Duality and Decay of Macroscopic *F* Strings

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We study the decay of fundamental string loops of arbitrary size $L/\min(n, m) \gg \sqrt{\alpha'}$, labeled by ($n, m$; $\lambda_n, \bar{\lambda}_m$), where n, m correspond to left- and right-mover harmonics and $\lambda_n, \bar{\lambda}_m$ to polarization tensors, and find that a description in terms of the recent coherent vertex operator construction of Hindmarsh and Skliros is computationally very efficient. We primarily show that the decay rates and mass shifts of vertex operators (n, m; $\lambda_n, \bar{\lambda}_m$) and their "duals" ($n, m$; $\lambda_n, \bar{\lambda}_m^*$) are equal to leading order in the string coupling, implying, for instance, that decay rates of epicycloids equal those of hypocycloids. We then compute the power and decay rates associated with massless IR radiation for the trajectory ($1, 1$; $\lambda_1, \bar{\lambda}_1$), and find that it is precisely reproduced by the low energy effective theory of Dabholkar and Harvey. Guided by this correspondence, we conjecture the result for arbitrary trajectories (n, m; $\lambda_n, \bar{\lambda}_m$) and discover a curious relation between gravitational and axion plus dilaton radiation. It is now possible to start exploring string evolution in regimes where a low energy effective description is less useful, such as in the vicinity of cusps.

There has been growing interest in the properties of macroscopic and/or excited string states, e.g., in the context of AdS/CFT, see Ref. [1] and references therein, where a precise understanding of the string-gauge theory spectrum is being explored, as a probe of black hole physics [2], in collider physics [3,4] when the string scale is in the TeV range, and in higher-spin gauge theories [4,5], where an intriguing suggestion is that massive Regge excitations may arise from spontaneous breaking of a higher-spin gauge symmetry. We draw our main motivation from yet another arena, the phenomenologically interesting possibility of cosmic superstrings [6,7], which provides a perfect context to probe stringy physics [7]. Recently, a description of generic excited strings in terms of covariant coherent vertex operators became available [8,9], and below we initiate a study of their interactions.

Total and partial decay widths of macroscopic and/or excited strings largely determine their phenomenological relevance at low energies. Most massive closed strings are unstable and decay [10–17], typical decay rates being (when massive radiation is negligible), $\Gamma \sim G_D \mu^2 (M/\mu)^{5-D}$, with G_D Newton's constant in D noncompact dimensions, $\mu = 1/(2\pi\alpha')$ the string tension, and M the mass of the decaying string. The associated lifetime is $\tau \sim (G_D\mu)^{-1}(M/\mu)^{D-3}$, and so low grand unified theory (GUT) scale strings the size of a galaxy, the Solar System, or an atom would survive $\tau \sim 10^{14}$ yr, 10^4 yr, and 10 ps, respectively, in $D = 4$. There are also more stable near-Bogomol'nyi-Prasad-Sommerfeld [18] trajectories (see, e.g., [13]) and approximate nonrenormalization theorems [15]. Stable Bogomol'nyi-Prasad-Sommerfeld states in turn provide a window into nonperturbative physics [19–22]. D-string decay at strong coupling can also be extracted from F-string decay at weak coupling [21], whereas a handle at finite coupling may also be within reach [23].

Once formed, unstable strings provide a source of radiation that generically is strongly anisotropic [24–26]. Given

a large enough sample of such strings, a classical analysis (which, however, neglects radiative backreaction [27] and massive emission) shows that we may be able to observe this radiation in the form of gravitational wave bursts [26], or cosmic rays, see, e.g., [7], and so predicting the precise spectrum is crucial.

