Origin of Cosmic Magnetic Fields

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We calculate, in the free Maxwell theory, the renormalized quantum vacuum expectation value of the two-point magnetic correlation function in de Sitter inflation. We find that quantum magnetic fluctuations remain constant during inflation instead of being washed out adiabatically, as usually assumed in the literature. The quantum-to-classical transition of super-Hubble magnetic modes during inflation allow us to treat the magnetic field classically after reheating, when it is coupled to the primeval plasma. The actual magnetic field is scale independent and has an intensity of few $\times 10^{-12}$ G if the energy scale of inflation is few $\times 10^{16}$ GeV. Such a field accounts for galactic and galaxy cluster magnetic fields.

Introduction.—The origin of the observed large-scale μG magnetic fields in galaxies and galaxy clusters is one of the major unsolved mysteries in cosmology (for reviews on cosmic magnetic fields, see [1–3]).

There are two main schools of thought about the generation of such cosmic magnetic fields, according to which magnetic fields we observe today are created either in the early Universe ("primordial hypothesis") or during the processes of large-scale structure formation and evolution ("astrophysical hypothesis"). According to the primordial hypothesis, large-scale magnetic fields have been created during an inflationary epoch of the Universe, or during primeval cosmic phase transitions (such as electroweak or quark-hadron phase transitions). Successively, these relic fields have been possibly amplified in galaxies and galaxy clusters by dynamo actions [1]. The astrophysical hypothesis, instead, supposes that seed fields are generated by plasma effects directly in galaxies and galaxy clusters, and then amplified by a dynamo mechanism. Both hypotheses meet with difficulties when their predictions are compared with observations.

It is believed that inflation-produced magnetic fields have large correlation scales λ but extremely low intensities, unless some nonstandard physics is introduced, e.g., by adding nonstandard terms to the photon field Lagrangian [2]. As shown in [4] (see [5] for a recent criticism to this work), this is the case only if the spatial curvature of the Universe is zero. However, in [4], the initial magnetic spectrum is that associated to "unrenormalized" vacuum fluctuations. This is a questionable assumption, since it gives a formally infinite, vacuum expectation value (VEV) of the two-point magnetic correlation function. It is the aim of this Letter to bring into question the physical correctness of using unrenormalized vacuum fluctuations and to show, contrary to what is believed, that strong inflationary magnetic fields are a natural consequence of standard quantum electrodynamics in curved space (in particular in a Friedmann spacetime with zero spatial curvature). This is possible if one, in order

to get a finite result, "renormalizes" the two-point magnetic correlator.

Phase-transition-generated fields can have astrophysically relevant intensities [3], but their correlation lengths are too small to explain cosmic magnetic fields, even allowing a possible amplification due to magnetohydrodynamic turbulent effects operating in the early Universe [6].

The generation of magnetic fields directly in galaxies and galaxy clusters is problematic due to the fact that it is very difficult to explain the presence of strong magnetic fields in galaxies at high redshift, since (large-scale) dynamo actions are inefficient on short time scales [1]. Moreover, the detected spectrum of distant blazars [7] seems to be compatible with the presence of magnetic fields in voids, whose nature can be then explained only in the framework of the primordial hypothesis.

Seed fields.—The observation of magnetic fields in galaxies and galaxy clusters could be explained if a sufficiently intense large-scale magnetic field, such as 10^{-13} G $\lesssim B_0 \lesssim$ few $\times 10^{-12}$ G with $\lambda \gtrsim$ few $\times$ Mpc, were present prior to their formation. The above comoving values take into account the amplification and stretching of magnetic fields inside galaxies and galaxy clusters, due essentially to the so-called Alfvén frozen flux effect [3] and to the Kelvin-Helmholtz instability of intracluster plasma flows [8].

In the following, we show that a primordial field with the above properties is a natural consequence of inflation. To set notations and to explain why this kind of field is believed not to be generated in the standard Maxwell theory, we consider first the case analyzed in the literature, to wit, that of unrenormalized magnetic fluctuations from inflation.

