Electromagnetic Duality Symmetry and Helicity Conservation for the Macroscopic Maxwell's Equations

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In this Letter, we show that the electromagnetic duality symmetry, broken in the microscopic Maxwell's equations by the presence of charges, can be restored for the macroscopic Maxwell's equations. The restoration of this symmetry is shown to be independent of the geometry of the problem. These results provide a tool for the study of light-matter interactions within the framework of symmetries and conservation laws. We illustrate its use by determining the helicity content of the natural modes of structures possessing spatial inversion symmetries and by elucidating the root causes for some surprising effects in the scattering off magnetic spheres.

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Symmetries, both continuous and discrete, are a powerful tool for studying nature. According to Noether's theorem [1], any continuous symmetry of a nondissipative system gives rise to a conserved quantity in the dynamic equations. In modern algebraic terms we say that when a system is invariant under the continuous transformation generated by a given operator, the observable represented by that operator is a conserved quantity. For example, rotational and translational invariance are associated with the conservation of angular momentum and linear momentum because, as transformations, rotations are generated by the components of angular momentum and translations are generated by the components of linear momentum.

In this Letter, we will study a nongeometrical symmetry in electromagnetism: electromagnetic duality. Electromagnetic duality is a transformation where the roles of electric and magnetic fields are mixed:

$$\begin{aligned} \mathbf{E} \to \mathbf{E}_\theta &= \mathbf{E}\cos\theta - \mathbf{H}\sin\theta, \\ \mathbf{H} \to \mathbf{H}_\theta &= \mathbf{E}\sin\theta + \mathbf{H}\cos\theta. \end{aligned} \tag{1}$$

The typical exchange, $\mathbf{E} \to \mathbf{H}$ and $\mathbf{H} \to -\mathbf{E}$, corresponds to setting $\theta = -(\pi/2)$. In the absence of charges and currents, Eq. (1) is a symmetry of Maxwell's equations: If the electromagnetic field ($\mathbf{E}$, $\mathbf{H}$) is a solution of the free space Maxwell equations, then the field ($\mathbf{E}_\theta$, $\mathbf{H}_\theta$) is also a solution. In 1965, Calkin [2] showed that helicity was the conserved quantity related to such symmetry.

Helicity is defined ([3], see Chap. 8.4.1) as the projection of the total angular momentum $\mathbf{J}$ onto the linear momentum direction $\mathbf{P}/|\mathbf{P}|$, i.e., $\Lambda = \mathbf{J}\cdot\mathbf{P}/|\mathbf{P}|$. In the case of photons ([4], see Chap. 2.5), helicity takes the values ± 1. It is possible to intuitively understand the meaning of helicity when considering the wave function of the particle in the momentum representation, that is, as a superposition of plane waves. In this representation, helicity is related to the handedness of the polarization of each and every plane wave. The helicity of the particle is well defined only when all the plane waves have the same handedness with respect to their momentum vector, including both propagating and evanescent plane waves ([5], see Appendices A and B). Calkin showed that, as an operator, helicity generates duality transformations. Since that seminal work, the role of helicity as the generator of duality symmetry transformations for the free space Maxwell's equations has been reported several times [6–8].

In 1968, Zwanziger [9] studied this free space invariance and conservation law in a quantum field theory with both electric and magnetic sources. He found that when a simultaneous transformation akin to Eq. (1) is allowed among the two kinds of sources, duality symmetry is maintained. Without this extra source transformation, Eq. (1) ceases to be a symmetry of the microscopic Maxwell's equations when sources are present. This is the current status of the duality symmetry in material systems. In this Letter, we show that the electromagnetic duality symmetry, broken for the microscopic Maxwell's equations by the presence of sources, can be restored for the macroscopic Maxwell's equations for material systems without free sources, characterized by scalar electric permittivities and magnetic permeabilities. The restoration condition for a system composed of different isotropic and homogeneous domains depends only on the materials and is independent of the shapes of the domains. When the system is dual, the helicity of the light interacting with it is preserved. After deriving the restoration condition, we provide two examples of its application. From now on, we will use a harmonic decomposition of the fields and assume a $\exp(-i\omega t)$ dependency with the angular frequency ω. Additionally, we will work in the representation of space dependent vectorial fields, also known as the real representation.