Below we study the decay of highly excited fundamental string loops. Traditionally, these are described in a mass eigenstate basis, and calculations almost exclusively [10–17] focus on leading Regge trajectory states which do not exhibit generic features (such as nondegenerate cusps [28]). Although physical vertex operators for arbitrary mass eigenstates are known [9], they are not a natural basis for highly excited strings as the resulting amplitudes generically become unwieldy. Below we show that a coherent state basis [8], a complete set of which was constructed in [9], is much more natural. For world sheet embeddings X^M: $\Sigma \rightarrow \mathbb{R}^{D-1,1} \times T^{26-D}$, in a flat string background, $G_{MN} = \eta_{MN}$, with vanishing Kalb-Ramond 2-form $B_{(2)} = 0$ and constant dilaton $\Phi = \Phi_0$ (with string coupling $e^{\Phi_0} \ll 1$), we consider the coherent vertex operators [8,9]:

$$\mathcal{O}(z, \bar{z}) =: C \int_0^{2\pi} \bar{d}s \exp\left(\frac{i}{n} e^{ins} \lambda_n \cdot D_z^n X e^{-inq \cdot X(z)}\right)$$
$$\times \exp\left(\frac{i}{m} e^{-ims} \bar{\lambda}_m \cdot \bar{D}_{\bar{z}}^m X e^{-imq \cdot X(\bar{z})}\right) e^{ip \cdot X(z,\bar{z})}, \quad (1)$$

when $\lambda_n \cdot p = 0$ and p^μ, $q^\mu \in \mathbb{R}^{D-1,1}$. The polarization tensors λ_n^i span a $\mathbb{R}^{D-2}$ subspace. $\mathcal{O}(z, \bar{z})$ will be a $(1, 1)$ conformal primary [29] when [9] $\lambda_n^2 = \lambda_n \cdot q = 0$, $p^2 = 2p \cdot q = 4/\alpha'$, and $q^2 = 0$. (Identical remarks hold for the antichiral half, $\bar{\lambda}_m^i$.)

We define $D_z^n \equiv \sum_{r=1}^n (S_{n-r}(a_s)/(r-1)!)\partial_z^r$, with $S_m(a_s)$ elementary Schur polynomials [30] and $a_s = -(1/s!)inq \cdot \partial^s X(z)$, and likewise for the antiholomorphic quantities [9]. The operator-product expansion, $\mathcal{O}^\dagger(z, \bar{z}) \cdot$

$\mathcal{O}(0,0) \cong g_D^2/|z|^4$, fixes the normalization [31] and unitarity [18] sets $(2\pi g_D)^2 = 8\pi G_D$.

The momentum expectation value of $\mathcal{O}(z,\bar{z})$ is $\langle \hat{p}^\mu \rangle = p^\mu - \ell^2 q^\mu$, with $\ell \equiv |\lambda_n| = |\bar{\lambda}_m|$. We work throughout in the "rest frame" [9], $\langle \hat{p}^N \rangle = M\delta_0^N$, with $M \equiv \mu L$ the invariant mass. The string length L is then determined from the Virasoro constraints, $L = 4\pi\sqrt{\alpha'\ell^2 - \alpha'}$.

When $\ell \gg 1$, rest-frame vertex operators (1) are in one-to-one correspondence with classical string trajectories [8], $X^N = (X^0, X^i, X^{D-1}, X^a)$, where $X^0 = -iM\ln z\bar{z}$,

$$X^i = \frac{i}{n}(\lambda_n^i z^{-n} - \lambda_n^{*i}z^n) + \frac{i}{m}(\bar{\lambda}_m^i \bar{z}^{-m} - \bar{\lambda}_m^{*i}\bar{z}^m), \quad (2)$$

and $X^a = X^{D-1} = 0$, which are closely related to the trajectories in [25]; here, $\alpha' = 2$. The indices $\mu = (0, i, D-1)$ and a span $\mathbb{R}^{D-1,1}$ and T^{26-D}, respectively. When the polarization tensors satisfy the aforementioned relations, the equations of motion, $\partial_z\partial_{\bar{z}}X^N = 0$, and constraints, $(\partial_z X)^2 = (\partial_{\bar{z}}X)^2 = 0$, are satisfied. For general harmonics, n, m, with greatest common divisor g, denoted by $[n,m] = g$, there exist unique relatively prime positive integers u, w, such that $n = gu$ and $m = gw$, with $[u,w] = 1$. Redefining z, $\bar{z} \to z^g$, $\bar{z}^g$, we learn that loops distinguished by different values of g (with u, w fixed) are self-similar: increasing the "winding" number g [32] decreases their size, $\sim L/g$, their shape remaining fixed.