Unrenormalized fluctuations.—The equation of motion for a magnetic field in a curved spacetime is homogeneous in the field, so one needs an initial field in order to have a today field different from zero. Quantum-mechanical effects during inflation give the unique possibility to have such an initial magnetic field. As shown a long time ago by

Parker [9], particles can be created by quantum-gravitational effects in an expanding universe. However, this is not the case for conformally invariant theories, a result known as "Parker theorem." Standard electromagnetism in a Friedmann universe is invariant under conformal transformations, so, in this case, the only other way to have an initial magnetic spectrum is to consider electromagnetic vacuum fluctuations, which are present even in conformally invariant theories.

The standard Maxwell Lagrangian for the electromagnetic field A_μ is $\mathcal{L} = -(1/4)\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$, where g is the determinant of the metric tensor and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. For the sake of simplicity, we assume that during inflation the Universe is described by a de Sitter spacetime with line element $ds^2 = a^2(d\eta^2 - d\mathbf{x}^2)$, where a is the expansion parameter, $\eta = -1/(aH)$ is the conformal time, and H is the (constant) Hubble parameter. Working in the Lorentz gauge, $\nabla_\mu A^\mu = 0$, we expand the transverse part of the vector potential as $\mathbf{A}_T(\mathbf{x}) = \sum_{\alpha=1}^2 \int d^3k (2\pi)^{-3} \boldsymbol{\varepsilon}_{\mathbf{k},\alpha} a_{\mathbf{k},\alpha} A_{k,\alpha} e^{i\mathbf{k}\mathbf{x}} + \text{H.c.}$, where the transverse polarization vectors $\boldsymbol{\varepsilon}_{\mathbf{k},\alpha}$ satisfy the completeness relation $\sum_\alpha (\varepsilon_{\mathbf{k},\alpha})_i (\varepsilon_{\mathbf{k},\alpha}^*)_j = \delta_{ij} - k_i k_j/\mathbf{k}^2$, with $\mathbf{k}$ being the comoving wave number. The annihilation and creation operators satisfy the usual commutation relations $[a_{\mathbf{k},\alpha}, a_{\mathbf{k}',\alpha'}^\dagger] = (2\pi)^3 \delta_{\alpha\alpha'}\delta(\mathbf{k} - \mathbf{k}')$, all the other commutators being null.

The equation of motion for $A_{k,\alpha}$ is $\ddot{A}_{k,\alpha} + k^2 A_{k,\alpha} = 0$ (a dot denotes differentiation with respect to the conformal time), whose solution is $A_{k,\alpha} = c_1(k)e^{-ik\eta} + c_2(k)e^{ik\eta}$, with $c_1(k)$ and $c_2(k)$ constants of integrations. These are fixed by the choice of the vacuum, which is taken to be the Bunch-Davies vacuum [9]. In this case, the above constants are $c_1(k) = 1/\sqrt{2k}$ and $c_2(k) = 0$, so that we have the standard plane-wave solution $A_{k,\alpha} = e^{-ik\eta}/\sqrt{2k}$. Let us introduce the magnetic field, $\mathbf{B}(\mathbf{x})$, in the usual way as $a^2\mathbf{B} = \nabla \times \mathbf{A}_T$. The vacuum expectation value of the squared magnetic field is then $\langle 0|\mathbf{B}(\mathbf{x})^2|0\rangle = \int_0^\infty dk\, k^{-1}\mathcal{P}(k)$, where $\mathcal{P}(k) = \sum_\alpha [k^5/(2\pi^2 a^4)]|A_{k,\alpha}|^2$ is the so-called magnetic power spectrum. For the plane-wave solution we have $\mathcal{P}(k) = k^4/(2\pi^2 a^4)$.

Introducing the comoving wavelength λ as $k = 2\pi/\lambda$, one usually defines the magnetic field strength B on the comoving scale λ as $B(\lambda) = \mathcal{P}(2\pi/\lambda)^{1/2}$. Accordingly, during de Sitter inflation the magnetic field scales adiabatically, $B \propto a^{-2}$, reducing (exponentially) its intensity. As a result, this field cannot explain cosmic magnetic fields, in agreement with the standard literature.

