

Chiral Plasma Instabilities

Yukinao Akamatsu

Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya 464-8602, Japan

Naoki Yamamoto

*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
and Maryland Center for Fundamental Physics, Department of Physics, University of Maryland,
College Park, Maryland 20742-4111, USA*

(Received 22 February 2013; published 1 August 2013)

We study the collective modes in relativistic electromagnetic or quark-gluon plasmas with an asymmetry between left- and right-handed chiral fermions, based on the recently formulated kinetic theory with Berry curvature corrections. We find that there exists an unstable mode, signaling the presence of a plasma instability. We argue the fate of this “chiral plasma instability” including the effect of collisions, and briefly discuss its relevance in heavy ion collisions and compact stars.

DOI: [10.1103/PhysRevLett.111.052002](https://doi.org/10.1103/PhysRevLett.111.052002)

PACS numbers: 12.38.Aw, 11.10.Wx, 12.38.Mh

Introduction.—Parity violating effects related to quantum anomalies play an important role in a wide range of physics from quantum Hall systems to cosmology. One example in the transport phenomena is the parity violating current in the presence of a magnetic field and an asymmetry between left and right-handed fermions, parametrized by the chiral chemical potential $\mu_5 \equiv \mu_R - \mu_L$. This is called the chiral magnetic effect (CME) [1–5]. Recently, hydrodynamics [6] and kinetic theory [7–11] have been appropriately modified to describe quantum anomalies and the CME (see also Refs. [12,13]); in the kinetic theory, essential corrections are the Berry curvature, the concept diversely applied in condensed matter physics [14,15]. These developments enable us to systematically understand the properties and dynamical evolution of various systems with hitherto neglected anomalous effects.

In this Letter, we study the collective modes and their consequences in relativistic electromagnetic or quark-gluon plasmas (QGP) at finite μ_5 [16] and temperature T based on the new kinetic theory. (For a review on the collective modes *without* Berry curvature corrections, see Ref. [18].) We show that the Berry curvature corrections dramatically modify the dispersion relation of the collective modes, and in particular, lead to unstable modes, signaling the presence of a plasma instability. We shall call it the chiral plasma instability. Closely related instability was studied within the electroweak theory at large lepton chemical potential [19,20] and in the context of the early Universe at $T \gg \mu_5$ [21–23] where it is ascribed to a possible origin of the primordial magnetic field.

Our main purpose of this Letter is to reveal the potential importance of the chiral plasma instabilities in heavy ion collisions and compact stars. It has been argued that the QGP created in noncentral heavy ion collisions may contain locally finite μ_5 for quarks and a large external

magnetic field to yield an observable CME [5,17,24]. It was also suggested that a degenerate electromagnetic plasma with finite μ_5 for electrons may exist inside neutron stars due to the parity violating weak process, where the CME is generated in a strong magnetic field [25]. We show within the kinetic theory that these QCD and QED plasmas at finite μ_5 are dynamically unstable and reduce μ_5 by converting it to (color) electromagnetic fields with magnetic helicity. We estimate the typical time scales of the chiral plasma instabilities for the QCD and QED plasmas as $\tau_{\text{QCD}} \sim 1/(\alpha_s^2 \mu_5 \ln \alpha_s^{-1})$ for $\mu_5 \sim T$ and $\tau_{\text{QED}} \sim 1/(\alpha^2 \mu_5)$, respectively.

Kinetic theory with parity violating effects.—Let us describe a chiral plasma within the kinetic framework. We first consider the collisionless kinetic theory and we shall argue the effect of collisions later. We also consider the regime of a sufficiently weak gauge field A^μ , where there is no essential difference between Abelian and non-Abelian gauge fields, up to color and flavor degrees of freedom [26,27]. For simplicity of notation, we first consider QED, and we will generalize it to QCD later (with some modifications mentioned below). We assume massless quarks and spatially homogeneous μ_5 below.

