Coupling Spin Qubits via Superconductors

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We show how superconductors can be used to couple, initialize, and read out spatially separated spin qubits. When two single-electron quantum dots are tunnel coupled to the same superconductor, the singlet component of the two-electron state partially leaks into the superconductor via crossed Andreev reflection. This induces a gate-controlled singlet-triplet splitting which, with an appropriate superconductor geometry, remains large for dot separations within the superconducting coherence length. Furthermore, we show that when two double-dot singlet-triplet qubits are tunnel coupled to a superconductor with finite charging energy, crossed Andreev reflection enables a strong two-qubit coupling over distances much larger than the coherence length.

Introduction.—Spin qubits defined in quantum dots [1] have emerged as a promising candidate for quantum information processing, and a number of recent experiments [2–5] have demonstrated long coherence times and fast one- and two-qubit gates. If a qubit is encoded in the spin of one electron, single-qubit rotations can be implemented with time-dependent magnetic fields [4], or with electric fields utilizing the spin-orbit coupling [5], and two-qubit gates by gated control of the exchange interaction between different dots [1]. An alternative is to encode a single qubit in a two-dimensional subspace of the spin state of two electrons in a double dot [3,6] (for example, the singlet and T_0 triplet). Then, the exchange interaction together with a static difference in the Zeeman splitting on the individual dots allow for arbitrary single-qubit rotations [7]. Readout is provided by spin-to-charge conversion, where one spin-qubit state is prohibited from tunneling out of the dot, either by a Zeeman splitting [8] (in a single-spin qubit) or by Pauli spin blockade [3,9] [in a singlet-triplet (ST) qubit].

However, exchange-based couplings are limited to qubits in direct proximity with each other. Any large scale spin-based quantum computer architecture will have to rely on an alternative coupling mechanism of spatially separated spin qubits. Recently, much progress has been made in coupling superconducting qubits via a microwave field [10–13] (circuit quantum electrodynamics). The same principles have been considered for spin qubits [14–19], but unfortunately the coupling between the electron spin and the cavity field is rather weak.

The Coulomb interaction provides a possible coupling of spin qubits which is reasonably long ranged. This requires coupling first the spin and charge degrees of freedom, e.g., via spin-orbit coupling [20,21]. Alternatively, a double-dot ST qubit can be operated in a regime of asymmetric level positions, where the singlet wave function has some weight with both electrons on the same dot [i.e., close to the $(1, 1) \rightarrow (0, 2)$ transition, where (m, n) refers to a state with m (n) electrons on dot 1 (2)]. The different charge configurations of the singlet and triplet states then enable a direct capacitive coupling of two ST qubits [7,22–24]. The range of the capacitive coupling might be increased by placing floating metallic gates between the quantum dots [25,26]. Recent proposals also suggest using ferromagnets [27] or Majorana bound states [28] for two-qubit couplings. However, achieving strong two-spin-qubit couplings remains challenging.

In this work, we introduce a proposal which uses a superconductor to couple spin qubits defined in spatially separated quantum dots. We show that crossed Andreev reflection (CAR) [29–32] allows for a large and long-ranged coupling. In the case of a one-dimensional superconductor, which can be realized in a semiconducting nanowire proximity coupled to a bulk superconductor, we show that the coupling remains large for quantum dot separations up to the superconducting coherence length. In addition, similar to Pauli spin blockade, CAR allows for qubit initialization and readout. Using a Cooper pair box (CPB), i.e., a floating superconductor with a finite charging energy, opens up even more possibilities. In a double-dot ST qubit, CAR introduces a coupling between the spin-qubit state and the charge on the CPB. This can be exploited for strong (hundreds of MHz) capacitive two-qubit couplings of ST qubits separated by many μm (the only separation restriction being that the CPB to which both qubits couple should have a sizable charging energy).