The expression of the helicity operator for monochromatic fields in the real representation can be obtained directly from the definition of helicity, using

$$\mathbf{J}\cdot\mathbf{P} = (\mathbf{S}+\mathbf{L})\cdot\mathbf{P} = \mathbf{S}\cdot\mathbf{P} = \nabla\times, \qquad (2)$$

where $\mathbf{S}$ and $\mathbf{L}$ are, respectively, the spin and orbital angular momentum operators. The third equality follows from the orthogonality of $\mathbf{L} = \mathbf{r}\times\mathbf{P}$ and $\mathbf{P}$. The last equality is given in [10] [see expression (XIII.93)]. Then, since for monochromatic fields $|\mathbf{P}|$ is equal to the wave number k: $\Lambda = k^{-1}\nabla\times$.

We start the derivation by setting convenient units of $\epsilon_0 = \mu_0 = 1$ for the vacuum electric and magnetic constants (thus $c_0 = 1$ and $k = \omega$). This convention will be used throughout the Letter. We can then use Eq. (2) to write the free space Maxwell equations as

$$\begin{aligned}\nabla\times\mathbf{E} &= i\omega\mathbf{H} \Rightarrow \mathbf{H} = -i\Lambda\mathbf{E},\\ \nabla\times\mathbf{H} &= -i\omega\mathbf{E} \Rightarrow \mathbf{E} = i\Lambda\mathbf{H}.\end{aligned} \qquad (3)$$

Equation (3) already reveals that Λ is an operator that transforms electric fields into magnetic fields and vice versa. In the same way that angular momentum generates rotation matrices [11], let us use Λ as the generator of a continuous transformation parametrized by the real number θ: $D_\theta = \exp(i\theta\Lambda)$. The explicit expression for the transformation that $D(\theta)$ performs on the fields is easily derived by noting that Λ^2 is the identity operator for Maxwell fields. This is a consequence of Eq. (3), valid for all $\mathbf{E}$ and $\mathbf{H}$. Using that $\Lambda^2 = I$, the 3×3 identity matrix, and the Taylor expansion of the exponential, the continuous transformation generated by helicity can be written as follows:

$$D_\theta = \exp(i\theta\Lambda) = \cos\theta I + i\sin\theta\Lambda. \qquad (4)$$

The application of D_θ to electromagnetic fields reads

$$\begin{aligned}\mathbf{E}_\theta &= (\cos\theta I + i\sin\theta\Lambda)\mathbf{E},\\ \mathbf{H}_\theta &= (\cos\theta I + i\sin\theta\Lambda)\mathbf{H},\end{aligned} \qquad (5)$$

which, after using Eq. (3) again, becomes the duality transformation of electromagnetic fields written in Eq. (1).

We will now show that duality symmetry can be restored in the macroscopic Maxwell's equations independently of the shapes of the material domains involved.

We consider an inhomogeneous medium Ω composed of several material domains with arbitrary geometry. We assume that each domain i is homogeneous and isotropic, and fully characterized by its electric ϵ_i and magnetic μ_i constants (we again use $\epsilon_0 = \mu_0 = 1$). In each domain, the constitutive relations are hence $\mathbf{B} = \mu_i\mathbf{H}$, $\mathbf{D} = \epsilon_i\mathbf{E}$, and the curl equations for monochromatic fields read

$$\nabla\times\mathbf{H} = -i\omega\epsilon_i\mathbf{E}, \qquad \nabla\times\mathbf{E} = i\omega\mu_i\mathbf{H}. \qquad (6)$$

Using $\Lambda = k^{-1}\nabla\times$ and $\omega = k_0 = k/\sqrt{\epsilon_i\mu_i}$, we obtain

$$\Lambda\mathbf{H} = -i\sqrt{\frac{\epsilon_i}{\mu_i}}\mathbf{E}, \qquad \Lambda\mathbf{E} = i\sqrt{\frac{\mu_i}{\epsilon_i}}\mathbf{H}. \qquad (7)$$

