Use of $B \to J/\psi\, f_0$ Decays to Discern the $q\bar{q}$ or Tetraquark Nature of Scalar Mesons

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(Received 28 May 2013; published 6 August 2013)

We consider the relative decay rates of $\bar{B}^0$ and $\bar{B}^0_s$ mesons into a J/ψ plus a light scalar meson, either the $f_0(500)$ (σ) or the $f_0(980)$. We show that it is possible to distinguish between the quark content of the scalars being quark-antiquark or tetraquark by measuring specific ratios of decay rates. Using current data we determine the ratio of form factors in $\bar{B}^0_s \to J/\psi f_0(980)$ with respect to $\bar{B}^0 \to J/\psi f_0(500)$ decays to be $0.99^{+0.13}_{-0.04}$ at a four-momentum transfer squared equal to the mass of the J/ψ meson squared. In the case where these light mesons are considered to be quark-antiquark states, we give a determination of the mixing angle between strange and light quark states of less than 29° at 90% confidence level. We also discuss the use of a similar ratio to investigate the structure of other isospin singlet states.

DOI: 10.1103/PhysRevLett.111.062001 PACS numbers: 14.40.Be, 12.39.−x, 13.25.Hw

Scalar mesons in general, and the $f_0(980)$ in particular, are not well understood. Their masses do not follow the expectation in the naïve quark model that the state containing two strange quarks is heavier than the state containing only one, in stark contrast to the vector mesons [1]. This has led to theories that the light J^{PC} equal to 0^{++} mesons may be combinations of diquarks and antidiquarks, e.g., $[qq][\bar{q}\bar{q}]$, called "tetraquarks" [2].

Recently there have been several studies of the $f_0(980)$ in heavy meson decays, some in the charm system [3]. Based on these data, the existence of the mode $\bar{B}^0_s \to J/\psi f_0(980)$ was predicted [4], discovered by the LHCb Collaboration [5], and confirmed [6]. The LHCb Collaboration also found the decay $\bar{B}^0 \to J/\psi f_0(500)$ and set an upper limit on the decay $\bar{B}^0 \to J/\psi f_0(980)$ [7]. From now on the $f_0(500)$ meson will be designated as σ and the $f_0(980)$ meson will be designed as f_0. The $\bar{B}^0_s \to J/\psi f_0(980)$ channel has also been used to measure *CP* violation [8], but Fleischer *et al.* have claimed that if the f_0 is a tetraquark state the measurement could be influenced by the presence of additional suppressed decay mechanisms [9]. Thus, a resolution of the problem of these states' structure would be helpful in several ways.

When the σ and f_0 are considered as $q\bar{q}$ states there is the possibility of their being mixtures of light and strange quarks that is characterized by a 2 × 2 rotation matrix with a single parameter, the angle ϕ, so that their wave functions are

$$
\begin{aligned}
|f_0\rangle &= \cos\phi|s\bar{s}\rangle + \sin\phi|n\bar{n}\rangle,\\
|\sigma\rangle &= -\sin\phi|s\bar{s}\rangle + \cos\phi|n\bar{n}\rangle, \qquad (1)
\end{aligned}
$$

where $|n\bar{n}\rangle \equiv \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$.

While there have been several attempts to measure the mixing angle ϕ, the model dependent results give a wide range of values. We describe here only a few examples. $D^\pm$ and $D^\pm_s$ decays into $f_0(980)\pi^\pm$ and $f_0(980)K^\pm$ give values of 31° ± 5° or 42° ± 7° [10]. $D^+_s \to \pi^+\pi^+\pi^-$ transitions give a range $35° < |\phi| < 55°$ [11]. In light meson radiative decays two solutions are found, either 4° ± 3° or 136° ± 6° [12]. Resonance decays from both $\phi \to \gamma\pi^0\pi^0$ and $J/\psi \to \omega\pi\pi$ give a value of $\simeq 20°$. On the basis of SU(3), a value of 19° ± 5° is provided [13]. Finally, Ochs [14], averaging over several processes, finds 30° ± 3°.