Recall the Maxwell equation

$$\partial_\nu F^{\nu\mu} = j_{\text{ind}}^\mu + j_{\text{ext}}^\mu, \quad (1)$$

where j_{ind}^μ is the induced current and j_{ext}^μ is the external current. For the small gauge field A^ν , the induced current can be expressed, via the linear response theory, as

$$j_{\text{ind}}^\mu(K) = \Pi^{\mu\nu}(K) A_\nu(K), \quad (2)$$

in the momentum space, where $\Pi^{\mu\nu}$ is the (retarded) self-energy and $K^\mu = (\omega, \mathbf{k})$ is the four-momentum. From Eqs. (1) and (2), we obtain

$$[K^2 g^{\mu\nu} - K^\mu K^\nu + \Pi^{\mu\nu}(K)] A_\nu(K) = -j_{\text{ext}}^\mu. \quad (3)$$

Because of the gauge invariance, which allows us to shift $A_\nu \rightarrow A_\nu + K_\nu$, we can choose the temporal gauge $A_0 = 0$. Then Eq. (3) can be rewritten, using the electric field, as

$$[\Delta^{-1}]^{ij} E^j \equiv [(k^2 - \omega^2)\delta^{ij} - k^i k^j + \Pi^{ij}] E^j = -i\omega j_{\text{ext}}^i \quad (4)$$

where $k \equiv |\mathbf{k}|$. The dispersion relation for the collective modes in the system can be found by computing the poles of Δ^{ij} ; a mode with the dispersion relation $\omega = \omega(k)$ satisfying $\text{Im}(\omega) > 0$, if exists, implies an instability.

The explicit form of $\Pi^{\mu\nu}$ including the parity violating effects can be found by using the kinetic theory with Berry curvature corrections [7,9,10]. To illustrate the point and for simplicity, we take the initial equilibrium distribution function $n_{\mathbf{p}}^0$ to be isotropic (up to the Zeeman effect in the presence of a magnetic field; see below). It should be remarked that though the same result for $\Pi^{\mu\nu}$ may be obtained in perturbation theory for the isotropic $n_{\mathbf{p}}^0$ [10] (see also Ref. [28]), the kinetic theory would be more versatile for this and later purposes; the topological origin of the parity violating tensor $\Pi^{\mu\nu}$ will be apparent in this framework. It is also easy to include the effect of collisions, as we shall do later. Moreover, it is suitable for other future applications, such as the inclusion of anisotropy of $n_{\mathbf{p}}^0$ and numerical simulations of the dynamical evolution of plasmas.

At the leading order in A^μ , the kinetic theory is given by [10]

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) n_{\mathbf{p}} + (e\mathbf{E} + \mathbf{v} \times e\mathbf{B} - \nabla_{\mathbf{x}} \epsilon_{\mathbf{p}}) \cdot \nabla_{\mathbf{p}} n_{\mathbf{p}} = 0, \quad (5)$$

where $\mathbf{v} = \mathbf{p}/p$, $\epsilon_{\mathbf{p}} = p(1 - e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}})$ with $\mathbf{\Omega}_{\mathbf{p}} = \pm \mathbf{p}/2p^3$ the Berry curvature for right- and left-handed fermions, respectively. The energy of the chiral fermion is shifted from $\epsilon_{\mathbf{p}} = p$ by the amount $-pe\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}$ due to the magnetic moment of chiral fermions at finite μ [10]. This correction makes the term involving $\nabla_{\mathbf{x}} \epsilon_{\mathbf{p}}$ in Eq. (5) nonvanishing for an inhomogeneous magnetic field, unlike the conventional Vlasov equation. In the presence of the Berry curvature flux, the definition of the current is also modified to [7,9,10]

$$\mathbf{j} = -e^2 \int_{\mathbf{p}} [\epsilon_{\mathbf{p}} \nabla_{\mathbf{p}} n_{\mathbf{p}} + (\mathbf{\Omega}_{\mathbf{p}} \cdot \nabla_{\mathbf{p}} n_{\mathbf{p}}) \epsilon_{\mathbf{p}} \mathbf{B} + \epsilon_{\mathbf{p}} \mathbf{\Omega}_{\mathbf{p}} \times \nabla_{\mathbf{x}} n_{\mathbf{p}}]. \quad (6)$$

