Test of the Universality of the Three-Body Efimov Parameter at Narrow Feshbach Resonances

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(Received 16 March 2013; published 1 August 2013)

We measure the critical scattering length for the appearance of the first three-body bound state, or Efimov three-body parameter, at seven different Feshbach resonances in ultracold ^{39}K atoms. We study both intermediate and narrow resonances, where the three-body spectrum is expected to be determined by the nonuniversal coupling of two scattering channels. Instead, our observed ratio of the three-body parameter with the van der Waals radius is approximately the same universal ratio as for broader resonances. This unexpected observation suggests the presence of a new regime for three-body scattering at narrow resonances.

DOI: 10.1103/PhysRevLett.111.053202 PACS numbers: 34.50.Cx

The Efimov effect [1] was first described in the context of nuclear physics but is now explored also in atomic, molecular, and condensed-matter systems [2–5]. Recent experiments on ultracold atoms with Feshbach resonances [6–18] have opened up a new path to study the universal spectrum of three-body Efimov states. The resonant interaction is expected to give rise to a three-body potential scaling as $1/R^2$, where R is the hyperradius that parameterizes the moment of inertia of the system. This leads to an infinite series of trimer states with an universal geometrical scaling for the binding energies. For a finite, negative two-body scattering length a, the three-body potential has a long-range cutoff at $R \simeq |a|$, and only a finite number of bound states exist. The critical scattering length a_- for the appearance of the first Efimov state at the three-body threshold, often called the three-body parameter, was expected to be the only parameter to be influenced by nonuniversal physics, i.e., by the microscopic details of two- or even three-body forces [1,3]. While clear evidence of the universal scaling of the Efimov spectrum is still missing, recent experiments on identical bosons suggested that also a_- might be universal [19]. This surprising result has been interpreted in a recent series of theoretical studies [20–22]. The underlying idea is that the sharp drop in the two-body interaction potential at a distance of the order of the van der Waals radius $R_{\rm vdW}$ results in an effective barrier in the three-body potential at a comparable distance [22]. This prevents the three particles from coming sufficiently close to explore nonuniversal features of the interactions at short distances and leads to a three-body parameter set by $R_{\rm vdW}$ alone, $a_- \simeq -9.5R_{\rm vdW}$ [19,21,22].

However, this scenario is realized only for the broad Feshbach resonances studied so far in most experiments, which can be described in terms of a single scattering channel, the so-called open channel. For narrow resonances, one must instead take into account the coupling of the open and a second closed channel [23]. It has been shown that in this case a new length scale that depends on the details of the specific Feshbach resonance, the so-called intrinsic length R^*, must be introduced to parameterize the two-body scattering. The three-body potentials are also modified, with an expected deviation from the Efimovian dependence into $1/(R^*R)$ for distances $R < R^*$ [24]. This tends to reduce the depth of the three-body potential and leads to the nonuniversal result $a_- = -12.90R^*$ [24,25], which is much larger than that obtained for broad resonances. This prediction is valid only close to resonance, where $|a| \gg R^*$. It is still unclear how a_- scales in the intermediate regime of $|a| \simeq R^*$ or generally for resonances of intermediate widths. Various general models have been proposed [26–31], but they either are not fully predictive or give contradicting results.

In this Letter, we address this problem by performing an experimental study of three-body collisions in ultracold bosonic ^{39}K atoms, where we determine the three-body parameter a_- at several Feshbach resonances of intermediate or narrow width. In particular, our measurements probe for the first time the regime of very small resonance strengths, $s_{\rm res} = 0.956R_{\rm vdW}/R^* \simeq 0.1$, where R^* might be expected to be the relevant length scale that determines a_-. Surprisingly, we find values of a_- that are around the same $-9.5R_{\rm vdW}$ measured for broad resonances, suggesting the existence of a novel intermediate regime of three-body scattering.

The investigation of closed-channel-dominated Feshbach resonances is particularly favored in ^{39}K, which has several resonances with moderate magnetic width Δ and relatively small background scattering length $-a_{\rm bg} \simeq (20–30)a_0$ [32]. These parameters, together with the difference of the magnetic moments of the closed and open channels, $\delta\mu$, determine the intrinsic length $R^* = \hbar^2/(ma_{bg}\Delta\delta\mu)$ [23], which can be related also to the on-resonance effective range (see the Supplemental Material [33]). In particular, we investigated seven different resonances with s_{res} in the range 0.1–2.8 in the three magnetic sublevels of the hyperfine ground state $F = 1$ [32].