Quantum-to-classical transition.—Before analyzing the problem of renormalization of inflationary quantum fluctuations, we notice that a transition from quantum to classical behavior of such fluctuations is generally expected to take place. Indeed, this occurs when quantum coherence is destroyed by its coupling to the environment. A quantum expectation value like $\langle 0|\mathbf{B}(\mathbf{x})^2|0\rangle$ becomes then indistinguishable from the corresponding classical ensemble average $\langle \mathbf{B}(\mathbf{x})^2\rangle$ [10].

Classicalization of a given quantum fluctuation is realized when it crosses outside the horizon during inflation, and this is understood in terms of its "squeezing" properties [10]. Once a given realization of the magnetic fluctuations has occurred during inflation, further evolution proceeds classically. For this reason, we can treat super-Hubble inflationary modes as classical stochastic fluctuations after inflation, and, in particular, after reheating; namely, after the energy associated to inflaton has been converted into ordinary matter and any magnetic field gets coupled to the newly formed plasma.

Renormalized fluctuations.—The standard approach in calculating the inflation-produced magnetic fluctuations is questionable since the quantity $\langle 0|\mathbf{B}(\mathbf{x})^2|0\rangle$ is formally infinite due to the ultraviolet divergence of the power spectrum. This divergence can be cured by renormalizing the magnetic correlator. It is worth noticing that the same situation appears in a very different context, namely in relation to the primeval power spectrum of the cosmic microwave background radiation, when quantizing the inflaton field fluctuations. Here, renormalizing the inflaton two-point correlator gives very significant effects on the amplitude and properties of perturbations from inflation [11].

In this Letter, we adopt the method of adiabatic renormalization [12] although, recently enough, there has been in the literature a critical discussion about the validity of this renormalization technique [13]. In the adiabatic renormalization procedure, one assumes that the expansion parameter is a slowly varying function of time. This is attained by replacing the expansion parameter $a(\eta)$ by a one parameter family of functions $a_T(\eta) = a(\eta/T)$, and taking the limit of large "slowness parameter" T. This allows us to find a WKB (or adiabatic) solution to the equation of motions to any desiderate order $(T^{-1})^n$ (with $n \geq 0$). The adiabatic expansion is a formal one, in the sense that it must be applied even if $a(\eta)$ is not a slowly varying function of time. This assures the conservation of the regularized energy-momentum tensor [12]. Then, the physical (i.e., renormalized) VEV of a given quantity is obtained from the unrenormalized one by subtracting mode by mode the corresponding adiabatic quantity up to the appropriate order, the minimum adiabatic order being determined by the degree of ultraviolet divergence of that quantity [9].

We assume that the physical VEV is a linear operator, in the sense that $\langle 0|\Psi_1[\psi(x)] + \Psi_2[\psi(x)] + \cdots |0\rangle_{\text{phys}} = \langle 0|\Psi_1[\psi(x)]|0\rangle_{\text{phys}} + \langle 0|\Psi_2[\psi(x)]|0\rangle_{\text{phys}} + \cdots$, for all functions Ψ_i of a given field ψ evaluated at the spacetime point x. This is a necessary condition we must impose on renormalized VEVs, since this property is verified by classical ensemble averages and, according to the above discussion, a possible classicalization of super-Hubble

quantum fluctuations makes them indistinguishable from each other. In order to cure ultraviolet divergences in the VEV of the energy-momentum tensor, $\langle 0|T_\nu^\mu|0\rangle$, one generally needs to subtract from that, and mode by mode, the corresponding adiabatic quantity up to the order $n = 4$ [12]. Since T_ν^μ is constructed starting from local quadratic quantities in the fields, the linearity of the $\langle 0|\ldots|0\rangle_{\text{phys}}$ operator requires the use of the fourth adiabatic order also for these quadratic quantities. In order to renormalize the two-point magnetic correlator then, we consider the WKB expansion up to fourth order.