For a moment, we concentrate on right-handed fermions with chemical potential μ . (This theory coupled to dynamical gauge fields itself is not well defined because of the gauge anomaly, but we shall eventually consider the theory with both right- and left-handed fermions, so that the gauge anomalies are cancelled out.) We solve the linearized equation of (5) in terms of the deviation $\delta n_{\mathbf{p}}$ from the thermal equilibrium state $n_{\mathbf{p}}^0 = 1/[e^{(\epsilon_{\mathbf{p}} - \mu)/T} + 1]$, where $n_{\mathbf{p}} = n_{\mathbf{p}}^0 + \delta n_{\mathbf{p}}$. Note that $n_{\mathbf{p}}^0$ does not obey $n_{\mathbf{p}}^0 = n_{-\mathbf{p}}^0$ and breaks parity owing to the Zeeman effect.

Substituting the solution $n_{\mathbf{p}}$ to the current \mathbf{j} and using the linear response theory (2), the self-energy of the gauge field can be expressed as $\Pi^{\mu\nu}(K) = \Pi_+^{\mu\nu}(K) + \Pi_-^{\mu\nu}(K)$, where [10]

$$\Pi_+^{\mu\nu}(K) = -m_D^2 \left[\delta^{\mu 0} \delta^{\nu 0} - \omega \int \frac{d\mathbf{v}}{4\pi} \frac{v^\mu v^\nu}{\mathbf{v} \cdot \mathbf{K} + i\epsilon} \right], \quad (7)$$

$$\Pi_-^{ij}(K) = \frac{e^2 \mu}{4\pi^2} i \epsilon^{ijk} k^k \left(1 - \frac{\omega^2}{k^2} \right) [1 - \omega L(K)], \quad (8)$$

are the parity-even and parity-odd self energies with

$$m_D^2 = e^2 \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \right), \quad (9)$$

$$L(K) = \frac{1}{2k} \ln \frac{\omega + k}{\omega - k}. \quad (10)$$

Here i, j, k denote the spatial indices [$\Pi_-^{\mu\nu}(K)$ is vanishing otherwise] and $v^\mu = (1, \mathbf{v})$. Note that Π_- is shown to be T independent from the topological nature of the Berry curvature (see the similar computation of the CME in Ref. [7]).

Collective modes.—We perform a tensor decomposition for the self-energy Π^{ij} . Recall that Π^{ij} is not symmetric with respect to the indices i and j due to the parity violating term Π_-^{ij} ; we need to use not only the longitudinal and transverse projectors, $P_L^{ij} = \hat{k}^i \hat{k}^j$ and $P_T^{ij} = \delta^{ij} - \hat{k}^i \hat{k}^j$, but also the antisymmetric tensor, $P_A^{ij} = i \epsilon^{ijk} \hat{k}^k$, to fully decompose the tensor structure of Π^{ij} , where $\hat{k}^i = k^i/k$. Then $[\Delta^{-1}]^{ij}$ can be written as

$$[\Delta^{-1}]^{ij} = C_L P_L^{ij} + C_T P_T^{ij} + C_A P_A^{ij}, \quad C_L = -\omega^2 + \Pi_L, \\ C_T = -\omega^2 + k^2 + \Pi_T, \quad C_A = \Pi_A, \quad (11)$$

where

$$\Pi_L = -m_D^2 \frac{\omega^2}{k^2} [1 - \omega L(K)], \quad (12a)$$

$$\Pi_T = m_D^2 \frac{\omega^2}{2k^2} \left[1 + \frac{k^2 - \omega^2}{\omega} L(K) \right], \quad (12b)$$

$$\Pi_A = \frac{e^2 \mu k}{4\pi^2} \left(1 - \frac{\omega^2}{k^2} \right) [1 - \omega L(K)]. \quad (12c)$$

