Cavity Optomechanics with Synthetic Landau Levels of Ultracold Fermi Gas

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Ultracold fermionic atoms placed in a synthetic magnetic field arrange themselves in Landau levels. We theoretically study the optomechanical interaction between the light field and collective excitations of such fermionic atoms in synthetic magnetic field by placing them inside a Fabry-Perot cavity. We derive the effective Hamiltonian for particle hole excitations from a filled Landau level using a bosonization technique and obtain an expression for the cavity transmission spectrum. Using this we show that the cavity transmission spectrum demonstrates cold atom analog of Shubnikov–de Haas oscillation in electronic condensed matter systems. We discuss the experimental consequences for this oscillation for such a system and the related optical bistability.

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By allowing an ultracold atomic ensemble to interact with a selected mode of a high-finesse cavity it is possible to probe the quantum many-body state of such an ultracold atomic system. The resulting cavity optomechanics or cavity quantum electrodynamics with ultracold atoms has recently witnessed a significant development [1]. The experimental successes include the coupling of collective density excitation of an ultracold bosonic condensate with a single-cavity mode and observation of the coupled dynamics through cavity transmission [2], a strongly coupled cavity mode with a highly localized ultracold atomic condensate trapped inside a single antinode of a cavity field [3], demonstration of strong ultracold atom-cavity-coupling-induced optical nonlinearity even at low photon density [4], and selected atom-photon coupling of a single atomic ensemble in a multiensemble system [5] to name a few.

In another development, there has been significant experimental and theoretical progress in studying the effect of optically induced artificial or synthetic gauge field [6] on such neutral atoms, making it a playground for quantum simulation of phenomena that occur when an electronic condensed matter system is placed in a real magnetic field. The experimental achievements in this direction include the observation of vortices, Abrikosov vortex lattice in trapped ultracold atomic superfluid initially by achieving the synthetic magnetic field through the rotation of the trap [7], and later by Raman laser induced spatially varying coupling of the hyperfine states of such ultracold atoms [8]. A more recent development in this direction includes the creation of optical flux lattices [9], realization of spin-orbit coupling for such neutral ultracold bosonic [10] and fermionic atoms [11] with the possibility of creating ultracold atomic analogs of topological condensed matter phases [12].

This Letter aims to combine these two developments by considering a system of such ultracold atoms trapped inside a high-finesse Fabry-Perot cavity interacting with a single-cavity mode (see Fig. 1), additionally, in the presence of a synthetic magnetic field. Specifically, we consider the case of ultracold fermionic atoms [13] in a synthetic magnetic field [14,15] such that a set of Landau levels (LLs) can be filled according to the Pauli principle. We consider the coupling between the bosonic particle-hole-like excitations from such filled Landau levels of fermionic atoms with the cavity mode. We find that the atom-photon coupling explicitly shows Landau level degeneracy and has finite discontinuities at certain values of artificial magnetic field strength that resembles the well-known Shubnikov–de Haas oscillation [16] in a condensed matter system. The cavity transmission spectrum shows optical bistability, a hallmark of optical nonlinearity in such a cavity system [17], but now the features of the bistable curve also reveal the Landau level structure. Our results suggest that cavity optomechanics with such atomic Landau levels can be a powerful probe for ultracold atoms in synthetic gauge field.

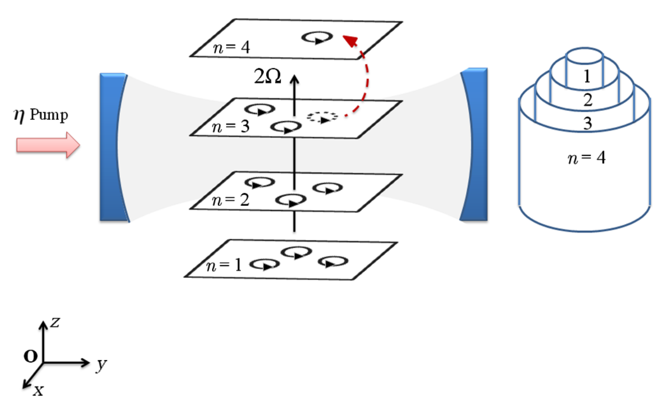

FIG. 1 (color online). Schematic diagram of the system considered. The concentric cylindrical surfaces represent the Fermi surfaces that correspond to Landau levels (LLs) of fermionic atoms inside the cavity with LL quantum number n. The particle hole excitation from the last filled Landau level is also shown.