In general, the adiabatic renormalization procedure applied to the stress tensor reduces to normal ordering in the limit of static $a(\eta)$ (the Minkowski case), and is completely equivalent to other renormalization schemes used in quantum theory in curved spacetime [9,18]. In particular, it gives the correct value of the so-called "conformal anomaly" in the case of conformally invariant theories [9,18,19]. Moreover, in all renormalization schemes, the removal of infinities in the VEV of the energy-momentum tensor corresponds to the renormalization of the coupling constants in the Einstein's equations [18].

To apply the adiabatic renormalization procedure to the two-point magnetic correlator, we first need to introduce a regulator photon mass, m, for the transverse part of the vector potential, which will be sent to zero at the end of the calculation. This is possible due to well-known fact that the transverse part of a Proca field (namely, a massive spin-1 vector field) smoothly tends to the electromagnetic field in the limit of vanishing mass [20]. This is also necessary since we must temporarily break conformal invariance of electromagnetism otherwise the adiabatic solution and the exact solution to the equation of motion would coincide. Generally, this would give incorrect results, such as a vanishing electromagnetic conformal anomaly. Similar breakdowns of conformal invariance happen in other regularization schemes [18], such as dimensional regularization, where conformal invariance is temporarily broken by letting the spacetime dimensions going away from $d = 4$, or in the ζ-function regularization scheme, where the conformal invariance is broken by the technique of analytic continuation. (Analogue situations appear also in quantum theory in Minkowski spacetime. For example, dimensional regularization may lead to the breaking of chiral invariance, giving a chiral anomaly.) In the case of a massive photon, the equation of motion becomes $\ddot{A}_{k,\alpha,m} + \omega^2 A_{k,\alpha,m} = 0$, where $\omega^2 = k^2 + m^2 a^2$. The solution corresponding to the Bunch-Davies vacuum is $A_{k,\alpha,m} = (\sqrt{\pi}/2)e^{i\pi(1+2\nu)/4}\sqrt{-\eta}H_\nu^{(1)}(-k\eta)$, where $\nu = \sqrt{1/4 - m^2/H^2}$ and $H_\nu^{(1)}(x)$ is the Hankel function of the first kind.

Second, we need the adiabatic solution $A_{k,\alpha,m}^{(A)}$ of the equation of motion. Because of the replacing of $a(\eta) \to a_T(\eta) = a(\eta/T)$ $(T \to \infty)$, the adiabatic order of the solution can be found by counting the number of time derivatives

of $a(\eta)$ [9]. Following the standard procedure [9], we write $A_{k,\alpha,m}^{(A)} = e^{-i\int_0^\eta d\eta' W(k,\eta')}/\sqrt{2W(k,\eta)}$. Expanding W up to the fourth adiabatic order, $W = \sum_{i=0}^4 \omega^{(i)}$, we get, from the equation of motion, $\omega^{(0)} = \omega$, $\omega^{(1)} = \omega^{(3)} = 0$, $\omega^{(2)} = (3/8)\omega^{-3}\dot{\omega}^2 - (1/4)\omega^{-2}\ddot{\omega}$, and $\omega^{(4)} = -(297/28)\omega^{-7}\dot{\omega}^4 - (99/32)\omega^{-6}\dot{\omega}^2\ddot{\omega} + (13/32)\omega^{-5}\ddot{\omega}^2 + (5/8)\omega^{-5}\dot{\omega}\dddot{\omega} - (1/16)\omega^{-4}\ddddot{\omega}$. The adiabatic expansion of W^{-1} up to the fourth order, $W^{-1} = \sum_{i=0}^4 (W^{-1})^{(i)}$, comes straightforwardly: $(W^{-1})^{(0)} = \omega^{-1}$, $(W^{-1})^{(1)} = (W^{-1})^{(3)} = 0$, $(W^{-1})^{(2)} = -(3/8)\omega^{-5}\dot{\omega}^2 + (1/4)\omega^{-4}\ddot{\omega}$, and $(W^{-1})^{(4)} = (315/28)\omega^{-9}\dot{\omega}^4 - (105/32)\omega^{-8}\dot{\omega}^2\ddot{\omega} + (15/32)\omega^{-7}\ddot{\omega}^2 + (5/8)\omega^{-7}\dot{\omega}\dddot{\omega} - (1/16)\omega^{-6}\ddddot{\omega}$.