In order to compute Δ^{ij} [the inverse of Eq. (11)], it is convenient to use the properties of the (mutually commutable) projectors,

$$P_L^2 = P_L, \quad P_T^2 = P_T, \quad P_L P_T = 0, \quad (13a)$$

$$P_A^2 = P_T, \quad P_L P_A = 0, \quad P_T P_A = P_A, \quad (13b)$$

from which we obtain

$$\Delta^{ij} = \frac{1}{C_L} P_L^{ij} + \frac{C_T}{C_T^2 - C_A^2} P_T^{ij} - \frac{C_A}{C_T^2 - C_A^2} P_A^{ij}. \quad (14)$$

Therefore, the dispersion relations of collective modes are given by $C_L = 0$ and $C_T \pm C_A = 0$, which, respectively, reduce to

$$\omega^2 = \Pi_L(K), \quad (15)$$

$$\omega^2 = k^2 + \Pi_T(K) \pm \Pi_A(K). \quad (16)$$

While the dispersion relation for the longitudinal component is identical to the case without Π_A , that for the transverse component is split up into two parts by the effect of Π_A .

Let us compute the dispersion relations of the collective modes based on Eqs. (15) and (16).

In the long-wavelength limit $|\omega| \gg k$, Π_A is negligible compared with $\Pi_{L,T}$ in the leading order, and

$$\Pi_{L,T} = \frac{1}{3} m_D^2 \equiv \omega_{\text{pl}}^2. \quad (17)$$

The dispersion relations for the longitudinal and transverse modes are both given by $\omega^2 = \omega_{\text{pl}}^2$, implying that nonstatic gauge fields oscillate with the plasma frequency ω_{pl} .

In the quasistatic limit $|\omega| \ll k$, using $\omega L(K) = \mp(i\pi/2)x + x^2 + (1/3)x^4 + \dots$ for positive and negative $\text{Im}(\omega)$ with $x \equiv |\omega|/k \ll 1$, the transverse dispersion relation can be obtained perturbatively in x . Equation (16) with the minus sign has the solutions,

$$\omega = \pm i\gamma(k), \quad \gamma(k) = \frac{4\alpha\mu}{\pi^2 m_D^2} k^2 \left(1 - \frac{\pi k}{\alpha\mu}\right) \quad (18)$$

for $0 \leq k \leq \alpha\mu/\pi$ with $\alpha = e^2/(4\pi)$, while that with the plus sign does not; see Fig. 1 for the numerical result of Eq. (16) in this regime. The mode $\omega = i\gamma(k)$ with $\gamma > 0$ is unstable and grows exponentially as $e^{-i\omega t} \sim e^{\gamma t}$, indicating the plasma instability. For $\mu \sim T$, the typical scales of the unstable mode are

$$k \sim \alpha\mu, \quad |\omega| \sim \frac{\alpha\mu k^2}{m_D^2} \sim \alpha^2\mu, \quad (19)$$

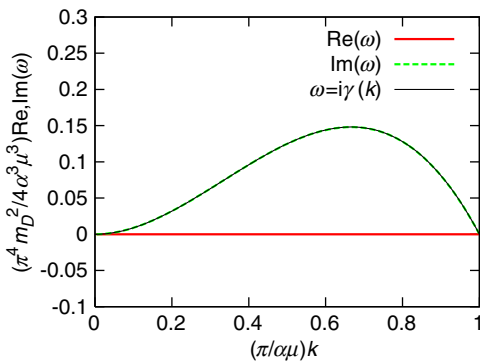


FIG. 1 (color online). Real and imaginary parts of the dispersion relation $\omega = \omega(k)$ for the unstable mode.

and the time scale of the plasma instability is $\tau_{\text{inst}} = 1/\gamma \sim 1/(\alpha^2\mu)$.