We consider a two-dimensional system of N ultracold neutral fermionic two-level atoms each of mass M subjected to a synthetic magnetic field [8,14,15,18], placed inside a Fabry-Perot cavity of area $\mathcal{A}$ which is driven at the rate of η by a pump laser of frequency ω_p and wave vector $\boldsymbol{K} = (K_x, K_y)$. The atoms have transition frequency ω_a, and interact strongly with a single standing wave empty cavity mode of frequency ω_c. We take the artificial magnetic field as $2\Omega\hat{z}$. We also ignore the effective trap potential assuming it is shallow enough in the bulk. The resulting single particle Hamiltonian is analogous to the Landau problem of a charged particle in a transverse magnetic field (for details, see Ref. [19]) that can be written as

$$H_L = \frac{1}{2M}\mathbf{\Pi}^2, \tag{1}$$

where $\mathbf{\Pi} = \mathbf{p} - M\mathbf{A}$ is the kinetic momentum, with the effective vector potential $\mathbf{A} = \mathbf{\Omega} \times \mathbf{r}$ in symmetric gauge. The eigenstates of this Hamiltonian are Landau levels with effective cyclotron frequency $\omega_0 = 2\Omega$ and eigenenergies $E_{n,m} = 2\hbar\Omega(n + 1/2)$. The effective magnetic length in this problem is $l_0 = \sqrt{\hbar/2M\Omega}$.

If the pump laser frequency ω_p is far detuned from the atomic transition frequency ω_a, the excited electronic state of the two level atoms can be adiabatically eliminated. It is assumed that such atoms interact dispersively with the cavity field, taken to be single mode. In the dipole and rotating wave approximation, we get the effective system Hamiltonian (details in Ref. [19])

$$\hat{H}_{\rm eff} = \hat{H}_L + \hat{H}_I + \hat{H}_C, \tag{2}$$

with

$$\hat{H}_L = \int d^2\mathbf{r}\hat{\Psi}^\dagger(\boldsymbol{r})[\hat{\mathbf{\Pi}}^2/2M]\hat{\Psi}(\boldsymbol{r}), \tag{3}$$

$$\hat{H}_I = \int d^2\mathbf{r}\hat{\Psi}^\dagger(\boldsymbol{r})[\hbar U_0 \cos^2(\mathbf{K}\cdot\mathbf{r})\hat{a}^\dagger\hat{a}]\hat{\Psi}(\boldsymbol{r}), \tag{4}$$

$$\hat{H}_C = \hbar\Delta_c\hat{a}^\dagger\hat{a} - \imath\hbar\eta(\hat{a} - \hat{a}^\dagger). \tag{5}$$

Here, $U_0 = g_0^2/\Delta_a$ is the effective light-matter coupling constant with g_0 a single photon Rabi frequency, $\Delta_a = \omega_p - \omega_a$. Here, $\hat{H}_L$ is the atomic Hamiltonian in the synthetic magnetic field, $\hat{H}_C$ captures the dynamics of the cavity photons with $\Delta_c = \omega_c - \omega_p$. $\hat{H}_I$ is the term that describes interaction between atom and the cavity mode. The atomic field operator in the Landau level basis (symmetric gauge) is given as

$$\hat{\Psi}(\boldsymbol{r}) = \sum_{m,n}\hat{c}_{n,m}\langle\mathbf{r}|n,m\rangle = \sum_{m,n}\frac{e^{-|z|^2/4l_0^2}}{\sqrt{2\pi l_0^2}}G_{m+n,n}(iz/l_0)\hat{c}_{n,m}, \tag{6}$$