Finally, the physical VEV of the squared magnetic field is defined by the mode-by-mode (namely under the integral sign) subtraction $\langle 0|\mathbf{B}(\mathbf{x})^2|0\rangle_{\text{phys}} = \lim_{m\to 0}\int_0^\infty dk k^{-1}\mathcal{P}_{\text{phys}}(k,m)$, where we have defined $\mathcal{P}_{\text{phys}}(k,m) = \mathcal{P}(k,m) - \mathcal{P}^{(A)}(k,m)$. Here, $\mathcal{P}(k,m) = \sum_\alpha [k^5/(2\pi^2 a^4)]|A_{k,\alpha,m}|^2$ is the exact magnetic power spectrum in the massive case, while $\mathcal{P}^{(A)}(k,m) = \sum_\alpha \sum_{i=0}^4 [k^5/(2\pi^2 a^4)](W^{-1})^{(i)}$ is the corresponding adiabatic expansion up to the fourth order. We find that only the fourth-order term determines the value of the renormalized magnetic correlator, giving $\langle 0|\mathbf{B}(\mathbf{x})^2|0\rangle_{\text{phys}} = 19H^4/(160\pi^2)$ [21]. This shows that vacuum magnetic fluctuations during de Sitter inflation are constant in time, and not adiabatically diluted by the cosmic expansion.

In order to study the correlation properties of these fluctuations, it is useful to consider the two-point magnetic correlator. It can be expressed in terms of the power spectrum as $\langle 0|\mathbf{B}(\mathbf{x})\mathbf{B}(\mathbf{y})|0\rangle = \int_0^\infty dk k^{-1}\mathcal{P}(k)j_0(k|\mathbf{x}-\mathbf{y}|)$, where $j_0(x)$ is the zeroth-order spherical Bessel function of the first kind. The physical two-point magnetic correlation function is, adopting again the adiabatic renormalization scheme, $\langle 0|\mathbf{B}(\mathbf{x})\mathbf{B}(\mathbf{y})|0\rangle_{\text{phys}} = \lim_{m\to 0}\int_0^\infty dk k^{-1}\mathcal{P}_{\text{phys}}(k,m)j_0(k|\mathbf{x}-\mathbf{y}|)$, giving

$$\langle 0|\mathbf{B}(\mathbf{x})\mathbf{B}(\mathbf{y})|0\rangle_{\text{phys}} = \frac{19H^4}{160\pi^2}. \tag{1}$$

This implies that $\mathcal{P}_{\text{phys}}(k,m)/k$, the double of the so-called magnetic energy density spectrum, is asymptotically proportional to a delta function, $\delta(k)$, in the limit $m \to 0$. Physically and in contrast to the case of unrenormalized fluctuations, this means that magnetic vacuum fluctuations do not depend on the comoving scale $\lambda = |\mathbf{x} - \mathbf{y}|$ [22,23]. Thereby, inflation "grows" quantum fluctuations equally on sub- and superhorizon scales [24].

Backreaction on inflation.—The above calculations have been carried out in a fixed de Sitter background, namely, assuming that backreaction of electromagnetic vacuum fluctuations on inflation is negligible. This is valid if the physical VEVs of the components of the electromagnetic energy-momentum tensor are much smaller than those associated to inflation, $(T_\nu^\mu)_{\text{inf}} = M^4\delta_\nu^\mu$, where δ_ν^μ

is the Kronecker delta. Here, we have introduced the energy scale of inflation, M, which is related to the energy density of inflation, ρ_{inf}, through $M^4 = \rho_{\text{inf}} = 3H^2/(8\pi G)$, where $G = 1/m_{\text{Pl}}^2$ is the Newton constant and m_{Pl} is the Planck mass.