Two-dot coupling via a superconductor.—We start by considering two quantum dots tunnel coupled to different points of a superconductor, sketched in Fig. 1, similar to a so-called Cooper pair splitter [32–36]. In the sketch, the quantum dots are defined in a semiconducting nanowire (e.g., InSb or InAs), which is an experimentally promising system for spin qubits [37,38] and can form good interfaces with a superconductor [39,40]. However, the results are not restricted to this particular qubit realization. The system is described by the Hamiltonian $H = \sum_{i=1,2}(H_{Di} + H_{Ti}) + H_S$.
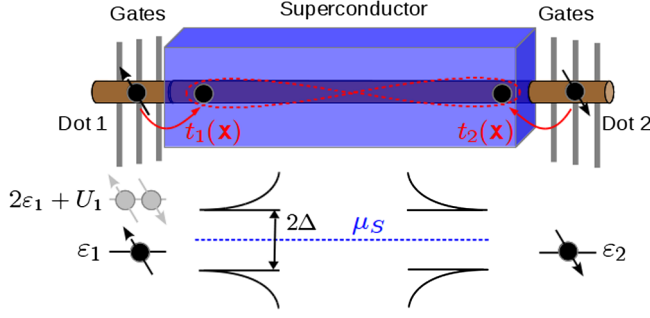
FIG. 1 (color online). Coupling quantum dot spins via a superconductor. Upper panel: Two quantum dots defined within a semiconducting nanowire, tunnel coupled to a superconductor (the main coupling is to the wire underneath the superconductor, which is a one-dimensional superconductor because of the proximity effect). The gray gates control the dot energy levels and the tunnel amplitudes. A CAR process is illustrated, where one electron from each dot tunnels into the superconductor and combines into a Cooper pair. Lower panel: Energy-level sketch of the system.

Here, $H_{Di} = \sum_\sigma n_{i\sigma}\varepsilon_{i\sigma} + U_i n_{i\uparrow}n_{i\downarrow}$ describes quantum dot i, which has a single level with energy $\varepsilon_{i\sigma} = \varepsilon_i \pm B_i$ for spin projection $\sigma = \uparrow, \downarrow$, occupation $n_{i\sigma} = d^\dagger_{i\sigma}d_{i\sigma}$, and Coulomb charging energy U_i. The BCS-type superconductor is described by $H_S = \sum_{\nu\sigma}E_{\nu\sigma}\gamma^\dagger_{\nu\sigma}\gamma_{\nu\sigma}$, where the Bogoliubov quasiparticle operators $\gamma_{\nu\sigma}$ are related to the electron operators $c_{\nu\sigma}$ and $c^\dagger_{\nu\sigma}$ in the standard way and $E_{\nu\sigma} = \sqrt{\Delta^2 + (\varepsilon_{\nu\sigma} - \mu_S)^2}$, with μ_S being the chemical potential and Δ the superconducting gap. The coupling between dot i and the superconductor is described by $H_{Ti} = \sum_{\nu\sigma}\int d\mathbf{x}\, t_{i\nu\sigma}(\mathbf{x})c^\dagger_{\nu\sigma}d_{i\sigma} + $ H.c., where $\mathbf{x}$ is a coordinate in the superconductor. For simplicity, spin-orbit interaction is not included in the model, but its qualitative effects will be discussed below.

We now want to calculate the superconductor-mediated coupling between electron spins on different dots, cf. Ref. [41], which discussed such coupling in a double-dot Josephson junction. Focusing on the (1,1) regime (each dot is occupied by a single electron), we should then calculate the energy shifts δE_α induced by the coupling to the superconductor, where $\alpha = S, T_0, T_\pm$ are the two-electron singlet and triplet states. We neglect all shifts which are the same for all α, leaving fourth order terms in the tunnel Hamiltonian $H_T = \sum_i H_{Ti}$ as the leading order contribution. We then find

$$\delta E_\alpha = \sum_n \frac{1}{E_\alpha - E_n}\left|\langle GS|\langle n|H_T \frac{1}{E_\alpha - H_0}H_T|\alpha\rangle|GS\rangle\right|^2, \quad (1)$$

where E_α and E_n are the unshifted energies of states α and n, $H_0 = \sum_i H_{Di} + H_S$, and $|GS\rangle$ is the ground state of the superconductor (it can be verified that only intermediate

states where the superconductor is in its ground state give an α-dependent contribution).