When these states are viewed as $q\bar{q}q\bar{q}$ states, the wave functions become

$$|f_0\rangle = \frac{1}{\sqrt{2}}([su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}]), \qquad |\sigma\rangle = [ud][\bar{u}\bar{d}]. \qquad (2)$$

In this Letter we assume the tetraquark states are unmixed, for which there is some justification [2,10,15], with a mixing angle estimate of <5° [9].

In general, the decay width for a $\overline{B}$ meson to decay into a J/ψ and light scalar state f can be expressed as [9,16,17]

$$\Gamma(\overline{B} \to J/\psi f) = C|F^f_B(m^2_{J/\psi})|^2|V_{\rm ci}|^2\Phi Z^2, \qquad (3)$$

where C is a constant, F^f_B is form factor evaluated at the four-momentum transfer q^2 equal to the mass of the J/ψ squared, and $V_{\rm ci}$ is the relevant Cabibbo-Kobaysashi-Maskawa element. The phase space factor $\Phi = [m_B E(x,y)]^3$, where $x = m_{J/\psi}/m_B$, $y = (m_f/m_B)$, and $E(x,y) = \sqrt{[1-(x+y)^2][1-(x-y)^2]}$ (the phase space is calculated taking into account the mass dependent line shapes). Z represents the coupling amplitude that depends on the quark configuration after the $\overline{B}$ meson decay and the quark content of the light meson in either the $q\bar{q}$ or tetraquark model. The values for Z are listed in Table I.

The diagrams for decays of $\bar{B}^0_s$ mesons into the σ and f_0 are shown in Fig. 1 for both $q\bar{q}$ and tetraquark models. The coupling amplitudes for the f_0 and σ in the $q\bar{q}$ model are $\cos\phi$ and $\sin\phi$, respectively, while in the tetraquark model the coupling is $\sqrt{2}$ for the f_0 and σ production is not allowed. Thus, a null test of the tetraquark model is

TABLE I. Values of the coupling amplitude $\mathcal{Z}$.

	$\bar{B}_s^0$		$\bar{B}^0$	
Model	f_0	σ	f_0	σ
$q\bar{q}$	$\cos\phi$	$\sin\phi$	$\sin\phi/\sqrt{2}$	$\cos\phi/\sqrt{2}$
Tetraquark	$\sqrt{2}$	0	$1/\sqrt{2}$	1

evident: if the decay $\bar{B}_s^0 \to J/\psi\sigma$ is observed, then the tetraquark model described here is ruled out.

The diagrams for decays of $\bar{B}^0$ mesons into the σ and f_0 are shown in Fig. 2 for both $q\bar{q}$ or tetraquark models [18].

There are measured branching fractions for some of these decays, which are summarized in Table II. [In order to minimize systematic uncertainties, we use only LHCb measurements even though other measurements of $\mathcal{B}(\bar{B}_s^0 \to J/\psi f_0)$ are available [1].] The branching fractions into final states with an f_0 have been corrected by their decay rates into $\pi^+\pi^-$ using measurements from BES [19] from which we obtain $\mathcal{B}(f_0(980) \to K^+K^-)/\mathcal{B}(f_0(980) \to \pi^+\pi^-) = 0.25^{+0.17}_{-0.11}$ [3], and from *BABAR* of $\mathcal{B}(f_0(980) \to K^+K^-)/\mathcal{B}(f_0(980) \to \pi^+\pi^-) = 0.69 \pm 0.32$ [20]. Averaging the two measurements gives

$$\frac{\mathcal{B}(f_0(980) \to K^+K^-)}{\mathcal{B}(f_0(980) \to \pi^+\pi^-)} = 0.35^{+0.15}_{-0.14}. \quad (4)$$

To determine the $\pi^+\pi^-$ branching fraction it is assumed that the $\pi\pi$ and KK decays are dominant, and that the ratios of $\pi^0\pi^0$ to $\pi^+\pi^-$ and $K^0\bar{K}^0$ to K^+K^- are given by isospin conservation as 1/2 and 1, respectively, leading to [7]

$$\mathcal{B}(f_0(980) \to \pi^+\pi^-) = (46 \pm 6)\%. \quad (5)$$

For σ decay we use $\mathcal{B}(\sigma \to \pi^+\pi^-) = 2/3$, which again results from isospin conservation and the assumption that the only decays are into two pions. The uncertainties in these rates are not included in Table II, but are introduced when comparisons between σ and f_0 are made.