The physical VEV of the electromagnetic energy-momentum tensor cannot be obtained as the massless limit of the total (transverse plus longitudinal) energy-momentum tensor of the Proca field. This is due to the fact that the longitudinal part of the energy-momentum tensor in the massive theory is not well behaved as $m \to 0$ [19]. In this case, to get the right result one needs to add a gauge-breaking term and a compensating complex ghost field to the standard Proca Lagrangian [19]. The final result is the usual one, $\langle 0|(T_\nu^\mu)_{\text{e.m.}}|0\rangle_{\text{phys}} = (31/480\pi^2)H^4\delta_\nu^\mu$ [9], and is strictly connected to the electromagnetic conformal anomaly. Consequently, backreaction on inflation is negligible if $(M/m_{\text{Pl}})^4 \ll 135/62$, which essentially means that the energy scale of inflation must be below the Planck scale $m_{\text{Pl}} \simeq 1.22 \times 10^{19}$ GeV.

The renormalized actual field.—To simplify the analysis we consider the case of instantaneous reheating; i.e., we assume that after inflation the Universe enters directly in the radiation dominated era. From the beginning of this era until the present time, quantum magnetic vacuum fluctuations are decohered and can be treated as classical stochastic fluctuations. In the presence of a plasma with conductivity σ, a classical magnetic field evolves according to the auto-induction equation [25] $\partial(a^2\mathbf{B})/\partial\eta = (1/\sigma)\nabla^2(a^2\mathbf{B})$. In the limit of (infinitely) high conductivity, we get $a^2\mathbf{B}(\mathbf{x}, \eta) = a_{\text{RH}}^2\mathbf{B}(\mathbf{x}, \eta_{\text{RH}})$, where RH indicates the time of reheating. Accordingly, we have $\langle\mathbf{B}(\mathbf{x}, \eta)\mathbf{B}(\mathbf{y}, \eta)\rangle = \langle\mathbf{B}(\mathbf{x}, \eta_{\text{RH}})\mathbf{B}(\mathbf{y}, \eta_{\text{RH}})\rangle(a_{\text{RH}}/a)^4$, where the classical ensemble average $\langle\mathbf{B}(\mathbf{x}, \eta_{\text{RH}})\mathbf{B}(\mathbf{y}, \eta_{\text{RH}})\rangle$ is indistinguishable from the quantum correlator $\langle 0|\mathbf{B}(\mathbf{x})\mathbf{B}(\mathbf{y})|0\rangle_{\text{phys}}$ on large (super-Hubble) scales, as explained above.

Since $a \propto g_{*S}^{-1/3}T^{-1}$ after reheating, where $g_{*S}(T)$ is the effective number of entropy degrees of freedom at the temperature T [26], the actual value of the magnetic field intensity is $B_0 = B_i(g_{*S,0}/g_{*S,\text{RH}})^{2/3}(T_0/T_{\text{RH}})^2\cos\theta_W$. Here, B_i is the root-mean-square value of the physical magnetic field at the end of inflation, $T_0 \simeq 2.37 \times 10^{-4}$ eV is the actual temperature, T_{RH} is the reheat temperature, $g_{*S,0} = g_{*S}(T_0) = 43/11$ [26], and $g_{*S,\text{RH}} = g_{*S}(T_{\text{RH}})$. Above the electroweak phase transition (when we assume inflation is taking place) the $U(1)$ gauge field which is quantum mechanically excited is indeed the hypercharge field, not the electromagnetic one. Below the electroweak phase transition, however, the hypercharge field is projected onto the electromagnetic field, and this gives the cosine of the Weinberg angle θ_W.