with

$$\begin{aligned}&\{\hat{c}^\dagger_{n,m}, \hat{c}^\dagger_{n',m'}\} = \{\hat{c}_{n,m}, \hat{c}_{n',m'}\} = 0,\\ &\{\hat{c}^\dagger_{n,m}, \hat{c}_{n',m'}\} = \delta_{n,n'}\delta_{m,m'},\end{aligned} \tag{7}$$

with $z = x + \imath y$. $|n,m\rangle$ is the Landau eigenket. $\langle\mathbf{r}|n,m\rangle$ is the symmetric-gauge wave function. $(e^{-|z|^2/4l_0^2}/\sqrt{2\pi l_0^2})G_{n+m,m}(iz/l_0)$ are two-dimensional harmonic oscillator wave functions whose properties are given in [19].

$\hat{c}^\dagger_{n,m}$ is the fermionic creation operator that creates the state $|n,m\rangle$, namely, a fermion in the nth LL, with the guiding center m obeying (7) with $n = 0, 1, 2, \ldots, \nu - 1$ and $m = 0, 1, 2, \ldots, N_\phi - 1$. $\nu = N/N_\phi$ is called the filling factor, where $N_\phi = \mathcal{A}/(2\pi l_0^2)$ is the degeneracy of each Landau level. The atomic Hamiltonian $(\hat{H}_L)$ can be diagonalized in the Landau level basis yielding $\hat{H}_L = \hbar\omega_0\sum_{m,n=0}^\infty(n + (1/2))\hat{c}^\dagger_{n,m}\hat{c}_{n,m}$. Using $4\cos^2(\mathbf{K}\cdot\mathbf{r}) = 2 + 2\cos(2\mathbf{K}\cdot\mathbf{r}) = 2 + e^{-\imath 2\mathbf{K}\cdot\mathbf{r}} + e^{\imath 2\mathbf{K}\cdot\mathbf{r}}$, $\hat{H}_I$ in the Landau basis can be written as

$$\begin{aligned}\hat{H}_I = \frac{\hbar U_0}{4}\Big\{\hat{N} + \sum\hat{c}^\dagger_{n',m'}\hat{c}_{n,m}e^{-2(|K|l_0)^2}\\ \times[G_{n',n}(2K^*l_0)G_{m',m}(2Kl_0)\\ + G_{n',n}(-2K^*l_0)G_{m',m}(-2Kl_0)]\Big\}\hat{a}^\dagger\hat{a},\end{aligned} \tag{8}$$

where $K = K_x + \imath K_y$, $K^* = K_x - \imath K_y$, $|K|^2 = K_x^2 + K_y^2$, and the summations are done over all available n, n', m, m'. If we assume that the interaction time between the cavity and ultracold fermions is much shorter than any time scale associated with the reorganization of the atomic ground state in the presence of the standing wave inside the cavity, the role of the interaction Hamiltonian (8) is restricted to transfer momentum $\pm 2|K|$ to the particle-hole excitation above the Fermi level.

By looking at the cavity transmission spectrum, we are interested in studying such low-energy excitations above an integer number of filled Landau level. Such particle-hole excitations are bosonic in nature and are known as magnetic exciton in the literature of quantum Hall systems [20]. In the absence of atom-photon interaction, the ground state of our system is a direct product state of photonic vacuum and excitonic vacuum, obtained by completely filling the first ν Landau levels of each guiding center:

$$|\mathrm{GS}\rangle = \prod_{m=0}^{N_\phi-1}\prod_{n=0}^{\nu-1}c^\dagger_{n,m}|0\rangle. \tag{9}$$

These inter-Landau-level excitations only involve the change in the Landau level index; they can be studied using the language of bosonization [21] by introducing the bosonic operator

$$\hat{b}_p^\dagger(\mathbf{k}) = \frac{1}{\sqrt{pN_\phi J_p^2(kR_\nu)}} e^{-(l_0|k|)^2/2} \sum_{n=0}^{\infty} \sum_{m,m'=0}^{\infty} \hat{c}^\dagger_{n+p,m'} \hat{c}_{n,m} \times [G_{n+p,n}(l_0 k^*) G_{m',m}(l_0 k)]. \quad (10)$$