Given a set of trajectories $X = X_L(z) + X_R(\bar{z})$, the distinct trajectories $X' = X_L(z) - X_R(\bar{z}^{-1})$ are also physical; we call the latter *dual* trajectories, obtained from (2) or (1) by $(n, m; \lambda_n, \bar{\lambda}_m) \to (n, m; \lambda_n, \bar{\lambda}_m^*)$; see also [33,34].

To visualize some of the trajectories captured by $\mathcal{O}(z,\bar{z})$, consider $D = 4$ and pick a coordinate system such that $\lambda_n = (1/\sqrt{2})\ell(\hat{\mathbf{x}} + i\hat{\mathbf{y}})$ and $\bar{\lambda}_m = (1/\sqrt{2})\ell(-\hat{\mathbf{x}} + i\cos\psi\hat{\mathbf{y}})$, with $\psi = 0$ or π. The angular momentum is [9] $\langle J^{xy} \rangle = (1/2) \times [(1/n) - (\cos\psi/m)][(\alpha'/2)M^2 + 2]$, giving rise to leading Regge and various "sister" trajectories. When $\ell \gg 1$, the rms radius of these strings is [9] $\mathcal{R} = (L/4\pi\sqrt{2}) \times \sqrt{(1/n^2) + (1/m^2) - (\delta_{n,m}/nm)(1 + \cos\psi)\cos(4\pi nX^0/L)}$. Trajectories self-intersect if both u, $w > 1$ [25]. We label solutions by $(n, m; \psi)$, with two subclasses: $(n, m; 0)$ and $(n, m; \pi)$. If m is a divisor of n (i.e., $g = m$), $(n, m; 0)$ are *epicycloids* (with $n/m - 1$ cusps [35]) and $(n, m; \pi)$ *hypocycloids* (with $n/m + 1$ cusps), the two being dual in the above sense [36]. The simplest example of the former is a pulsating circle, $(1, 1; 0)$, followed by a rotating cardioid, $(2, 1; 0)$, etc., whereas the simplest example of the latter is a rotating folded string, $(1, 1; \pi)$, followed by a rotating deltoid, $(2, 1; \pi)$, etc., and similarly for $g = n$. In $D = 4$, all trajectories except $(n, n; 0)$ have permanent cusps; in $D > 4$, if $\lambda_n \cdot \bar{\lambda}_m^*$ and $\lambda_n \cdot \bar{\lambda}_m$ are *either* real or imaginary, cusps generically appear at discrete instants [37].

Decay rates may be extracted from the imaginary part of two-point amplitudes [10–15], $\Gamma = (1/M)\text{Im}\mathcal{M}$. The amplitudes $\mathcal{M}$ are in turn obtained by integrating over complex moduli τ and vertex insertion points z of a compact Riemann surface, and over matter X and b, c ghost contributions [29,38,39]. When strings in the loops go on shell, they can appear in final states. In the usual approach [10–15], dependence on the loop momenta is not manifest, and so one does not have a handle on the frequency and isotropy of the emitted radiation. We bypass this obstacle by considering instead the fixed (A-cycle) loop momentum amplitude [38] $\mathcal{M}(\mathbb{P})$ defined (at genus-1) by $\mathcal{M} \equiv \int(d^D\mathbb{P}/(2\pi)^D)\mathcal{M}(\mathbb{P})$, in terms of which,

$$\Gamma = \frac{1}{\mu L}\int\frac{d^D\mathbb{P}}{(2\pi)^D}\text{Im}\mathcal{M}(\mathbb{P}). \quad (3)$$

The total power emitted P is in turn obtained by means of $dP = \mathbb{P}^0 d\Gamma$. The quantities of interest will be the decay rates $d\Gamma/d\Omega_{S^{D-2}}$ and power emitted $dP/d\Omega_{S^{D-2}}$ per unit solid angle. We extract the contribution associated with decay channels of interest by factorizing $\mathcal{M}(\mathbb{P})$ on the corresponding poles, e.g., $\mathbb{P}^2 = 0$ for massless radiation.