Note that to arrive at this result, the fact that the wave number in each medium is $k = k_0\sqrt{\epsilon_i\mu_i}$ has to be used in the expression of the helicity operator. Now, we can normalize the electric field $\mathbf{E} \to \sqrt{\epsilon_i/\mu_i}\mathbf{E}$, to show that inside each of the domains, we can recover the exact form of Maxwell's equations in free space, Eq. (3). Clearly, if we want to have a consistent description for the whole medium Ω, the normalization can only be done when all the different materials have the same ratio $\epsilon_i/\mu_i = \alpha\ \forall\ i$. In this case, the electromagnetic field equations on the whole medium Ω are invariant under the duality transformations of Eq. (1).

We need to study the matching of the fields at the interfaces between the different domains, where the material constants are discontinuous. In the absence of free currents and charges, the electromagnetic boundary conditions impose the following restrictions on the fields: $\hat{\mathbf{n}}\times(\mathbf{E}_1-\mathbf{E}_2) = \mathbf{0}$, $\hat{\mathbf{n}}\times(\mathbf{H}_1-\mathbf{H}_2) = \mathbf{0}$, $\hat{\mathbf{n}}\cdot(\mathbf{D}_1-\mathbf{D}_2) = 0$, and $\hat{\mathbf{n}}\cdot(\mathbf{B}_1-\mathbf{B}_2) = 0$, where $\hat{\mathbf{n}}$ is the unit vector perpendicular to the interface. The boundary conditions can be seen as a real space, point to point transformation of the fields. For example, at a particular point $\mathbf{r}$ on the interface between domains 1 and 2, the boundary conditions may be interpreted as the following linear transformation:

$$\begin{bmatrix}\mathbf{E}_2(\mathbf{r})\\ \mathbf{H}_2(\mathbf{r})\end{bmatrix} = \mathrm{diag}\left(1, 1, \frac{\epsilon_1}{\epsilon_2}, 1, 1, \frac{\mu_1}{\mu_2}\right)\begin{bmatrix}\mathbf{E}_1(\mathbf{r})\\ \mathbf{H}_1(\mathbf{r})\end{bmatrix}, \qquad (8)$$

where we have oriented our Cartesian reference axis so that $\hat{\mathbf{z}} = \hat{\mathbf{n}}$.

On the other hand, the duality transformation, Eq. (1), may also be written in matrix form, rewriting Eq. (5):

$$\begin{bmatrix}\mathbf{E}_\theta\\ \mathbf{H}_\theta\end{bmatrix} = \begin{bmatrix}I\cos\theta & -I\sin\theta\\ I\sin\theta & I\cos\theta\end{bmatrix}\begin{bmatrix}\mathbf{E}\\ \mathbf{H}\end{bmatrix} = U_\theta\begin{bmatrix}\mathbf{E}\\ \mathbf{H}\end{bmatrix}.$$

It is a trivial exercise to check that the transformation matrix of Eq. (8) commutes with U_θ if and only if $\epsilon_1/\mu_1 = \epsilon_2/\mu_2$. In such a case, the fields in each of the two media can be transformed as in Eq. (1) while still meeting the boundary conditions at point $\mathbf{r}$. We can now vary $\mathbf{r}$ to cover all the points of the interface and repeat the same argument: The fact that U_θ does not depend on the spatial coordinates allows us to reorient the reference axis as needed to follow the shape of the interface between two media ($\hat{\mathbf{n}} = \hat{\mathbf{z}}$). The derivation is hence independent of the shape of the interface, and we can say that the boundary conditions are invariant under duality transformations when $\epsilon_1/\mu_1 = \epsilon_2/\mu_2$. The above derivations show that both the equations and the boundary conditions in Ω are invariant under (1) when

$$\epsilon_i/\mu_i = \text{constant} \quad \forall \text{ domain } i. \qquad (9)$$

As a conclusion, we can state that independent of the shapes of each domain, a piecewise homogeneous and isotropic system has an electromagnetic response that is invariant under duality transformations if and only if all the materials have the same ratio of electric and magnetic constants. In this case, since helicity is the generator of duality transformations, the system preserves the helicity of the electromagnetic field interacting with it.