This operator creates a bosonic particle-hole excitation by shifting an atom from the nth LL to the $n+p$th LL, where J_p is the Bessel function of the first kind, $R_\nu = \sqrt{2\nu} l_0$, and obeys

$$[\hat{b}_p(\mathbf{k}), \hat{b}_q(\mathbf{k}')] = [\hat{b}_p^\dagger(\mathbf{k}), \hat{b}_q^\dagger(\mathbf{k}')] = 0,$$
$$[\hat{b}_p(\mathbf{k}), \hat{b}_q^\dagger(\mathbf{k}')] = \delta(\mathbf{k}-\mathbf{k}')\delta_{p,q}. \quad (11)$$

Using the commutators of the bosonic operator (11), the bosonized version of the Landau level Hamiltonian (3) of ultracold fermions can be written as

$$\hat{H}_L = \hbar \sum_{p=1}^{\infty} \sum_{\mathbf{k}} p\omega_0 b_p^\dagger(\mathbf{k}) b_p(\mathbf{k}). \quad (12)$$

The atom-photon interaction (8) can similarly be rewritten in terms of the bosonic operator (10) as

$$\hat{H}_I = \frac{\hbar U_0}{4} \hat{N} \hat{a}^\dagger \hat{a} + \frac{\hbar U_0}{4} \sum_{p=1}^{\infty} \sqrt{N_\phi p J_p^2(2KR_\nu)} \times [\hat{b}_p^\dagger(2\mathbf{K}) + \hat{b}_p(2\mathbf{K})] \hat{a}^\dagger \hat{a}. \quad (13)$$

The derivation of the Hamiltonians (12) and (13) from (8) is given in the Supplemental Material [19]. The bosonized effective Hamiltonian (2) of the atom-photon system thus becomes

$$\hat{H}_{\text{eff}} = \hbar \sum_{p=1}^{\infty} \left\{ \sum_{\mathbf{k}} p\omega_0 \hat{b}_p^\dagger(\mathbf{k}) \hat{b}_p(\mathbf{k}) + \delta_p^\nu \sqrt{p} [\hat{b}_p^\dagger(2\mathbf{K}) + \hat{b}_p(2\mathbf{K})] \hat{a}^\dagger \hat{a} \right\} + \hbar\Delta \hat{a}^\dagger \hat{a} - \imath\hbar\eta(\hat{a} - \hat{a}^\dagger), \quad (14)$$

$$\delta_p^\nu = \frac{U_0}{4} \sqrt{N_\phi J_p^2(2KR_\nu)}. \quad (15)$$

The above Hamiltonian is one of the central result of this Letter. Here the operator $\hat{N}$ is replaced with its steady-state expectation value; subsequently, the term $[(N\hbar U_0)/2]\hat{a}^\dagger a$ is incorporated into $\hbar\Delta_c a^\dagger a$ of H_C to get the effective cavity detuning $\Delta = \omega_c - \omega_p + (NU_0/2)$.

δ_p^ν is the atom-photon coupling constant that couples the excited levels with the photon field. Figure 2 depicts its variation with the field strength when an atom gets excited from the filled LL to the next unoccupied LL. For that purpose we choose experimentally achievable parameters: $\lambda = 500$ nm$(K \simeq 10^7\ \text{m}^{-1})$, $\mathcal{A} \simeq (30\ \mu\text{m})^2$, $\kappa = 2\pi$ MHz, atomic mass $M = 1.5 \times 10^{-25}$ kg, $N = 2000$, $g_0 = 2\pi \times 10$ MHz, pump-atom detuning $\omega_p - \omega_a = 2\pi \times 50$ GHz. It linearly depends on atom-photon coupling constant U_0 and is enhanced by the Landau level degeneracy $\sqrt{N_\phi}$, which is different as compared to the case of ordinary fermions [17] and akin to the scaling of the atom-photon