A detailed description of the experimental setup is given elsewhere [34]. The three-body parameter was determined by finding the maximum of the three-body loss coefficient K_3 in the region of negative a at each Feshbach resonance, as in previous experiments [6–18]. In the presence of three-body losses, both the atom number N and temperature T evolve according to $dN/dt = -K_3\langle n^2\rangle N$ and $dT/dt = (K_3/3)\langle n^2\rangle T$, where $\langle n^2\rangle = (1/N)\int n(\vec{x})^3 d^3x$ is the mean square density [35]. The temperature increase is due to the preferential removal of atoms in the high-density region around the trap center. The typical starting condition was a noncondensed sample with 3–80 × 10^4 atoms in a temperature range of 20–400 nK, depending on the spin channel and Feshbach resonance (see the Supplemental Material [33]). The atoms were held in a purely optical trap (or with an additional magnetic confinement, depending on the specific resonance) at sufficiently low density to have a negligible mean-field interaction energy. Care was taken to have a trap depth sufficiently large to avoid evaporation associated to the heating. The samples were initially prepared at small negative a in proximity of the Feshbach resonances; the measurements started 10 ms after a was ramped to the final value in about 2 ms.

Figure 1 shows a typical evolution of N and T, as measured by absorption imaging after a free expansion. They were simultaneously fitted with

$$N(t) = N_0 \Big/ \left(1 + \frac{3\beta^2}{\sqrt{27}}\frac{N_0^2}{T_0^3}K_3 t\right)^{1/3}, \quad (1)$$

$$T(t) = T_0\left(1 + \frac{3\beta^2}{\sqrt{27}}\frac{N_0^2}{T_0^3}K_3 t\right)^{1/9}. \quad (2)$$

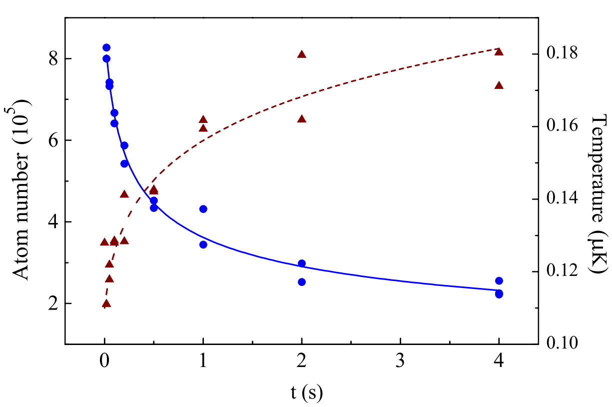

FIG. 1 (color online). Example of the time evolution of the atom number (circles) and temperature (triangles), fitted to Eq. (1) (solid line) and Eq. (2) (dashed line) to determine the three-body loss coefficient K_3.

Here N_0 and T_0 are the initial atom number and temperature, respectively, and $\beta = (m\bar{\omega}^2/2\pi k_B)^{3/2}$, with $\bar{\omega}$ the mean trap frequency. In such a fit, one-body losses were neglected, since they occur on a much longer time scale.

Crucial ingredients for a reliable measurement of the K_3 dependence on the scattering length were an accurate calibration of the magnetic field B and the use of a high-quality coupled-channel (CC) model for $a(B)$, based on a large number of experimental observations for the positions and widths of the Feshbach resonances [32,33]. The centers and widths of the Feshbach resonances were redetermined in the present work, finding a good agreement with the theoretical ones. An additional confirmation of the CC model was derived from a direct measurement of the dimer binding energy at the two narrowest resonances by radio-frequency spectroscopy.

We observed for all Feshbach resonances a clear peak in K_3 in the region of $|a| = (600–1000)a_0$, as shown in Figs. 2 and 3. We compared the observations to the known