It is worth emphasizing that the fixed-loop momenta amplitudes $\mathcal{M}(\mathbb{P})$ are also more natural than their integrated manifestations $\mathcal{M}$ in light of a chiral splitting theorem [40], and this will play an important role below.

To determine $\mathcal{M}(\mathbb{P})$, note that the ghost contribution amounts to an overall factor $|\eta(\tau)|^4$, with $\eta(\tau)$ the Dedekind eta function [38]. We then extract $\mathcal{M}(\mathbb{P})$ by inserting a momentum-conserving delta function into the path integral which only allows strings of momentum $\hat{\mathbb{P}} = \mu \oint_A(\partial X - \bar{\partial}X)$ to propagate through the loop [41],

$$i\delta^D(0)\mathcal{M}(\mathbb{P}) = \frac{1}{2}\int_{\mathcal{M}_1}d^2\tau|\eta(\tau)|^4\int_\Sigma d^2z$$
$$\times \int \mathcal{D}Xe^{-S_X}\delta^D(\mathbb{P} - \hat{\mathbb{P}})\mathcal{O}^\dagger(z,\bar{z})\mathcal{O}(z',\bar{z}'),$$
$$(4)$$

with z', $\bar{z}'$ fixed. Our gauge slice choice leads to a moduli space [42] $\mathcal{M}_1 \equiv \{|\tau|^2 \geq 1, -(1/2) \leq \text{Re}\tau \leq (1/2), \text{Im}\tau > 0\}$ and $\Sigma = \mathbb{C}/(\mathbb{Z} + \tau\mathbb{Z})$, with metric $ds^2 = dzd\bar{z}$. The remaining $\mathbb{Z}_2$ isometry leads to the factor of $1/2$ in Eq. (4), although the overall normalization is fixed by unitarity [43]. The Euclidean string action is $S_X = \mu \int_\Sigma d^2z\partial_z X^\mu \partial_{\bar{z}}X^\nu \delta_{\mu\nu}$, and Hermitian conjugation takes $(p^\mu, q^\mu; \lambda_n, \bar{\lambda}_m) \to (-p^\mu, -q^\mu; \lambda_n^*, \bar{\lambda}_m^*)$ in Eq. (1).

We use point splitting to integrate out X [44]. Paying careful attention to the combinatorics and normalization, the chirally split [40] two-point coherent state amplitude reads [45]:

$$\mathcal{M}(\mathbb{P}) = \sum_{K,W}\frac{1}{2}\int_{\mathcal{M}_1}d^2\tau\int_\Sigma d^2z\int_0^{2\pi}ds\,\mathcal{F}_n(z|\tau)\bar{\mathcal{F}}_m(\bar{z}|\bar{\tau}),$$
$$(5)$$

where, writing Fay's prime form [38] as $E = E(z, z')$, the chiral half reads:

$$\mathcal{F}_n(z|\tau) \equiv C\eta(\tau)^{-24} e^{\pi i \tau \mathbb{P}^2} E^{-p^2} e^{-2\pi i \mathbb{P} \cdot p(z-z')}$$

$$\times \exp\left\{ e^{ins} \frac{1}{n^2} |\lambda_n|^2 e^{2\pi i \mathbb{P} \cdot nq(z-z')} E^{2n} \mathcal{D}_z^n \mathcal{D}_{z'}^n \ln E \right\}$$

$$\times I_0\left(2 e^{ins/2} \frac{1}{n} |\mathbb{P} \cdot \lambda_n| e^{\pi i \mathbb{P} \cdot nq(z-z')} 2\pi E^n \mathcal{S}_{n-1} \right). \tag{6}$$