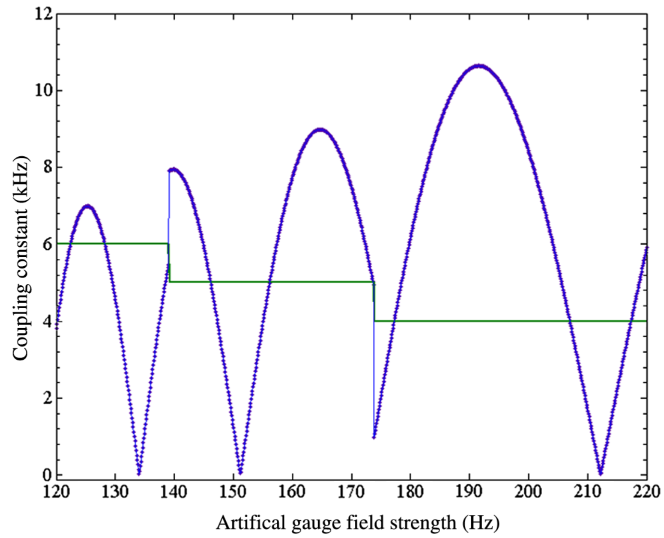

FIG. 2 (color online). Variation of coupling constant (for $p = 1$) with synthetic field strength. The green steps in the background correspond to the corresponding first empty LL, $\nu = 6, 5, 4$.

coupling constant by $\sqrt{N}$ for an N-boson condensate [2,3]. In Fig. 2, with increasing Ω, the coupling constant oscillates along with jump discontinuities. This is the usual Shubnikov–de Haas effect [16], now occurring for a synthetic magnetic field when the Fermi level makes a jump to the previous level at some increased value of the field.

The other important feature, the oscillatory behavior of δ_p^ν, can be attributed to the length scales associated with the current problem. In the presence of synthetic gauge field the cyclotron radius of the ultracold atoms ($l_0 \sim 200$–800 nm) is comparable with the wavelength of the probing photon ($\lambda \sim 600$ nm). With an increase in field strength the cyclotron radius decreases and the number of wavelengths that fit within this radius also changes, leading to the oscillatory behavior of the atom-photon coupling strength as a function of the field strength. In comparison, in the corresponding electronic problem the electron cyclotron radius is much smaller ($l_0 \sim 20$ nm), so the incident photon cannot actually *see* the individual cyclotron orbit, making such oscillation hard to observe in electronic LL spectroscopy [22,23].

The oscillation of the coupling constant as a function of the strength of the synthetic gauge field can be obtained from the steady-state cavity transmission spectrum, an experimentally measurable quantity. The Hamiltonian (14) represents a coupled system of particle-hole excitations (magnetic exciton) and photons. The steady-state solution of Heisenberg equations for operators associated with the dynamics of exciton and photon yields the cavity transmission spectrum. For this purpose we introduce phase space quadrature variables

$$\hat{X}_L = [\hat{b}_p^\dagger(2\mathbf{K}) + \hat{b}_p(2\mathbf{K})]/\sqrt{2};$$
$$\hat{P}_L = \imath[\hat{b}_p^\dagger(2\mathbf{K}) - \hat{b}_p(2\mathbf{K})]/\sqrt{2},$$

