{Lb}$ [57], which disagrees with the best-fit value for δg_{Lb}, $\mathcal{O}(10^{-3})$ [68]. Prospects for measuring $b\bar{b}$ and $t\bar{t}$ asymmetries at future linear colliders are examined in [70].

Conclusions.—In summary, we computed $A_{\rm FB}^{b\bar{b}}$ in the SM and for several NP scenarios, carefully accounting for the Z pole, which is in the signal region for the $b\bar{b}$ asymmetry. The largest SM contribution to $A_{\rm FB}^{b\bar{b}}$ near the Z pole comes from tree level exchanges of Z and $\gamma^\star$. While at higher invariant mass, NLO QCD dominates the SM asymmetry. Light NP, $M_{\rm NP} \lesssim 150$ GeV, is needed to generate a $b\bar{b}$ asymmetry, which CDF would be able to distinguish from the SM. $A_{\rm FB}^{b\bar{b}}$ can be used to distinguish between competing NP explanations of $A_{\rm FB}^{t\bar{t}}$ based on how the NP interferes with s-channel gluon and Z exchange.

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