Equation (9) can be verified in two of the few analytically solvable electromagnetic scattering problems: a planar multilayer system and a sphere. When imposing condition (9), the Fresnel coefficients are identical for the two (TE and TM) polarizations for any plane wave impinging on the multilayer. This implies preservation of the circular polarization handedness of any plane wave, i.e., helicity preservation. The same is true for the Mie coefficients representing the scattering of magnetic and electric multipoles off a sphere: They are identical when Eq. (9) is met, implying preservation of the multipoles of well-defined helicity which are linear combinations of the electric and magnetic ones [[3], see expression (11.4-19)]. These derivations are included in the Supplemental Material [12].

In order to illustrate the independence of helicity conservation from geometry in more complex systems we performed numerical simulations. We analyzed the helicity change (Fig. 1) for two different dielectric structures in free space: A circular cylinder, which is symmetric under rotations along its axis, and a curved pan-flute-like structure without any rotational, translational, or spatial inversion symmetry. Two versions of each structure were simulated, corresponding to two different materials: The first one models the properties of silica by setting $\epsilon = \epsilon_{\text{glass}} = 2.25$ and $\mu = \mu_{\text{glass}} = 1$. In the second material we enforce duality, Eq. (9), by setting $\epsilon = \mu = \epsilon_{\text{glass}} = 2.25$. The incident field is a circularly polarized plane wave (i.e., it has well-defined helicity) with a momentum vector parallel to the red arrows in the figure. In the case of the cylinder, the momentum direction is aligned with the axis of the cylinder. Figure 1 shows that helicity is conserved independently of the spatial symmetries, whenever Eq. (9) is fulfilled, i.e., under conditions of duality symmetry.

The result in Eq. (9) is in agreement with Bialynicki-Birula's wave equation for photons propagating in a linear, time-independent, isotropic, and inhomogeneous medium. In [13] (see Sec. 2), he shows that the two helicities of the photon are only coupled through the gradient of $\sqrt{\mu(\mathbf{r})/\epsilon(\mathbf{r})}$. Relation (9) is often referred to as the surface impedance matching condition. To the best of our knowledge, its simultaneous connection with helicity preservation and duality symmetry has not been considered before. This connection allows the relationship between helicity and duality to be used as a tool for the study of light-matter interactions. The fact that the restoration condition in Eq. (9) is independent of geometry makes the use of this tool very simple as we will now show.