Without Π_A originating from Berry curvature corrections, there was no unstable mode for isotropic n_{p}^0 . Note also that the criteria in Ref. [27] for the conventional (Weibel) plasma instability [29] is not applicable here. This is because the assumptions there, $n_{\text{p}}^0 = n_{\text{p}}^0$ and $\Pi^{ij} = \Pi^{ji}$, are violated by the parity violating effects.

Now these results can easily be extended to a plasma at finite vector and chiral chemical potentials, $\mu_V = \mu_R + \mu_L$ and $\mu_5 = \mu_R - \mu_L$, by the following replacements: $\mu^2 \rightarrow \mu_R^2 + \mu_L^2 = (1/2)(\mu_V^2 + \mu_5^2)$ in m_D^2 , and $\mu \rightarrow \mu_R - \mu_L = \mu_5$ in Π_-, Π_A and Eq. (18).

The fate of chiral plasma instabilities.—What is the fate of chiral plasma instabilities above? Below we shall argue that this instability reduces μ_5 so that it is weakened. We first recall the relation of triangle anomalies,

$$\partial_t Q_5 = \frac{2\alpha}{\pi} \int_{\mathbf{x}} \mathbf{E} \cdot \mathbf{B}, \quad (20)$$

where Q_5 is the global chiral charge. Equivalently,

$$\partial_t \left(Q_5 + \frac{\alpha}{\pi} \mathcal{H} \right) = 0, \quad \mathcal{H} = \int_{\mathbf{x}} \mathbf{A} \cdot \mathbf{B}, \quad (21)$$

where \mathcal{H} is the Chern-Simons number (which is also called the magnetic helicity in plasma physics). In QCD, there is an additional A^3 term in \mathcal{H} , but it is negligible under the assumption of small A . Equation (21) is the conservation of the helicity.

We now show the reduction of μ_5 within the kinetic theory in the regime under consideration. We concentrate on the unstable mode, $C_T \simeq C_A$ at $|\omega| \ll k$, for which the longitudinal part is negligible. Using Eq. (14), $A^i = \Delta^{ij} j_{\text{ext}}^j$ [see Eq. (4)] reduces to

$$A^i(K) \simeq \frac{i\beta(k)}{\omega - i\gamma(k)} J_{\text{ext}}^i(K), \quad (22)$$

where we defined $\beta(k) = -2k/(\pi m_D^2)$ and $J_{\text{ext}}^i(K) \equiv (P_T^{ij} - P_A^{ij}) j_{\text{ext}}^j(K)$. In the (t, \mathbf{k}) space, A^i satisfies

$$[\partial_t - \gamma(k)] A^i(t, \mathbf{k}) = \beta(k) J_{\text{ext}}^i(t, \mathbf{k}). \quad (23)$$

This can be solved in terms of A^i for $j_{\text{ext}}(t, \mathbf{k}) = j_{\text{ext}}(\mathbf{k}) \delta(t)$:

$$A^i(t, \mathbf{k}) = \beta e^{\gamma t} \theta(t) J_{\text{ext}}^i(\mathbf{k}). \quad (24)$$

We then obtain $E^i = -\partial_t A^i$ and $B^i = -k P_A^{ij} A^j$ as

$$E^i(t, \mathbf{k}) = -\beta [\gamma e^{\gamma t} \theta(t) + \delta(t)] J_{\text{ext}}^i(\mathbf{k}), \quad (25)$$

$$B^i(t, \mathbf{k}) = k \beta e^{\gamma t} \theta(t) J_{\text{ext}}^i(\mathbf{k}), \quad (26)$$

where we used $P^A J_{\text{ext}} = -J_{\text{ext}}$ that follows from Eq. (13b). Hence, we find $\mathbf{E}(t, \mathbf{k}) \cdot \mathbf{B}^*(t, \mathbf{k}) < 0$ and $\mathbf{A}(t, \mathbf{k}) \cdot \mathbf{B}^*(t, \mathbf{k}) > 0$. From Eq. (20), this unstable mode

decreases μ_5 for fixed T and increases the magnitude of \mathcal{H} . As a result, it weakens the instability.