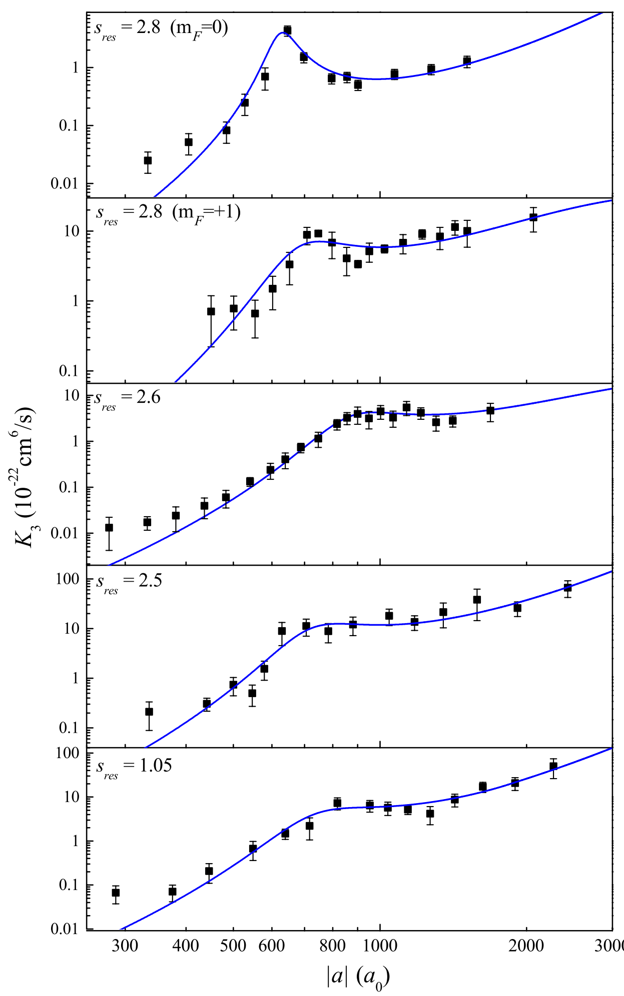

FIG. 2 (color online). Three-body loss rate measured in the proximity of five Feshbach resonances of intermediate strength (see Table I for the assignment of the spin state). The experimental data (squares) are fitted to Eq. (3) (solid line).

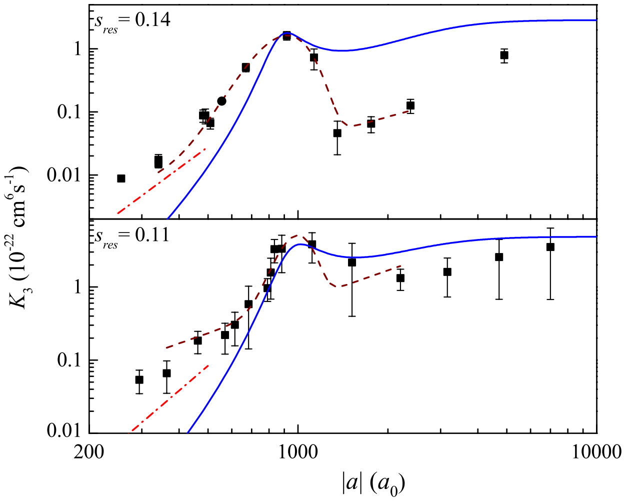

FIG. 3 (color online). Three-body loss rate measured in the proximity of two narrow Feshbach resonances in the $m_F = 0$ state. The experimental data (squares) are fitted with a Gaussian (dashed line) to determine a_- from the position of the loss maximum, and also compared with Eq. (3), using $\eta_- = 0.1$ (solid line). The dash-dotted lines provide a comparison to a $|a|^{7/2}$ behavior for low $|a|$.

relation for identical bosons at zero collision energy and in the zero-range approximation, for $a < 0$:

$$K_3(a) = 4590 \frac{3\hbar a^4}{m} \frac{\sinh(2\eta_-)}{\sin^2[s_0 \ln(a/a_-)] + \sinh^2 \eta_-}. \quad (3)$$

Here $s_0 \simeq 1.00624$ is an universal constant, and η_- is the decay parameter which sets the width of the Efimov resonance and incorporates short-range inelastic transitions to deeply bound molecular states [3]. At the finite temperature of the experiment, there is a limitation in the maximum observable K_3 set by unitarity at $K_3^{\max} = 36\sqrt{3}\pi^2\hbar^5/(k_B T)^2 m^3$ [36,37]. Therefore, we fitted the data with an effective rate of the form $[1/K_3(a) + 1/K_3^{\max}]^{-1}$ [7,13,38], with a_-, η_-, and $K_3^{\max}$ as fitting parameters. The experimental $K_3(a)$ for the five broadest resonances, shown in Fig. 2, is in good agreement with Eq. (3), besides a multiplicative factor of the order of 3 that can be justified with the experimental uncertainty in the determination of the density (see the Supplemental Material [33]).