There are two distinct contributions to δE_α, which both lower the energy of the singlet relative to the triplet states: (i) exchange, where an electron tunnels from one dot to the other via the superconductor and then back again, and (ii) CAR, where one electron from each dot tunnels into the superconductor to briefly combine into a Cooper pair before tunneling back again. These processes differ in the dot charge configuration in the intermediate state, where for exchange processes one dot is empty and the other doubly occupied ($|n\rangle = |02\rangle$ or $|n\rangle = |20\rangle$), while for CAR both dots are empty ($|n\rangle = |00\rangle$). For simplicity, we consider first zero magnetic field $B_1 = B_2 = 0$. For the CAR contribution, Eq. (1) gives $\delta E_T^{CAR} = 0$ for any of the triplet states $T = T_0, T_\pm$, while $\delta E_S^{CAR} = |\gamma|^2/\varepsilon_\Sigma$, where $\varepsilon_\Sigma = \varepsilon_1 + \varepsilon_2 - \mu_S$. The CAR amplitude is given by

$$\gamma = \sum_{\nu i}\frac{\Delta}{\sqrt{2}E_\nu}\frac{1}{E_\nu - \varepsilon_i}\int d\mathbf{x}_1 d\mathbf{x}_2\, t_{1\nu\uparrow}(\mathbf{x}_1)t_{2\nu\downarrow}(\mathbf{x}_2). \quad (2)$$

Assuming a finite tunnel coupling only between dot i and a single point $\mathbf{x}_{i,0}$ in the superconductor, we can write $t_{i\nu\sigma} = t_i\delta(\mathbf{x}_i - \mathbf{x}_{i,0})\psi_{\nu\sigma}(\mathbf{x}_{i,0})$, where $\psi_{\nu\sigma}(\mathbf{x})$ is a real space wave function in the superconductor. With the approximation $E_\nu - \varepsilon_i \approx E_\nu$, an analytic solution can easily be found (we come back to the full ε-dependent result below). For a three-dimensional ballistic superconductor [32,42], one finds $\gamma \propto (k_F\delta x)^{-1}\exp(-\delta x/\pi\xi_0)$, where $\delta x = |\mathbf{x}_{1,0} - \mathbf{x}_{2,0}|$, ξ_0 is the superconducting coherence length, and k_F is the Fermi wave vector of the superconductor. Although ξ_0 can be hundreds of nm, the algebraic prefactor limits the range of the interaction, since in typical metals $k_F^{-1} \sim$ Å. The situation might be slightly better for a diffusive superconductor, where this prefactor is replaced by [43] $(k_F\delta x)^{-1/2}(k_F l)^{-1/2}$, where l is the mean-free path. However, the exponential suppression is more severe in this case, with ξ_0 being replaced by $\xi \approx \sqrt{\xi_0 l}$.

To achieve a long-ranged coupling, one should reduce the dimensionality of the superconductor. This makes the algebraic prefactor less severe, since it arises from summing over different paths between $\mathbf{x}_{1,0}$ and $\mathbf{x}_{2,0}$ with varying phases. For a single-channel ballistic one-dimensional superconductor, we find [keeping ε_i in Eq. (2)]

$$\gamma = \sqrt{2}t_1 t_2 \rho \sum_i \frac{\Delta}{\sqrt{\Delta^2 - \varepsilon_i^2}}\left[\pi + 2\tan^{-1}\left(\frac{\varepsilon_i}{\sqrt{\Delta^2 - \varepsilon_i^2}}\right)\right]$$
$$\times \sin(k_F\delta x)e^{-\delta x/\pi\xi_0}, \quad (3)$$