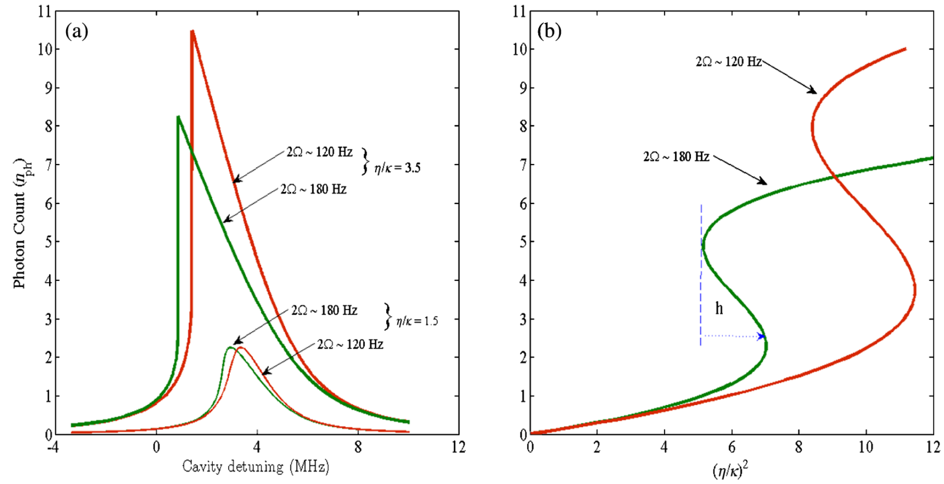

FIG. 3 (color online). Steady-state interactivity photon number as a function of (a) pump cavity detuning for a set of synthetic field and η/κ; (b) pump rate for the same set of synthetic fields and cavity detuning $\Delta = 2\pi \times 2.5$ MHz.

which obey the standard commutator $[\hat{X}_L, \hat{P}_L] = \imath$. The resulting Heisenberg equations are

$$\begin{aligned} \frac{d\hat{X}_L}{dt} &= p\omega_0 \hat{P}_L, \quad \frac{d\hat{P}_L}{dt} = -p\omega_0 \hat{X}_L - \delta_p^\nu \sqrt{2p}\,\hat{a}^\dagger \hat{a}, \\ \frac{d\hat{a}}{dt} &= -\imath \sum_{p=1}^{\infty} \delta_p^\nu \sqrt{2p}\,\hat{X}_L \hat{a} - \imath \Delta \hat{a} + \eta - \kappa \hat{a} + \sqrt{2\kappa}\,\hat{a}_{in}. \end{aligned} \tag{16}$$

Here, κ is the cavity decay rate and $\hat{a}_{in}$ denotes a Markovian noise operator [24] with zero mean, correlation $\langle \hat{a}_{in}^\dagger(t) \hat{a}_{in}(t') \rangle = 2\kappa \delta(t - t')$, and $\langle \hat{a}_{in}(t) \hat{a}_{in}(t') \rangle = 0$, so it can be dropped for steady-state analysis. The steady-state solutions are given by

$$\begin{aligned} \hat{P}_L^{(s)} &= 0, \qquad \hat{X}_L^{(s)} = -\frac{\delta_p^\nu \sqrt{2p}}{p\omega_0} \hat{a}^{\dagger(s)} \hat{a}^{(s)}, \\ \hat{a}^{(s)} &= \frac{\eta}{\kappa + \imath(\Delta - S_\nu \hat{a}^{\dagger(s)} \hat{a}^{(s)})}, \end{aligned} \tag{17}$$

with [25]

$$S_\nu = \frac{2}{\omega_0} \sum_{p=1}^{\infty} (\delta_p^\nu)^2 = \frac{U_0^2 \mathcal{A} M}{32\pi\hbar} [1 - J_0^2(2KR_\nu)].$$

Therefore, the steady-state intercavity photon number is

$$\hat{n}_{ph} = \hat{a}^{\dagger(s)} \hat{a}^{(s)} = \frac{\eta^2}{\kappa^2 + (\Delta - S_\nu \hat{n}_{ph})^2}. \tag{18}$$

The cavity transmission spectrum is given by its expectation value that follows

$$S_\nu^2 n_{ph}^3 - 2S_\nu \Delta n_{ph}^2 + (\kappa^2 + \Delta^2) n_{ph} = \eta^2. \tag{19}$$

Such a nonlinear cubic equation is characteristic of optical multistability [2,4]. Figure 3 shows the behavior of a steady-state mean photon number as a function of pump rate, and Δ. To understand this multistability we study the fluctuation around the steady state through a linear stability analysis. To that end, we write $n_{ph} = n_{ph}^s + \delta n_{ph}$ in Eq. (19), where n_{ph}^s corresponds to the steady-state intercavity photon number plotted in Fig. 3(b). Then only the