The quantities $\mathcal{S}_{n-\ell}$, $\mathcal{D}_z^n$ are equal to $S_{n-\ell}$, D_z^n above, but now $a_s \equiv (n/s!)\partial_z^s[\ln|E|^2 - 4\pi\mathbb{P} \cdot q \, \mathrm{Im}(z-z')]$. The $I_0(2x)$ are modified Bessel functions. The integers $K, W \in \mathbb{Z}^{26-D}$ are momentum and winding modes of A-cycle strings associated with T^{26-D}. The momenta are $\mathbb{P}^a \equiv (K/R)^a + [(WR)^a/2]$ and $\bar{\mathbb{P}}^a \equiv (K/R)^a - [(WR)^a/2]$, where $x^a \sim x^a + 2\pi R^a$. The antichiral half $\bar{\mathcal{F}}_m(\bar{z}|\bar{\tau})$ is in turn obtained by complex conjugation from $\mathcal{F}_n(z|\tau)$ and the replacements $(n, \lambda_n, \mathbb{P}) \to (m, \bar{\lambda}_m, \bar{\mathbb{P}})$ [46].

The normalization of (5) has been fixed by unitarity [45]; we factorize $\mathrm{Im}\mathcal{M}(\mathbb{P})$ on two tachyon poles, take $\ell \to 0$, and relate [43] this to the modulus squared of the three-tachyon amplitude [18], $\mathcal{M}_{ttt} = 8\pi g_D/\alpha'$.

Notice that (5) is invariant under $(n, m, \lambda_n, \bar{\lambda}_m) \to (n, m, \lambda_n, \bar{\lambda}_m^*)$, a powerful (albeit tree-level) statement relating decay rates (and mass shifts) of *distinct* states,

$$\frac{d\Gamma}{d\Omega_{S^{D-2}}}\bigg|_{(n,m,\lambda_n,\bar{\lambda}_m)} = \frac{d\Gamma}{d\Omega_{S^{D-2}}}\bigg|_{(n,m,\lambda_n,\bar{\lambda}_m^*)}; \tag{7}$$

so for instance, in $D = 4$, decay rates of epicycloids $(gu, g; 0)$ and the dual hypocycloids $(gu, g; \pi)$, remarkably, are equal, for all $g, u \in \mathbb{Z}^+$. This also generalizes the observation of Iengo and Russo [14], that decay rates of pulsating circles and folded strings are equal, and remains true (at one loop) for all vertex operators, $\mathcal{O}(z, \bar{z})$.

As an example, let us compute (3) explicitly for massless radiation for the trajectory $(1, 1, \lambda_1, \bar{\lambda}_1)$ and its dual $(1, 1, \lambda_1, \bar{\lambda}_1^*)$. The integrand of (5) consists of a linear superposition of terms $|e^{2\pi i(\tau - z)}|^{\mathbb{P}^2 + m_1^2} |e^{2\pi i z}|^{(k-\mathbb{P})^2 + m_2^2}$, there being a sum over on-shell masses m_1^2, m_2^2 of states that propagate in the loop, and over mass eigenstates of momentum $k = p - aq$, with $a = 0, 1, 2, \ldots$, of which the decaying coherent state is composed. We search for branch cuts in the complex k^0 plane, the discontinuity across which yields an imaginary part, $\mathrm{Im}\mathcal{M}(\mathbb{P})$ [47], whereby we analytically continue back to the Minkowski signature. Only the $m_1^2 = 0$ and $m_2^2 = 0$ terms contribute to massless radiation. We write the associated on-shell D momentum of the radiation as $\mathbb{P}^\mu = \mathbb{P}^0(1, \hat{\mathbb{P}}^j)$, with $\hat{\mathbb{P}}^2 = 1$. Omitting the details of the computation [45], in the IR region of the emission spectrum, $\mathbb{P}^0 \ll \mu L$, the resulting series can be resummed, leading precisely to the following expression for the power per unit solid angle in a direction $\hat{\mathbb{P}}$ into massless radiation of energy $\mathbb{P}^0 = 4\pi Nuwg/L$, with $N = 1, 2, \ldots$:

$$\frac{dP_N}{d\Omega_{S^{D-2}}}\bigg|_{m^2=0} = \frac{16\pi G_D \mu^2}{(2\pi)^{D-4}} \left(\frac{4\pi Nuwg}{L} \right)^{D-4-\delta} (Nuwg)^2 \left[J_{Nw}'^2(Nw\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_n|) + \left(\frac{1}{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_n|^{-2} - 1 \right) J_{Nw}^2(Nw\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_n|) \right]$$
$$\times \left[J_{Nu}'^2(Nu\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_m|) + \left(\frac{1}{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_m|^{-2} - 1 \right) J_{Nu}^2(Nu\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_m|) \right], \tag{8}$$

with $u, w, g = 1$, $\lambda_n = \ell\hat{\lambda}_n$, $\bar{\lambda}_m = \ell\hat{\bar{\lambda}}_m$, and $\delta = 0$. When $\delta = 1$, the above yields a decay rate ($d\Gamma_N/d\Omega_{S^{D-2}}$, probability per unit time per unit solid angle that a massless particle of energy $\mathbb{P}^0$ is emitted in direction $\hat{\mathbb{P}}$). We conjecture that Eq. (8) holds for arbitrary non-self-intersecting trajectories $(n, m; \lambda_n, \bar{\lambda}_m)$ provided perturbation theory remains valid. [The integers u, w, g are defined in terms of n, m above and the $J_n(z)$ are Bessel functions.]

The L dependence, $\Gamma \sim G_D \mu^2 L^{5-D} g^{D-3}$, agrees with [11] in $D = 10$. When massless radiation is the dominant decay channel, the L dependence of the total lifetime, $\tau \sim (G_D \mu g)^{-1}(L/g)^{D-3}$, is in agreement with [16] in $D = 10$ and [48] in $D = 4$; but see also [10].

The characteristic cusp signal [24,26] comes from the region $\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_n| = \sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_m| = 1$, which for large N leads to $dP_N/d\Omega_{S^{D-2}} \propto (Nuw)^{D-4-\delta}(uw/N)^{2/3} g^{D-2-\delta}$, generalizing the $D = 4$ result [24], $dP_N/d\Omega_{S^2} \propto N^{-2/3}$; see also [49]. For $D = 4$, the explicit polarization tensors are given below (2), and we write $\hat{\mathbb{P}}^i = \hat{\mathbf{x}}\sin\theta\cos\varphi + \hat{\mathbf{y}}\sin\theta\sin\varphi + \hat{\mathbf{z}}\cos\theta$ in (8), from which $\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_n| = \sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_m| = \sin\theta$,

with $\theta \in [0, \pi]$—the radiation is axially symmetric about the z axis.

Remarkably, Eq. (8) (with $\delta = 0$) is precisely equal [37] to the power per unit solid angle associated with $G + B + \Phi$ emission, as derived from the following effective action in the absence of radiative backreaction [20,50]:

$$S_{\mathrm{eff}} = \frac{1}{16\pi G_D} \int d^D x \sqrt{-G} e^{-2\Phi}$$
$$\times \left[R_{(D)} + 4(\nabla\Phi)^2 - \frac{1}{12}H_{(3)}^2 + \cdots \right]$$
$$- \mu \int_{S^2} \partial X^\mu \wedge \bar{\partial} X^\nu (G_{\mu\nu} + B_{\mu\nu}) + \cdots. \tag{9}$$

In particular, we perturb around the string background, $G_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}$, $B_{\mu\nu} \simeq b_{\mu\nu}$, and $\Phi \simeq \Phi_0 + \phi$, in (9) and evaluate the source term on the trajectories (2). The power is extracted from the energy momentum $T_{G+B+\Phi}^{\mu\nu}$ carried away from the string source by the perturbations $h_{\mu\nu}$, $b_{\mu\nu}$ and ϕ, according to [25,51]

$dP/d\Omega_{S^{D-2}} = \langle r^{D-2}\hat{\mathbb{P}}^i T^{0i}_{G+B+\Phi} \rangle$, with r the distance to the observer. The averaging is over spacetime regions much larger than the wavelengths under consideration [51], and the resulting expression is precisely Eq. (8) (in *both* the string and Einstein frame [52], the two being related by $G_{\mu\nu} = e^{4\phi/(D-2)}G^{\mathrm{E}}_{\mu\nu}$).