The concrete time evolution of the plasma beyond the leading order in A^μ can be described by the full kinetic theory [see Eq. (15) in Ref. [10]] together with Eqs. (1), (6), and (20), which would require a numerical analysis. Still the saturation of the instability itself may be understood from the energy and helicity conservations. We assume $\mu_5 \sim T$ or $\mu_5 \gg T$. As the energy and magnetic helicity of electromagnetic fields, $\sim (kA)^2$ and $\sim \alpha kA^2$, come from those of chiral fermions, $O(\mu_5^4)$ and $O(\mu_5^3)$, we get the typical scales of k and A relevant to the instability:

$$k \sim \alpha \mu_5, \quad A \sim \frac{\mu_5}{\alpha}, \quad B \sim \mu_5^2. \quad (27)$$

The electromagnetic fields cannot grow beyond this and will saturate in the end. At the saturation, the gauge fields become nonperturbatively large. Thus, if μ_5 for electrons is produced during the evolution of neutron stars [25], chiral plasma instabilities would provide a mechanism to generate the large magnetic helicity which plays an important role for the stability of the large magnetic field (see, e.g., Ref. [30]).

Effect of collisions.—For $\mu_5 \sim T$, the mean free time for electric charge transfer (or large-angle scatterings) of the plasma constituents in QED, $\tau_{\text{large}} \sim 1/(\alpha^2 T \ln \alpha^{-1})$ [31], is shorter than the time scale of the plasma instability, $\tau_{\text{inst}} \sim 1/(\alpha^2 \mu_5)$. On the other hand, the mean free path, $l_{\text{large}} \sim \tau_{\text{large}}$, is larger than the typical wavelength of the plasma instability, $1/k_{\text{inst}} \sim 1/(\alpha \mu_5)$, and one expects that the effect of collisions is irrelevant for it.

To see this explicitly, consider the effect of collisions in the relaxation time approximation: we add the term $-\delta n_{\mathbf{p}}/\tau_{\text{rel}}$ in the right-hand side of Eq. (5), where τ_{rel} is the relaxation time assumed to be of the same order as τ_{large} . Then ω in the denominator of Eq. (7) is replaced by $\omega + i/\tau_{\text{rel}}$. Repeating a similar computation to above, one finds that the modification to Eq. (18) by the effect of collisions is subleading in $1/(k_{\text{inst}}\tau_{\text{rel}}) \ll 1$ and is negligible. (From this argument, the effect of collisions is clearly negligible for $\mu_5 \gg T$.) As we shall see below, however, this is not the case in QCD.

Quark-gluon plasma with μ_5 .—Suppose μ_5 ($\sim T$) for quarks is initially generated in the QGP before the thermalization, whose evolution is described by the kinetic theory. (The case without μ_5 but with the anisotropy of $n_{\mathbf{p}}$ was analyzed in Refs. [26,27].) The above arguments are then extended to color electromagnetic fields, within the leading order in A , by replacing $\alpha \rightarrow \alpha_s = g^2/(4\pi)$ and $m_D^2 \rightarrow (N_f + 2N_c)g^2T^2/6 + N_f g^2 \mu^2/(2\pi^2)$ with N_f and N_c the number of flavors and colors. The qualitative difference from the QED plasma is that the mean free time (path) for color charge transfer (or small-angle scatterings) is now $\tau_{\text{small}} \sim l_{\text{small}} \sim 1/(\alpha_s T \ln \alpha_s^{-1})$ because colored gluons, exchanged between quarks, can change the color