The reheat temperature can be related to the energy scale of inflation by observing that the energy density of radiation at the beginning of radiation era, $\rho_{\text{rad}} = (\pi^2/30)g_{*,\text{RH}}T_{\text{RH}}^4$, where $g_{*,\text{RH}}$ is the effective number of

degrees of freedom at the time of reheating and can be taken equal to $g_{*S,\text{RH}}$ [26], must be equal to the energy density at the end of inflation. We get $T_{\text{RH}} = [30/(\pi^2 g_{*,\text{RH}})]^{1/4}M$. Taking $g_{*S,\text{RH}} = 427/4$ [26], referring to the massless degrees of freedom of the standard model of particle physics, the actual, scale-independent magnetic field is $B_0 \simeq 3 \times 10^{-13}(M/10^{16} \text{ GeV})^2$ G. If the energy scale of inflation is around $M \simeq 10^{16}$ GeV, this field explains the cosmic magnetic fields.

Conclusions.—We have shown, in the framework of the standard free Maxwell theory, that the renormalized quantum VEV of the two-point magnetic correlation function does not evolve adiabatically but remains constant during de Sitter inflation. Quantum magnetic fluctuations are scale independent and their intensity depends on the scale of inflation. Super-Hubble quantum magnetic fluctuations decohere during inflation, and can be then treated as classical stochastic fluctuations in radiation and matter eras, when they are coupled to the cosmic plasma. The actual magnetic field is scale independent on large scales and, if the scale of inflation is of order of $M \sim 10^{16}$ GeV, it has the right intensity to explain the magnetization of galaxies and galaxy clusters.

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[13] The validity of the adiabatic renormalization procedure has been recently brought into question in [14,15], but then counterarguments have been given in [16]. Some of the results in [14,15] are, however, in contrast with basic principles of renormalization in curved spacetime. For example, the introduction of a infrared cutoff proposed in [14,15] to eliminate subhorizon modes for which adiabatic expansion is not mathematically accurate, gives the

physically unacceptable result of having a nonconserved renormalized stress tensor (see, e.g., [17]).

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[22] A delta-function spectrum of the form $\mathcal{P}(k) \propto k\delta(k)$, is the only type of spectrum that gives a true scale-independent magnetic field. A scale-invariant spectrum $\mathcal{P}(k) \propto constant$ usually used in the literature to define a scale-independent field is both infrared and ultraviolet divergent. Introducing an infrared and an ultraviolet cutoff, $k_{\min}$ and $k_{\max}$, respectively, one straightforwardly finds that the associated field is not constant but scales logarithmically as $\ln(1/\lambda)$ in the range $2\pi/k_{\max} \lesssim \lambda \lesssim 2\pi/k_{\min}$.

[23] The fact that the physical magnetic VEV is (scale) and time independent, should not come as a surprise. Indeed, the same is true for the renormalized electromagnetic energy density (see below). Moreover, this result is also confirmed in the ζ-function renormalization approach. To see this, we observe that $\langle 0|\mathbf{B}(\mathbf{x})^2|0\rangle \sim \int_0^\infty dk k^3/a^4 \sim H^4 \int_0^\infty dx x^3 \sim H^4 \sum_{n=1}^\infty n^3$, where we introduced the dimensionless variables $x = -k\eta$, and roughly approximated the integral by a sum. The divergent sum can now be cured by analytical continuation [18] by replacing it with $\zeta(-3) = 1/120$, where $\zeta(x)$ is the Riemann ζ function. This gives the renormalized and finite result $\langle 0|\mathbf{B}(\mathbf{x})^2|0\rangle_{\mathrm{phys}} \sim H^4$.

[24] We expect that the results obtained in de Sitter inflation hold, with minor modifications, also in more realistic models of inflation. This because while the Hubble parameter sets the scale of energy during inflation, giving $\langle 0|\mathbf{B}(\mathbf{x})^2|0\rangle_{\mathrm{phys}} \sim H^4$, the mass of the field sets the scale of correlation. For the photon case, then, the renormalized vacuum magnetic fluctuations are allowed to be correlated on arbitrarily large distances.

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