The power into gravitational waves, in $D = 4$, was computed in [25], and for $D \geq 4$ in [37]. Let us write $\mathcal{O} = (n, m, \lambda_n, \bar{\lambda}_m)$, $\tilde{\mathcal{O}} = (n, m, \lambda_n, \bar{\lambda}^*_m)$. Intriguingly, the power into massless radiation equals the sum of the gravitational wave results [25] for the dual trajectories,

$$\frac{dP}{d\Omega}\bigg|^{\mathcal{O}}_{m^2=0} = \frac{dP}{d\Omega}\bigg|^{\tilde{\mathcal{O}}}_{m^2=0} = \frac{dP}{d\Omega}\bigg|^{\mathcal{O}}_{G} + \frac{dP}{d\Omega}\bigg|^{\tilde{\mathcal{O}}}_{G}, \quad (10)$$

where we note that

$$\frac{dP}{d\Omega}\bigg|_{m^2=0} = \frac{dP}{d\Omega}\bigg|_{G} + \frac{dP}{d\Omega}\bigg|_{B} + \frac{dP}{d\Omega}\bigg|_{\Phi}.$$

To conclude, we have constructed the two-point amplitudes (5), associated with coherent vertex operators (1), labeled by quantum numbers $(n, m, \lambda_n, \bar{\lambda}_m)$, in D noncompact dimensions. There is an invariance under $(n, m, \lambda_n, \bar{\lambda}_m) \rightarrow (n, m, \lambda_n, \bar{\lambda}^*_m)$, implying a symmetry (7) [and (10)]. This is possibly broken by higher loop effects: the quantity $(\mathbb{P}_I \cdot \lambda^*_n)\mathcal{D}^n_z \int^z \omega_I (\mathbb{P}_J \cdot \lambda_n)\mathcal{D}^n_{z'} \int^{z'} \omega_J$ that would appear in the argument of the Bessel function in (6) when $h \geq 1$, with ω_I ($I = 1, \ldots, h$) the holomorphic Abelian differentials of the genus-h Riemann surface and $\mathbb{P}^\mu_I$ the momentum through the A^{th}_I homology cycle [38], is only invariant when $h = 1$.

We have derived the power and decay rate (8) associated with massless IR radiation for the trajectories $(1, 1; \lambda_1, \bar{\lambda}_1)$, when backreaction is neglected, and find that the result can be precisely reproduced by a low energy effective theory [20]. We conjecture the result for general (non-self-intersecting) trajectories $(n, m; \lambda_n, \bar{\lambda}_m)$, provided perturbation theory remains valid [53].

The coherent state basis [8] and fixed-loop momenta approach is seemingly very efficient for string calculations, and makes it possible to start exploring stringy effects (such as cusp emission [26,49] and the effects of massive string modes) in regimes where the low energy effective description is less useful. Because of the universality of (9) [18], we expect that (8) holds also for the superstring.

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[31] Here, $C^{-2} = g_D^{-2} \int_0^{2\pi} (ds/2\pi) \exp[e^{ins}(|\lambda_n|^2/n)] \times \exp[e^{-ims}(|\bar{\lambda}_m|^2/m)]$. In terms of the compactification volume, $g_D \equiv (\alpha'^6 e^{\Phi_0}/\sqrt{V_{26-D}})$.

[32] Notice that (2) is invariant under $g \rightarrow g + 1$, up to a rescaling of L and a world sheet reparametrization. Furthermore, $M \sim g\mathcal{R}/\alpha'$, which is characteristic of winding modes; here, $\mathcal{R}$ is determined by the string dynamics.

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