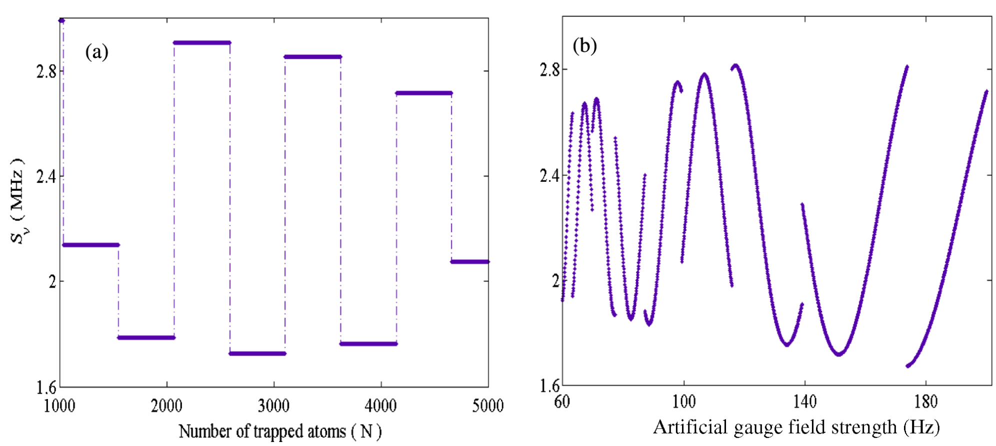

FIG. 4 (color online). Variation of S_ν with (a) number of trapped atoms and (b) gauge field strength.

terms linear in $\delta n_{\rm ph}$ are kept in the resulting equation, and the steady-state solution (19) is substituted in it to get

$$[3S_\nu^2(n_{\rm ph}^s)^2 - 4S_\nu \Delta n_{\rm ph}^s + (\kappa^2 + \Delta^2)]\delta n_{\rm ph} = 0. \tag{20}$$

The solution of this equation defines the upper and lower bounds of the unstable regime as the turning points of the plot in Fig. 3(b). In the region between these two turning points, the intercavity photon number is a decreasing function of the cavity parameter $(\eta/\kappa)^2$ and corresponds to the unstable solution [26–28].

A more formal way of doing the linear stability analysis is through the Heisenberg equation of motion of the operators, namely, setting $\mathcal{O}(t) = \mathcal{O}^{(s)} + \delta\mathcal{O}(t)$. However, for the current problem, $\mathcal{O}(t) = [\hat{X}_L(t)^{p=1}, \hat{P}_L(t)^{p=1}, \hat{X}_L(t)^{p=2}, \hat{P}_L(t)^{p=2}, \ldots, \hat{X}(t), \hat{P}(t)]^T$, with $\hat{X} = (\hat{a}^\dagger + \hat{a})/\sqrt{2}$, $\hat{P} = \imath(\hat{a}^\dagger - \hat{a})/\sqrt{2}$ being the cavity quadratures. The linear stability analysis gets contributions from all p's and the resulting stability matrix is infinite dimensional and cannot be handled in the same way as one in the absence of such synthetic gauge field [17]. A more detailed discussion on this issue is given in the Supplemental Material [19]. The distance between the two turning points of the bistability curve in Fig. 3(b) is calculated to be

$$h(S_\nu) = \frac{4(\Delta^2 - 3\kappa^2)^{3/2}}{27S_\nu}. \tag{21}$$

An experimentally obtained cavity spectrum [2–4] can be used to extract the corresponding $h(S_\nu)$, and hence the corresponding S_ν, which can be compared with the theoretical value obtained from Eq. (17). Figure 4 provides the informations about the Landau levels inside the cavity.

To summarize, we have shown that cavity optomechanics could be a very useful tool to explore an ultracold atomic system in a synthetic gauge field, providing us direct access to the atomic Landau levels. Future studies may consider the system in the limit when atom-photon interaction will lead to the reorganization of the many-body atomic state in the presence of the standing wave in the cavity as well as a similar problem for the bosonic systems, where bosons will prefer to stay in the lowest Landau level.

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