charges of the scatterers [32–35]. As $\tau_{\text{small}} \ll \tau_{\text{inst}}$ and $l_{\text{small}} \ll 1/k_{\text{inst}}$ in this case, the collisions can affect the plasma instability. In the relaxation time approximation, $L(\omega + i/\tau_{\text{rel}}, k) \simeq -i\tau_{\text{rel}}[1 - (k\tau_{\text{rel}})^2/3]$ with $\tau_{\text{rel}} \sim \tau_{\text{small}}$, one indeed finds that Eq. (18) is modified to

$$\gamma(k) = \frac{3N_f \alpha_s \mu_5 k}{\pi m_D^2 \tau_{\text{rel}}} \left(1 - \frac{\pi k}{\alpha_s \mu_5}\right). \quad (28)$$

The unstable modes in $0 \leq k \leq \alpha_s \mu_5/\pi$ still exist but grow more rapidly with $\tau_{\text{QCD}} \sim 1/(\alpha_s^2 \mu_5 \ln \alpha_s^{-1})$ [36].

Conclusion and outlook.—We have shown that the relativistic electromagnetic plasma and QGP with finite μ_5 are dynamically unstable and μ_5 is damped by the exponential growth of (color) electromagnetic fields at the early stage. As the gauge field grows, the nonlinearity of the gauge field becomes important [39]. To understand the roles of the chiral plasma instability at the later stage quantitatively, more detailed studies of the dynamical evolution of plasmas based on the kinetic theory need to be worked out. It would also be an interesting question whether the instability persists even at strong coupling where the kinetic description breaks down.

There are also other directions in which one can extend or improve our analysis. (i) It is straightforward to include the anisotropy of $n_{\mathbf{p}}^0$ analogous to Ref. [26]. This allows for the study of the coexistence (or competition) with the Weibel instability. (ii) The effect of the quark mass can be incorporated in the chirality evolution in Eq. (20), which makes μ_5 smaller. These analyses would be important to understand whether the CME remains an observable effect in the realistic situations.

We thank T.D. Cohen, S. Mahmoodifar, M.E. Shaposhnikov, and D.T. Son for useful conversations, and K. Fukushima for comments. N.Y. is supported by JSPS Research Fellowships for Young Scientists. N.Y. thanks the hospitality of the Quark-Hadron Theory Group and Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI) at Nagoya University, where this work was initiated.

-
- [1] A. Vilenkin, *Phys. Rev. D* **22**, 3080 (1980).
 - [2] H.B. Nielsen and M. Ninomiya, *Phys. Lett.* **130B**, 389 (1983).
 - [3] A. Y. Alekseev, V. V. Cheianov, and J. Frohlich, *Phys. Rev. Lett.* **81**, 3503 (1998).
 - [4] D. T. Son and A. R. Zhitnitsky, *Phys. Rev. D* **70**, 074018 (2004).
 - [5] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, *Phys. Rev. D* **78**, 074033 (2008).
 - [6] D. T. Son and P. Surowka, *Phys. Rev. Lett.* **103**, 191601 (2009).
 - [7] D. T. Son and N. Yamamoto, *Phys. Rev. Lett.* **109**, 181602 (2012).
 - [8] I. Zahed, *Phys. Rev. Lett.* **109**, 091603 (2012).