Also, the two narrowest Feshbach resonances feature a maximum in K_3 around $-1000a_0$, as shown in Fig. 3. There is, however, a slower background variation of K_3 with a, not reproduced by Eq. (3). It was shown that for narrow resonances one should expect a slower evolution in the regime $|a| < R^*$, with $K_3 \propto |a|^{7/2}$ [24], but also this does not seem to reproduce the data at small $|a|$. In the absence of a better model and in analogy with the broad resonances, we determined the position of the measured maximum in K_3 with a Gaussian fit, as shown in Fig. 3, and we interpreted it as the a_- parameter. As uncertainty, we conservatively took the $1/e^2$ half-width of the Gaussian.

For all the resonances in excited spin states, there is in principle also a contribution of two-body losses, which have a slower dependence on a [23]. While it was not possible to distinguish in a reliable way two- from three-body losses in the experiment, we have verified that the calculated two-body losses from the CC models are typically negligible, besides some large-a regions close to the two narrow resonances (see the Supplemental Material [33]).

A summary of our analysis is reported in Table I. For the calculation of $a(B)$, we used the experimentally determined Feshbach resonance centers $B_0^{\rm expt}$ and the resonance widths and the background scattering lengths from the CC model. The uncertainties in $B_0^{\rm expt}$ include those in the calibration of B and in the determination of B_0 from the loss resonances. Particular care was put in the determination of B_0 for the two narrowest resonances, where we found a rather good agreement between independent measurements of the atom losses and of the binding energy (see the Supplemental Material [33]). The uncertainties in a_- include the statistical uncertainties from the fit of the K_3 data and from the determination of $a(B)$. For the two narrowest resonances, the dominant source of uncertainty comes from the determination of B_0. These two resonances are coupled, and $a(B)$ can be represented only over an

TABLE I. Theoretical and experimental parameters at Feshbach resonances in the m_F spin channels: measured resonance center $B_0^{\rm expt}$; intrinsic length R^* and strength $s_{\rm res}$ of the Feshbach resonances from the CC model; measured three-body parameter a_- and decay parameter η_-; initial temperature T. For ^{39}K, $R_{\rm vdW} = 64.49a_0$. Figures in parentheses represent one standard deviation.

m_F	$B_0^{\rm expt}$ (G)	R^* (a_0)	$s_{\rm res}$	$-a_-$ (a_0)	η_-	T (nK)
0	471.0 (4)	22	2.8	640 (100)	0.065 (11)	50 (5)
+1	402.6 (2)	22	2.8	690 (40)	0.145 (12)	90 (6)
−1	33.64 (15)	23	2.6	830 (140)	0.204 (10)	120 (10)
−1	560.72 (20)	24	2.5	640 (90)	0.22 (2)	20 (7)
−1	162.35 (18)	59	1.1	730 (120)	0.26 (5)	40 (5)
0	65.67 (5)	456	0.14	950 (250)		330 (30)
0	58.92 (3)	556	0.11	950 (150)		400 (80)

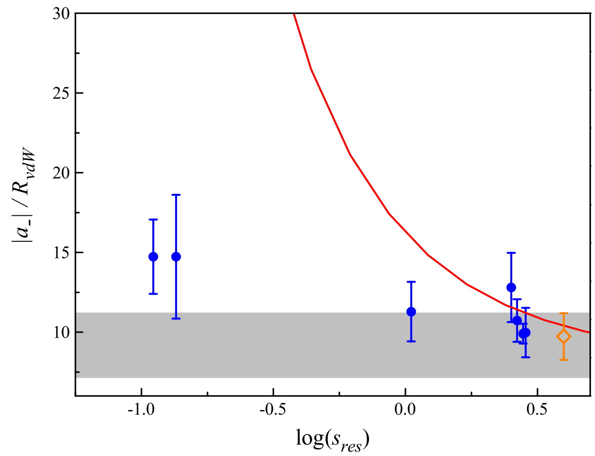

FIG. 4 (color online). Ratio of the measured $|a_-|$ to $R_{\rm vdW}$ as a function of the strength of the Feshbach resonances (filled circles). Our data are compared to predictions for $s_{\rm res} \gg 1$ [21] (open diamond) and for generic $s_{\rm res}$ [26] (solid line). The gray shaded region shows the scatter in the experimental data for $|a_-|/R_{\rm vdW}$ measured in other atomic species.

extended range of magnetic fields in terms of a two-pole expression containing two widths and a single $a_{\rm bg}$ [32]. The reported values of R^* are determined on resonance, from a comparison of the predictions of our CC calculation to a generalization of the quantum-defect model of Ref. [39] to the case of coupled resonances. The coupling causes a dependence of R^* on B, which is, however, limited to about 20% in the experimental range (see the Supplemental Material [33]).

