Marqués and Sáenz Reply: In the preceding article [1], Ruffner and Grier (RG) raise some objections to the conclusions of our Letter [2]. In contrast with the traditional assumption that the scattering force is proportional to the Poynting vector, we had shown that the curl of the spin angular momentum (SAM) density contributes an additional term to the radiation pressure experienced by small illuminated particles. However, RG claim that this additional term plays no role in the time-averaged optical forces on small particles. In this Reply, we show that the apparent contradiction with RG's conclusion is related to the actual physical significance of the "full" Poynting vector.

We first consider the same vector potential $\mathbf{A}(\mathbf{r})$ discussed by RG [see Eqs. (1) and (2) in [1]]. The time-averaged force on a small object (with polarizability $\alpha = \alpha' + i\alpha''$) is approximately given by [3]

$$\langle \mathbf{F} \rangle = \frac{\omega^2}{2} \mathrm{Re}\left\{\alpha \sum_{j=1}^{3} A_j(\mathbf{r}) \nabla A_j^*(\mathbf{r})\right\} = \frac{\omega^2}{4}\alpha' \nabla |\mathbf{A}|^2 - \frac{\omega^2}{2}\alpha'' \mathrm{Im}\left\{\sum_{j=1}^{3} A_j \nabla A_j^*\right\}, \quad (1)$$

where the first term is the intensity-gradient force and the second term, proportional to the extinction cross section $\sigma = (\omega/c)(\alpha''/\epsilon_0)$, can be associated to "scattering" forces, $\langle \mathbf{F} \rangle_{\mathrm{scatt}}$. As shown by RG, substituting the vector potential in Eq. (1) yields [see Eq. (4) in Ref. [1]]

$$\langle \mathbf{F} \rangle_{\mathrm{scatt}} = \frac{\omega^2}{2}\alpha'' u^2(\mathbf{r}) \sum_{j=1}^{3} a_j^2(\mathbf{r}) \nabla \varphi_j(\mathbf{r}), \quad (2)$$

which shows that, in general, the scattering force can be written as a "phase-gradient" force [4] which, for linearly polarized fields, is proportional to the Poynting vector. For arbitrary polarizations, however, the time-averaged momentum density in vacuum $\mathbf{g}(\mathbf{r})$ obtained from Poyinting's theorem can be written as

$$\langle g \rangle = \frac{\omega}{2\mu_0 c^2}\left[-\mathrm{Im}\left\{\sum_{j=1}^{3} A_j \nabla A_j^*\right\} + i\frac{\nabla \times (\mathbf{A}\times\mathbf{A}^*)}{2}\right]. \quad (3)$$

Taking into account that the SAM density $\mathbf{s}(\mathbf{r})$ is given by [see Eq. (5) in Ref. [1]]

$$\langle \mathbf{s}(\mathbf{r}) \rangle = \frac{\omega}{2\mu_0 c^2} i(\mathbf{A}(\mathbf{r}) \times \mathbf{A}^*(\mathbf{r})), \quad (4)$$

it is easy to see that the phase-gradient force [Eq. (2)] can be rewritten in terms of the time-averaged momentum and SAM densities as

$$\langle \mathbf{F} \rangle_{\mathrm{scatt}} = \sigma c \langle \mathbf{g}(\mathbf{r}) \rangle - \frac{\sigma c}{2} \nabla \times \langle \mathbf{s}(\mathbf{r}) \rangle. \quad (5)$$

Using the identity $\mathbf{E} = i\omega \boldsymbol{A}$ and taking into account that the SAM density $\langle \mathbf{L}_S \rangle$ defined in Ref. [2] differs from $\langle \mathbf{s} \rangle$ in Eq. (4) above by $\langle \mathbf{L}_S \rangle = -\langle \mathbf{s} \rangle/2$, we recover exactly the scattering force in Eq. (13) of Ref. [2]. Notice that Eqs. (2) and (5) are "mathematically" identical. The controversy arises in the physical interpretation of these equations.

The concept of spin-curl force is linked to the identification of light's momentum density as proportional to the full Poynting vector. If we assign a physical meaning to the full Poynting vector times the extinction cross section as part of the radiation pressure, we then must consider the SAM contribution to the force as having its own physical meaning [2].

As an alternative approach, it is tempting to interpret the two contributions to the Poynting vector in Eq. (3) separately, as representing the orbital and spin parts of the light's momentum density [5] (notice, however, that this decomposition is not unique [5]). With this interpretation, Eqs. (3) and (5) show that the spin-curl component of the light's momentum density does not play any role in the optical forces, in agreement with RG's statement. The total scattering force given by the phase-gradient term is then simply proportional to the orbital component of the Poynting vector. From this point of view, the scattering force does not depend on the spin-curl, but, importantly, neither does it depend on the full Poynting vector.

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