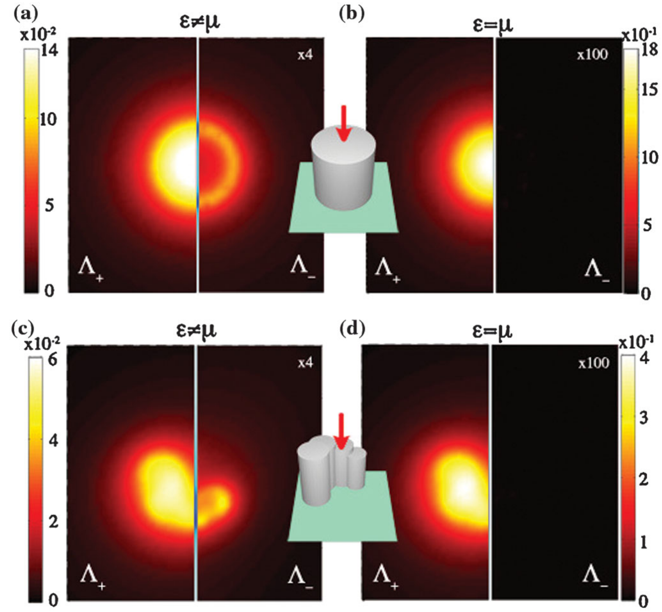

FIG. 1 (color). Impact of the different symmetries on the field scattered by two dielectric structures. The upper row shows the scattered intensity for a symmetric cylinder and the lower row for a pan-flute-like shape without any rotational, translational, or spatial inversion symmetry. The length and diameter of the cylinder are 200 nm. The pan flute is made of cylinders of different lengths and diameters; the longest one is 200 nm long and the total pan flute's width is around 200 nm. In panels (a) and (c), the structures have $\epsilon = 2.25$, $\mu = 1$, while in panels (b) and (d) we enforced duality symmetry by setting $\epsilon = \mu = \epsilon_{\text{glass}} = 2.25$. The incident field is a plane wave of well-defined helicity equal to 1, a momentum vector pointing to the positive z axis, and a wavelength of 633 nm. Its electric field is $(\hat{\mathbf{x}} + i\hat{\mathbf{y}})/\sqrt{2}\exp(kz - \omega t)$. The left-half side of each subfigure corresponds to the scattered field with helicity equal to the incident plane wave Λ_+; the right half is for the opposite helicity Λ_-. The intensities of the two helicities ($\pm$) are computed as $|\mathbf{E} \pm i\mathbf{H}|^2$. In [13] (see Sec. 2.1), it is shown that $\mathbf{E} \pm i\mathbf{H}$ (with our choice of units) separates the two helicity components. The calculation plane is perpendicular to the z axis and 20 nm away from the surface of the scatterers opposite to the one where the incident field comes from. The calculation area is 700×700 nm. For color scaling purposes, the right-half side is multiplied by the factor in the upper right corner. The (lack of) cylindrical symmetry of the structures results in (non-)cylindrically symmetric field patterns, which is consistent with the geometry of each case. On the other hand, both scatterers behave identically with respect to the conservation of helicity, which is seen to depend exclusively on the electromagnetic properties of the material. The Supplemental Material [12] contains another simulation illustrating the fact that helicity preservation is independent of the angle of incidence of the plane wave.

We will now turn to demonstrate the practical value of this result. We will show that it can be applied to better understand systems with spatial inversion symmetries and also to calculate properties of the scattering off magnetic spheres.

Consider a general linear system interacting with the electromagnetic field, which remains invariant under a spatial inversion transformation T, i.e., either a point inversion ($T = \Pi$, with Π the parity operator) or a mirror operation across the plane perpendicular to $\hat{\mathbf{u}}$ [$T = M_{\hat{\mathbf{u}}} = \Pi R_{\hat{\mathbf{u}}}(\pi)$, where $R_{\hat{\mathbf{u}}}(\pi)$ is a rotation of π radians along the $\hat{\mathbf{u}}$ axis]. Many structures of fundamental and technological interest possess one or several symmetries of this kind, for instance nanoapertures in metallic films [14] and split-ring resonators [15]. We now show that, quite generally, the resonant natural modes of the system are linear superpositions of states of well-defined helicity. Since T leaves the system invariant, its eigenstates (natural modes) can be chosen to be eigenstates of T. Then, because T is a Hermitian operator and $T^2 = 1$, it follows that T has two eigenvalues equal to ± 1. The eigenstates of T, and hence those of the system, are of two different kinds, symmetric (s) and antisymmetric (a) under the action of T. Now, since parity and helicity anticommute and Λ commutes with any rotation, then T and Λ also anticommute. T, as parity, flips the helicity of any state it acts on. As a consequence, recalling that $\Lambda^2 = 1$ as well, the eigenstates of T are sum and subtractions of the eigenstates of Λ. The converse is also true. Symbolically,

$$|+\rangle = \frac{1}{\sqrt{2}}(|s\rangle + |a\rangle), \qquad |-\rangle = \frac{1}{\sqrt{2}}(|s\rangle - |a\rangle), \quad (10)$$

$$|a\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle), \qquad |s\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle). \quad (11)$$

The eigenstates of the system under consideration will be decomposable as indicated by Eq. (11). In general, the symmetric and antisymmetric eigenstates of the system will be nondegenerate, and their excitation will maximally mix the two helicity eigenstates. For example, in planar multilayer systems, which have mirror symmetries for all planes containing the stacking direction, the eigenmodes can be chosen to be either symmetric or antisymmetric with respect to one of the mirror operations. When the multilayer system presents resonances, as in the case of the surface modes in metal-dielectric interfaces, this leads to symmetric or antisymmetric modes. These modes mix both helicities maximally. The same physical phenomena happen in cylindrical scatterers, such as cylindrical nanoholes in metallic layers. Again, the cylindrical symmetry also implies mirror symmetries and the eigenmodes of the structure maximally mix the two helicity modes. This effect can be used to experimentally test plasmonic resonances by monitoring helicity changes.