In this Letter we present information obtainable from ratios of the $\bar{B}_s^0$ and $\bar{B}^0$ decay rates into σ and f_0 mesons. Using the ratios allows cancellation of many of the experimental and theoretical uncertainties. The ratios we will

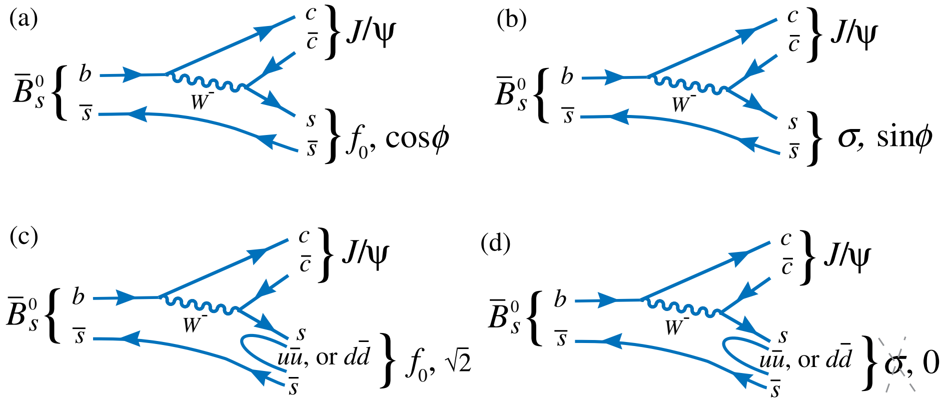

FIG. 1 (color online). Decays of the $\bar{B}_s^0$ meson to a J/ψ and (a) f_0 in the $q\bar{q}$ model, (b) σ in the $q\bar{q}$ model, (c) f_0 in the tetraquark model, and (d) σ in the tetraquark model. The factor next to the scalar resonance name indicates the coupling amplitude $\mathcal{Z}$.

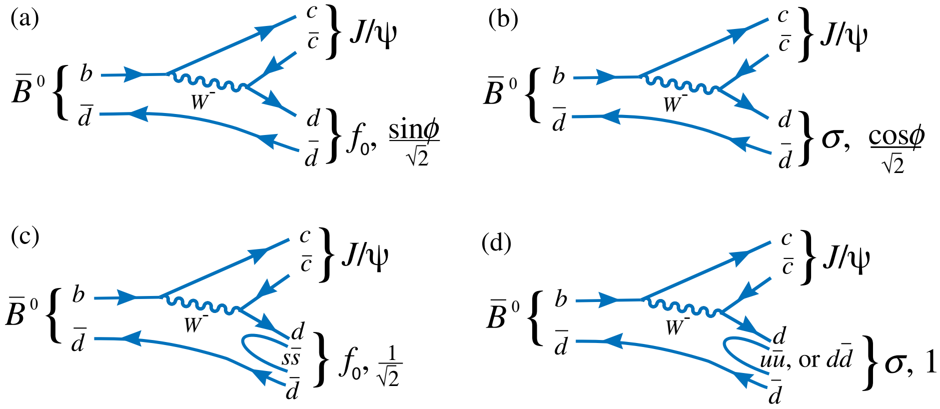

FIG. 2 (color online). Decays of the $\bar{B}^0$ meson to a J/ψ and (a) f_0 in the $q\bar{q}$ model, (b) σ in the $q\bar{q}$ model, (c) f_0 in the tetraquark model, and (d) σ in the tetraquark model. The factor next to the scalar resonance name indicates the coupling amplitude $\mathcal{Z}$.

TABLE II. Experimental branching fractions from LHCb for $B \to J/\psi f$ meson final states. The uncertainties on $\mathcal{B}(f \to \pi^+\pi^-)$ are not included.

Final state	$\bar{B}_s^0$ [22]	$\bar{B}^0$ [7]
σ	$\cdots$	$9.60^{+3.79}_{-1.70} \times 10^{-6}$
f_0	$3.40^{+0.63}_{-0.16} \times 10^{-4}$	$<1.7 \times 10^{-6}$

consider are listed in Table III for both $q\bar{q}$ and tetraquark models.