where ρ is the normal-state density of states at the Fermi energy, and we have assumed $\hbar^2 k_F^2/2m \gg \Delta$ and linearized the spectrum around k_F. There is no algebraic suppression, and γ remains large as long as $\delta x \lesssim \xi_0$. In a setup as in Fig. 1, a one-dimensional superconductor
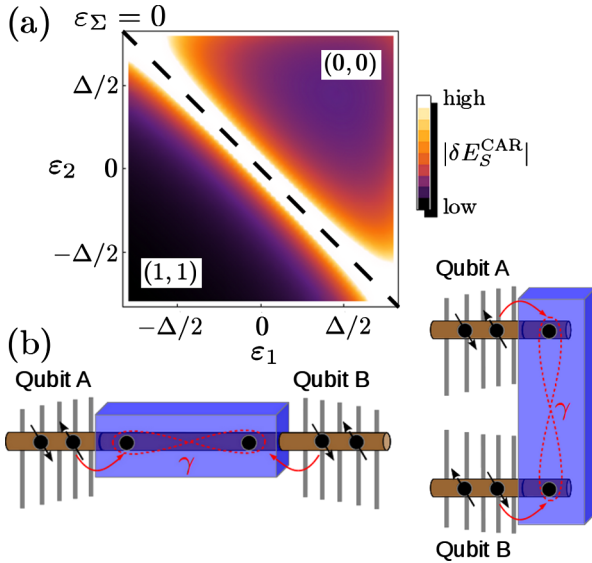
FIG. 2 (color online). (a) $|\delta E_S^{\text{CAR}}|$ as a function of ε_1 and ε_2, with γ from Eq. (3), $\mu_S = 0$, and $B_1 = B_2 = 0$. At the CAR resonance (dashed line at $\varepsilon_\Sigma = 0$), the perturbatively calculated shift diverges. We focus on the regime $\varepsilon_\Sigma < 0$, where the (1,1) charge configuration has lowest energy. (b) Two different systems with CAR-induced coupling between two ST qubits, defined either in the same wire or in different wires (the CAR amplitude is reduced in the latter case).

results from proximity-inducing superconductivity in a semiconducting nanowire [39,40]. In fact, the experiments in Ref. [35] revealed an unexpectedly large CAR amplitude, likely due to the reduced dimensionality of the proximity-induced superconducting wire connecting the dots. Figure 2(a) shows the CAR-induced singlet-triplet splitting $|\delta E_S^{\text{CAR}}|$ as a function of ε_1 and ε_2, with γ given by Eq. (3). The shift is of course largest close to the CAR resonance at $\varepsilon_\Sigma = 0$, but we note that at a given ε_Σ, it can be increased by asymmetrically gating the dots, i.e., by operating at $\varepsilon_1 > \varepsilon_2$ or $\varepsilon_2 > \varepsilon_1$. An analogous calculation of the exchange contribution gives $\delta E_T^{\text{ex}} = 0$ and $\delta E_S^{\text{ex}} = \sum_i |\eta_i|^2/(\varepsilon_\Sigma - E_{02,i})$, where $E_{02,i} = 2\varepsilon_i + U$ and the exchange amplitude η_i is given by an expression similar to Eq. (2). U is typically large, several meV in nanowire dots, and we neglect the exchange contribution in the following (it merely gives rise to an additional singlet-triplet splitting, which is small and lacks significant gate dependence around our operating point with large CAR).

Finite magnetic field and/or spin-orbit interaction does not qualitatively change the above results. The situation is analogous to coupling between (1,1) states and the (0,2) singlet in double dots without coupling to a superconductor [44]. At zero magnetic field, only one of the four (1,1) states can undergo CAR. At finite magnetic field, two (1,1) states split off in energy similar to $T_\pm$ but in the presence of spin-orbit interaction acquire a finite CAR amplitude, except for at specific field angles [44,45]. CAR splits the remaining two degenerate states into one which can

undergo CAR and one which cannot, and we can still refer to the former as S and the latter as T_0.

In a setup as in Fig. 1, CAR can be used for controlled spin manipulation by for a certain time either moving the dot energy levels close to $\varepsilon_\Sigma = 0$ or always staying close to resonance but controlling the tunnel couplings $t_{1,2}$. However, to minimize decoherence due to charge noise, one should avoid moving too close to $\varepsilon_\Sigma = 0$, where the singlet state acquires a significant (0,0) component. Initialization can be done by moving both initially empty dot levels down through the CAR resonance adiabatically (with respect to γ), which fills the double dot with two electrons in a singlet state (Cooper pair splitting). If each dot is tunnel coupled to a normal electrode biased to drive an electron current into the superconductor, the triplet states act as blocking states since they do not allow for CAR [46]. Similar to Pauli spin blockade, this can be used to prepare triplet states and for readout through spin-to-charge conversion. Readout can also be done through charge detection by adiabatically moving both dot levels up through the CAR resonance, thereby selectively emptying the singlet state.