We observe a whole range of values of η_- for the different Efimov resonances; this is probably a consequence of the different measurement temperatures but possibly also of the nonuniversal nature of η_- [3,40].

A comparison of the results in Table I leads to the striking conclusion that the three-body parameter a_- stays around values of the order of $-10R_{\rm vdW}$ for all the Feshbach resonances explored in ^{39}K, including the ones with R^* as large as $\sim 600a_0$, hence much larger than $R_{\rm vdW}$. We note that in the earlier measurement at the $m_F = 1$ resonance, we found two K_3 resonances at $|a| \simeq 700a_0$ and $|a| \simeq 1500a_0$, which we tentatively identified as a four- and a three-body resonance, respectively [7]. We now think that the previous resonance around $1500a_0$, which we no longer observe, was an artifact of the analysis of the limited time-dependent data, and we reassign the one around $700a_0$ as the three-body resonance (see the Supplemental Material [33]).

Figure 4 shows the measured $|a_-|/R_{\rm vdW}$ as a function of $s_{\rm res}$. We observe just a moderate deviation of our data from the mean value 9.73(3) measured for open-channel-dominated resonances [17,19,21,41] and also for other intermediate resonances [9–11,18,19,41]. This observation is far from the already mentioned prediction for narrow resonances [24,25], which indicates that the Efimov resonances should appear at scattering lengths that are multiples of $a_- = -12.9R^*$ by a factor $\exp(\pi/s_0) \simeq 22.7$. One might note that this result is expected to be valid only in the limit of a scattering length larger than any other length scale, $|a| \gg R^* \gg |a_{\rm bg}|$, where the three-body potential at large hyperradii $R > R^*$ has an Efimovian character [24]. The present experiment does not access this extreme limit but is in an intermediate regime also for the two narrowest resonances, which show indeed $R^* \simeq |a_-|$.

Other models for the three-body physics at Feshbach resonances of intermediate strength have been proposed [26–31]. The specific problem of connecting the results for the three-body parameter in the open-channel-dominated regime, where a_- is determined by $R_{\rm vdW}$, and the closed-channel limit, where it is R^* which sets the scale for a_-, has been addressed recently [26,31], finding, however, considerably different results. In particular, the model of Ref. [26] predicts that a crossover between the two regimes of broad and narrow resonances would take place around $s_{\rm res} \simeq 1$, as shown in Fig. 4. Additionally, the regime of $a_- = -12.9R^*$ should be reached only for excited Efimov states, while the first one has a slightly smaller $a_- = -10.3R^*$. Although an increase of $|a_-|$ with decreasing $s_{\rm res}$ might be present in the experimental data, there is a clear disagreement with such predictions. Experiments on ^{7}Li and ^{133}Cs have also measured similar values for a_- at three intermediate resonances with $s_{\rm res} = 0.5$–1 [10,11,18,19], indicating that this behavior might not be peculiar of ^{39}K. Also, a system without Feshbach resonances like metastable ^{4}He might be consistent with these results [42].

We note that for the two narrowest resonances $|a_-|$ is only a factor of 2 larger than R^*. This observation seems to indicate that the three-body potential can support a bound state that resides only in the region with hyperradius $R \leq 2R^*$. This is a regime that was not accessible in previous one-channel models, and a multichannel approach [43], possibly comprising the coupled-resonance aspect, will presumably be necessary to model the observations.

In conclusion, our study showed an apparent universal behavior of the three-body parameter on several different Feshbach resonances of the same atomic species, down to a resonance strength $s_{\rm res} \simeq 0.1$. This gives important information on the three-body physics in this narrow-resonance regime, where an interplay of the open and the closed molecular channels is expected.

We thank R. Schmidt, M. Zaccanti, and W. Zwerger for helpful discussions. This work was supported by INFM (MICRA), ERC (Grants No. 203479, No. 247371, and No. 258325), the Agence Nationale de la Recherche (Contract No. ANR-12-BS04-0020-01), and MIUR (PRIN 2009FBKLNN).

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