To calculate the width ratios from the branching fractions when both $\bar{B}^0$ and $\bar{B}_s^0$ initial states are present, we use values of the lifetimes of 1.530 ± 0.007 ps and 1.622 ± 0.0023 ps [21], respectively. (Since the $\bar{B}_s^0$ modes are all negative *CP* eigenstates, we use the value provided for τ_{long}.) Input on the form-factor ratios is needed to reach quantitative conclusions. For $r_{0\sigma}^{sf_0}$, both the $q\bar{q}$ and tetraquark models predict identical ratios, and this ratio is independent of ϕ. Using the data in Table II we find

$$\frac{|F_{B_s^0}^{f_0}(m_{J/\psi}^2)|}{|F_{B^0}^{\sigma}(m_{J/\psi}^2)|} = 0.99^{+0.13}_{-0.04}. \tag{6}$$

The ratio $r_{sf_0}^{s\sigma}$ was suggested as a way of measuring $\tan\phi$ by Li *et al.* [17]. The form-factor ratio calculated by Li *et al.* is very close to unity, $|F_{B_s^0}^{\sigma}(m_{J/\psi}^2)|^2/|F_{B_s^0}^{f_0}(m_{J/\psi}^2)|^2 = 1$. Assuming that the similar form-factor ratio $|F_{B^0}^{f_0}(m_{J/\psi}^2)|/|F_{B^0}^{\sigma}(m_{J/\psi}^2)|$ is unity, LHCb used their data to set an upper limit on $\phi < 31°$ at 90% C.L. [7].

Measurement of the branching fraction of $\bar{B}^0 \to J/\psi f_0$ was suggested by Fleischer *et al.* [9] as a way of investigating the tetraquark structure of the f_0. In the $q\bar{q}$ model they use the form-factor ratio $|F_{B^0}^{f_0}(m_{J/\psi}^2)|/|F_{B_s^0}^{f_0}(m_{J/\psi}^2)|$ that was computed by El-Bennich *et al.* [16] of 0.69 using dispersion relations (in the covariant light front dynamics model El-Bennich *et al.* compute 0.58). They find results that are mixing angle dependent. In the tetraquark model they use a unit form-factor ratio and predict $\mathcal{B}(\bar{B}^0 \to J/\psi f_0, f_0 \to \pi^+\pi^-) \sim (1\text{–}3) \times 10^{-6}$. The measured upper limit from LHCb is 1.1×10^{-6} at 90% C.L., which is barely consistent. It is also interesting that, using the upper limit on the measured ratio $r_{sf_0}^{0f_0}$ and a unit form-factor ratio, we find an upper limit $\phi < 29°$ in the $q\bar{q}$ model, slightly more restrictive than the LHCb determined limit of $\phi < 31°$ using $r_{sf_0}^{s\sigma}$; this evaluation does not depend on any properties of the σ, nor on $\mathcal{B}(f_0 \to \pi^+\pi^-)$. The ratio $r_{sf_0}^{0f_0}$ was also suggested by Ochs [14] as a way of investigating the properties of the $f_0(980)$ and the $f_0(1500)$; he also takes a unit form-factor ratio.

TABLE III. Ratios of decay widths. The rate ratio is multiplied by the value for $\mathcal{Z}^2$ in either the $q\bar{q}$ model or the tetraquark model. The form factors are notated as F_j^i, and the phase space factor Φ_j^i, where i indicates either σ or f_0 and j indicates either $\bar{B}^0$ or $\bar{B}_s^0$.