The geometry discussed here can be used to couple two single-spin qubits, in which case the shift δE_S allows for two-qubit gates, or to define a nonlocal ST qubit, in which case δE_S together with a dot-dependent Zeeman splitting allows for universal single-qubit rotations. Alternatively, we can consider one of the geometries suggested in Fig. 2(b) with two different ST qubits A and B, where each double-dot ST qubit is internally coupled by normal exchange, while there is a long-distance coupling between one dot in each qubit via the superconductor. This situation is analogous to two exchange-coupled ST qubits [47] but with a more long-ranged interaction.

Cooper pair box and two-qubit couplings.—A single ST qubit coupled to a superconductor can be described by the effective Hamiltonian

$$H_{\text{ST}} = \varepsilon_\Sigma |S\rangle\langle S| + \varepsilon_\Sigma |T_0\rangle\langle T_0| + \gamma |S\rangle\langle 00| + \text{H.c.,} \quad (4)$$

where we have integrated out the quasiparticle degrees of freedom of the superconductor, leaving an effective CAR amplitude $\gamma(\varepsilon_1, \varepsilon_2)$, given by Eq. (3) for a one-dimensional superconductor.

We now consider coupling instead to a small floating superconductor (CPB), with a well-defined number of Cooper pairs N and a finite charging energy E_C. The Hamiltonian (4) remains valid in this case, but we need to also keep track of the superconducting charge state $|N\rangle$, and because charge is conserved in the total system, $|S/T_0\rangle$ should be interpreted as $|S/T_0\rangle|N_0\rangle$, while $|00\rangle$ corresponds to $|00\rangle|N_0 + 1\rangle$. ε_Σ is now measured relative to the energy of the $|N_0\rangle \rightarrow |N_0 + 1\rangle$ transition. Next, we consider two different ST qubits coupled to the same CPB (see Fig. 3), where the dots of the different qubits are separated by more than ξ_0, such that there is no CAR
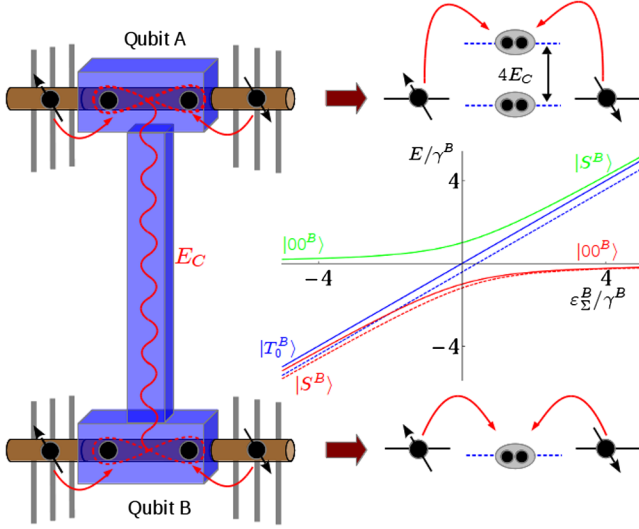
FIG. 3 (color online). Proposed setup for two-qubit gates between two ST qubits. The energy-level sketch to the right shows the principle of the two-qubit interaction: If the energy of qubit B is adjusted so that the singlet component of the wave function (partially) leaks into the CPB, through E_C this affects the energy at which the singlet component of qubit A can do the same. Inset: Eigenspectrum of qubit B as a function of ε_Σ^B with qubit A operated at $\delta^A = -|\gamma^A|/\varepsilon_\Sigma^A = 1/2$ and $E_C/10 = \gamma^A = \gamma^B$, and neglecting the gate dependence of γ^B. The solid (dashed) lines represent the result when qubit A is in state $|T_0^A\rangle$ ($|\tilde{S}^B\rangle$): The difference in singlet-triplet splitting of qubit B depending on the state of qubit A allows two-qubit operations.