- [9] M. A. Stephanov and Y. Yin, *Phys. Rev. Lett.* **109**, 162001 (2012).
- [10] D. T. Son and N. Yamamoto, *Phys. Rev. D* **87**, 085016 (2013).
- [11] J.-W. Chen, S. Pu, Q. Wang, and X.-N. Wang, *Phys. Rev. Lett.* **110**, 262301 (2013).
- [12] R. Loganayagam and P. Surowka, *J. High Energy Phys.* **04** (2012) 097.
- [13] J.-H. Gao, Z.-T. Liang, S. Pu, Q. Wang, and X.-N. Wang, *Phys. Rev. Lett.* **109**, 232301 (2012).
- [14] D. Xiao, M.-C. Chang, and Q. Niu, *Rev. Mod. Phys.* **82**, 1959 (2010).
- [15] G.E. Volovik, *The Universe in a Helium Droplet* (Clarendon Press, Oxford, 2003).
- [16] It should be remarked that μ_5 is not associated with any conserved charge owing to the axial anomaly, so that it cannot be treated as the usual chemical potential. Still μ_5 can be introduced as an external parameter of a system if its variation is slow enough, as will be verified *a posteriori* below (see also Ref. [17]).
- [17] D. Kharzeev and A. Zhitnitsky, *Nucl. Phys.* **A797**, 67 (2007).
- [18] J.-P. Blaizot and E. Iancu, *Phys. Rep.* **359**, 355 (2002).
- [19] A. N. Redlich and L. C. R. Wijewardhana, *Phys. Rev. Lett.* **54**, 970 (1985).
- [20] V. A. Rubakov, *Prog. Theor. Phys.* **75**, 366 (1986).
- [21] M. Joyce and M. E. Shaposhnikov, *Phys. Rev. Lett.* **79**, 1193 (1997).
- [22] M. Laine, *J. High Energy Phys.* **10** (2005) 056.
- [23] A. Boyarsky, J. Frohlich, and O. Ruchayskiy, *Phys. Rev. Lett.* **108**, 031301 (2012).
- [24] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, *Nucl. Phys.* **A803**, 227 (2008).
- [25] J. Charbonneau and A. Zhitnitsky, *J. Cosmol. Astropart. Phys.* **08** (2010) 010.
- [26] P. Romatschke and M. Strickland, *Phys. Rev. D* **68**, 036004 (2003).
- [27] P. B. Arnold, J. Lenaghan, and G. D. Moore, *J. High Energy Phys.* **08** (2003) 002.
- [28] D. E. Kharzeev and H. J. Warringa, *Phys. Rev. D* **80**, 034028 (2009).
- [29] E. S. Weibel, *Phys. Rev. Lett.* **2**, 83 (1959).
- [30] H. C. Spruit, *AIP Conf. Proc.* **983**, 391 (2008).
- [31] G. Baym, H. Monien, C. J. Pethick, and D. G. Ravenhall, *Phys. Rev. Lett.* **64**, 1867 (1990).
- [32] A. Selikhov and M. Gyulassy, *Phys. Lett. B* **316**, 373 (1993).
- [33] H. Heiselberg, *Phys. Rev. Lett.* **72**, 3013 (1994).
- [34] D. Bodeker, *Phys. Lett. B* **426**, 351 (1998).
- [35] P. B. Arnold, D. T. Son, and L. G. Yaffe, *Phys. Rev. D* **59**, 105020 (1999).
- [36] We note that $\tau_{\text{QCD}} \sim \tau_{\text{large}}$ and $1/k_{\text{inst}} \ll l_{\text{large}}$, where $\tau_{\text{large}} \sim l_{\text{large}} \sim 1/(\alpha_s^2 \mu_5 \ln \alpha_s^{-1})$ is the mean free time (path) for momentum transfer (or large-angle scatterings); the typical time and length scales of the QCD chiral plasma instability are thus smaller than those of hydrodynamics, and hydrodynamic modes, such as the chiral magnetic wave [37,38], are irrelevant at least in the weak-coupling regime.
- [37] G. M. Newman, *J. High Energy Phys.* **01** (2006) 158.
- [38] D. E. Kharzeev and H.-U. Yee, *Phys. Rev. D* **83**, 085007 (2011).
- [39] At this stage, the magnetic mass of order $O(\alpha_s T)$ may affect the scenario of the instability for $\mu_5 \sim T$, while it would be irrelevant for $\mu_5 \gg T$. In the former case, the nonperturbative dynamics at $O(\alpha_s T)$ is described by a local Langevin-type effective theory [34,35] with parity violating effects. This will be reported elsewhere.