Label	Mode ratio	Rate ratio	$\mathcal{Z}^2$ $q\bar{q}$	$\mathcal{Z}^2$ Tetraquark
$r_{sf_0}^{0f_0}$	$\frac{\Gamma(\bar{B}^0 \to J/\psi f_0)}{\Gamma(\bar{B}_s^0 \to J/\psi f_0)} =$	$\frac{\lvert F_{B^0}^{f_0}(m_{J/\psi}^2)\rvert^2}{\lvert F_{B_s^0}^{f_0}(m_{J/\psi}^2)\rvert^2} \frac{\lvert V_{\text{cd}}\rvert^2 \Phi_{B^0}^{f_0}}{\lvert V_{\text{cs}}\rvert^2 \Phi_{B_s^0}^{f_0}}$	$\frac{1}{2}\tan^2\phi$	$\frac{1}{4}$
$r_{0\sigma}^{0f_0}$	$\frac{\Gamma(\bar{B}^0 \to J/\psi f_0)}{\Gamma(\bar{B}^0 \to J/\psi \sigma)} =$	$\frac{\lvert F_{B^0}^{f_0}(m_{J/\psi}^2)\rvert^2}{\lvert F_{B^0}^{\sigma}(m_{J/\psi}^2)\rvert^2} \frac{\Phi_{B^0}^{f_0}}{\Phi_{B^0}^{\sigma}}$	$\tan^2\phi$	$\frac{1}{2}$
$r_{sf_0}^{s\sigma}$	$\frac{\Gamma(\bar{B}_s^0 \to J/\psi \sigma)}{\Gamma(\bar{B}_s^0 \to J/\psi f_0)} =$	$\frac{\lvert F_{B_s^0}^{\sigma}(m_{J/\psi}^2)\rvert^2}{\lvert F_{B_s^0}^{f_0}(m_{J/\psi}^2)\rvert^2} \frac{\Phi_{B_s^0}^{\sigma}}{\Phi_{B_s^0}^{f_0}}$	$\tan^2\phi$	0
$r_{0\sigma}^{sf_0}$	$\frac{\Gamma(\bar{B}_s^0 \to J/\psi f_0)}{\Gamma(\bar{B}^0 \to J/\psi \sigma)} =$	$\frac{\lvert F_{B_s^0}^{f_0}(m_{J/\psi}^2)\rvert^2}{\lvert F_{B^0}^{\sigma}(m_{J/\psi}^2)\rvert^2} \frac{\lvert V_{\text{cs}}\rvert^2 \Phi_{B_s^0}^{f_0}}{\lvert V_{\text{cd}}\rvert^2 \Phi_{B^0}^{\sigma}}$	2	2

A further elucidation of the null prediction of $r_{sf_0}^{s\sigma}$ in the tetraquark model is in order. In addition to the caveat that there could be a small amount of mixing, <5°, between the σ and f_0 tetraquark states, there also could be higher order diagrams that couple to the σ in the $\bar{B}_s^0$ decay. In terms of the topological diagrams illustrated in Ref. [9], both the tree and leading penguin diagrams do not couple to the σ, as well as three other higher order diagrams. On the other hand, three diagrams involving penguin annihilation and W exchange would couple to the σ. As these are expected to have a very small rate compared to the tree diagram, we do not expect that they could induce a rate corresponding to a mixing angle of more than a few degrees.

In conclusion, we discuss the importance of branching fraction ratios in $(\bar{B}_s^0 \text{ or } \bar{B}^0) \to J/\psi$ (σ or f_0) decays. These measurements can discern whether or not the σ and f_0 are $q\bar{q}$ or tetraquarks. To aid in these tests we have determined the form-factor ratio $|F_{B_s^0}^{f_0}(m_{J/\psi}^2)|/|F_{B^0}^{\sigma}(m_{J/\psi}^2)| = 0.99^{+0.13}_{-0.04}$, based on LHCb data. If the σ is a tetraquark state we do not expect to see it $\bar{B}_s^0$ decays at a level of more than a percent of the $f_0(980)$ rate. For the σ and f_0 being $q\bar{q}$ states, we provide a limit on the mixing angle of <29° at 90% C.L. Furthermore, we note that these tests could be extended to other systems. For example, if an isospin equal zero meson $f_{I=0}$ was found in both $\bar{B}^0 \to J/\psi f_{I=0}$ and $\bar{B}_s^0 \to J/\psi f_{I=0}$ decays, its mixing angle with another meson could be determined using a ratio similar to $r_{sf_0}^{0f_0}$ (see also Ref. [14]). It is interesting that the square of the coupling amplitude would be $1/4$ in the tetraquark model, and in the $q\bar{q}$ model its mixing angle with some other, possibly unknown, meson could be determined.

We gratefully acknowledge support from the National Science Foundation, and thank Jack Laiho, and Joe Schechter for useful discussions.

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