(or exchange) between them. However, because of the conservation of total charge, a state where both double dots are empty must correspond to two Cooper pairs being added to the CPB. This gives rise to a capacitive coupling term

$$V^{AB} = 4E_C|00^A\rangle|00^B\rangle\langle00^A|\langle00^B|, \qquad (5)$$

where superscripts refer to the two different ST qubits (we will leave out the superscripts when referring to either one of the two qubits). Equation (5) contains a projector unto the state where both double dots are empty, which is not a part of the normal computational space for an ST qubit. However, a qubit initially in $|S\rangle$ can be brought into $|00\rangle$ by adiabatically (with respect to γ) increasing ε_Σ (moving along a red line in the spectrum in Fig. 3). If both qubits A and B are brought across the $|S\rangle \rightarrow |00\rangle$ avoided crossing for a finite time and then back again, Eq. (5) gives rise to a $\sigma_z^A \otimes \sigma_z^B$-type coupling. The coupling strength is in principle only limited by the charging energy of the CPB, but operating above the $|S\rangle \rightarrow |00\rangle$ anticrossing means that the spin superposition states have been fully converted into superpositions of charge degrees of freedom, resulting in short coherence times. It is likely better to stay below the anticrossing, where we can define effective computational states $|T_0\rangle$ and $|\tilde{S}\rangle \approx (1/\sqrt{1 + \delta^2})|S\rangle + \delta|00\rangle$, where

$\delta = -|\gamma|/\varepsilon_\Sigma \ll 1$ (note that $\varepsilon_\Sigma < 0$). Because of this small gate-dependent fraction of empty states which is added to the singlets, the two-qubit interaction to leading order in δ and E_C/ε_Σ is given by

$$V^{AB} \approx 4E_C(\delta^A)^2(\delta^B)^2|\tilde{S}^A\rangle|\tilde{S}^B\rangle\langle\tilde{S}^A|\langle\tilde{S}^B| \qquad (6)$$

$$\sim 4E_C(\delta^A)^2(\delta^B)^2\sigma_z^A \otimes \sigma_z^B, \qquad (7)$$

where the second line holds up to single-qubit rotations. With $\gamma^A = \gamma^B = 50$ μeV [48], and operating at $\delta^A = \delta^B = 1/3$, we find $V^{AB} \sim 0.5$ μeV ~ 100 MHz if $4E_C = 50$ μeV. This tiny value for E_C would allow for a very long CPB. Increasing E_C increases V^{AB}, but Eq. (6) only holds for $|E_C/\varepsilon_\Sigma| \ll 1$ and V^{AB} saturates for larger E_C [with the same γ and δ as above, $V^{AB} \approx 300$ MHz for $4E_C = 500$ μeV is found from Eqs. (4) and (5)]. To reach large values while keeping δ small requires a large γ, which can be achieved in a few-channel nanowire with a high-quality interface with the superconductor, as discussed above. Note that it is not necessary to have E_C larger than temperature, as long as the CPB is well isolated from outside sources of quasiparticles, such that N is conserved on the time scale of the two-qubit operations.

Conclusions.—We have demonstrated that a superconductor can be used to mediate a long-distance coupling between spin qubits through CAR, which in a one-dimensional superconductor remains large for distances up to the superconducting coherence length. Thus, a superconductor can be used to couple spatially separated spin qubits or to define a nonlocal double-dot ST qubit. CAR-based initialization, manipulation, and readout would also provide definite evidence of the entanglement of electron spins in a Cooper pair splitter. Furthermore, coupling two different ST qubits to the same CPB introduces a coupling between the spin qubits and the charge on the CPB, which can be used for truly long-distance (many μm) two-spin-qubit gates with potential for fast operation (hundreds of MHz).

Finally, we mention that this charge-spin coupling has the potential to allow coupling of even more distant ST qubits through circuit quantum electrodynamics [10,12,13]. The two ST qubits in Fig. 3 could be coupled to two different CPBs, which are then capacitively coupled to each other via a superconducting transmission line. This would allow the very large dipole moment of superconducting islands to be exploited to couple spins.

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