As we have shown, though, when the system can be characterized by scalar electric and magnetic constants, the duality symmetry can be restored independently of the geometry of the system. The system can hence possess both duality symmetry and some spatial inversion symmetry, which implies that the symmetric and antisymmetric eigenstates have the same eigenvalue. We now analyze an interesting example of such a degenerate case.

Consider the unusual scattering effects for magnetic spheres reported by Kerker [16]. He found that upon scattering off a vacuum embedded sphere with $\epsilon/\mu = 1$, the state of polarization of light is preserved independently of the scattering angle. The root cause of such interesting phenomenon is related to our previous discussion: the simultaneous invariance of the system with respect to duality transformations due to the materials, and mirror operations through planes containing the origin of coordinates due to the geometry. In the helicity basis, the 2×2 scattering matrix between an incident and a scattered plane wave ([4], see Chap. 3) must be diagonal because of helicity preservation. Additionally, it must also preserve the linear polarizations parallel and perpendicular to the plane containing the two plane wave momentum vectors, because a mirror operation across such a plane leaves the sphere and both momentum vectors invariant. Then, using Eq. (10) it can be easily shown that all the 2×2 scattering matrices are indeed diagonal and hence preserve the state of polarization between any pair of incident and scattered plane waves.

In the same paper, Kerker finds that a plane wave impinging on such a dual sphere does not produce any backscattered field (at a 180 deg scattering angle). This effect, which has been referred to as an anomaly [17], can be easily understood using our result. A backscattered plane wave will have a linear momentum equal to minus the linear momentum of the impinging plane wave. Because of duality symmetry, the helicities of the two plane waves should be equal. Since $\Lambda = \mathbf{J} \cdot \mathbf{P}/|\mathbf{P}|$, the angular momentum along the plane waves axes must then change sign to compensate for the linear momentum sign change, but this is impossible due to the rotational symmetry of the sphere. The only solution is that no backscattered plane wave can exist.

In this Letter, we have shown that the restoration of duality symmetry is possible for the macroscopic Maxwell's equations, even though the microscopic equations are rendered asymmetric by the presence of charges. The restoration of the symmetry is independent of the geometry of the problem: a system made of piecewise isotropic and homogeneous domains of different materials characterized by electric and magnetic constants (ϵ_i, μ_i) is invariant under duality transformations if and only if $\epsilon_i/\mu_i = \alpha \ \forall\, i$. This result is independent of the shapes of the domains. With this result, the known relationship between helicity and duality transformations, namely, that the former is the generator of the latter, is turned into a simple and powerful tool for the practical study of light-matter interactions using symmetries and conserved quantities. In particular, in this Letter we have used it to establish that, quite generally, the eigenstates of a system with some kind of spatial inversion symmetry are sums and subtractions of modes with well-defined helicity. Also, interesting scattering effects of magnetic spheres have been shown to arise from simultaneous duality, cylindrical, and mirror symmetries. Additionally, in [5], this tool allowed us to prove the inconsistency of the concept of optical spin to orbital angular momentum conversion in focusing and scattering, and to propose a substitute framework based on helicity.

Our results may be useful in other fields. For example, they may prove important in the field of metamaterials and transformation optics [18], which is dramatically extending the range of wavelengths where effective electric and magnetic constants can be engineered. The transfer of helicity between light and matter remains an open line of research, which could have importance in the fields of plasmonics and "spintronics" [19], where the control of the helicity of electrons is crucial. Finally, it can be seen that the same tool that we have developed here can be successfully used to explain effects in electron beams [20]. This parallelism is an encouraging sign toward the possibility of simulating particle interactions on an optical table [21].

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