Quantum Discord Cannot Be Shared

Alexander Streltsov[1,2,*] and Wojciech H. Zurek[2,3]

[1]Heinrich-Heine-Universität Düsseldorf, Institut für Theoretische Physik III, D-40225 Düsseldorf, Germany
[2]Los Alamos National Laboratory, Theoretical Division, MS-B213, Los Alamos, New Mexico 87545, USA
[3]Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA

(Received 15 January 2013; published 22 July 2013)

Bohr proposed that the outcome of a measurement becomes objective and real, and, hence, classical, when its results can be communicated by classical means. In this work we revisit Bohr's postulate using modern tools from quantum information theory. We find a full confirmation of Bohr's idea: if a measurement device is in a nonclassical state, the measurement results cannot be communicated perfectly by classical means. In this case some part of the information in the measurement apparatus is lost in the process of communication: the amount of this lost information turns out to be the quantum discord. The information loss occurs even when the apparatus is *not* entangled with the system of interest. The tools presented in this work allow us to generalize Bohr's postulate: we show that for pure system-apparatus states quantum communication does not provide any advantage when measurement results are communicated to more than one recipient. We further demonstrate the superiority of quantum communication to two recipients on a mixed system-apparatus state and show that this effect is fundamentally different from quantum state cloning.

DOI: 10.1103/PhysRevLett.111.040401 PACS numbers: 03.65.Ta, 03.65.Yz, 03.67.Hk

The quantum measurement problem arises because our Universe is quantum to the core, so the interaction between the (quantum) apparatus $\mathcal{A}$ and the measured quantum system $\mathcal{S}$ creates a correlation that is also quantum. The joint states of the apparatus and the system can be even entangled (as is illustrated by Schrödinger's cat [1]) which leads to interpretational problems such as the basis ambiguity [2].

Quantum entanglement is a hallmark of quantum correlations: Pure states that possess it violate Bell's inequality, showing that quantum mechanics contradicts classical intuition resting on the assumption (expressed by Einstein, Podolsky, and Rosen [3]) that state is local—a property of an individual system. Yet, as our daily experience convincingly demonstrates, Einstein's intuition about reality is respected on the macroscopic level: A state is a property of an individual system, and while correlations exist, a completely known state of composite classical system can be always expressed as a "Cartesian product" of pure local states, so that a state of each component is also completely known. Entanglement depends on the tensor structure of composite quantum states that allows and even mandates [4–6] ignorance of the ingredients for a completely known composite state.

Entanglement is simply impossible in the classical realm. In general (for mixed—i.e., incompletely known—states) quantum entanglement is defined by specifying how a composite state can be put together. When it can be assembled from classical ingredients by mixing direct (ultimately Cartesian) products of quantum states of individual subsystems, a composite state can be prepared by observers that employ only local operations and classical communication. Such states are known as separable, and, by definition, they are not entangled [7].

Here we show that quantum discord can be defined by considering the opposite of the process of assembling a state. That is, when one attempts to pull apart a quantum state so that, in the end, all the ingredients are classical and can be communicated classically to distant recipients, the cost of such an operation is given by quantum discord. Thus, discord is the information lost when a composite quantum state is disassembled. It is amusing to note that this disparity between how hard it is to pull a state apart compared to how difficult it is to put it together has the opposite "sign" than in everyday experience, as it is generally easier to take apart a device—e.g., a clock—than it is to put it back together.

Quantum discord is the difference between the quantum mutual information $I(\mathcal{S}:\mathcal{A}) = S(\rho^{\mathcal{S}}) + S(\rho^{\mathcal{A}}) - S(\rho^{\mathcal{S}\mathcal{A}})$ and the information J accessible via the measurement with outcomes $\{E_i^{\mathcal{A}}\}$, record states of the apparatus $\mathcal{A}$: $J(\mathcal{S}:\mathcal{A})_{\{E_i^{\mathcal{A}}\}} = S(\rho^{\mathcal{S}}) - \sum_i p_i S(\rho_i^{\mathcal{S}})$, where S is the von Neumann entropy, $p_i = \mathrm{Tr}[E_i^{\mathcal{A}}\rho^{\mathcal{S}\mathcal{A}}]$ is the probability of outcome i, and $\rho_i^{\mathcal{S}}$ is the state of the system after the outcome i has been obtained: $\rho_i^{\mathcal{S}} = \mathrm{Tr}_{\mathcal{A}}[E_i^{\mathcal{A}}\rho^{\mathcal{S}\mathcal{A}}]/p_i$. In the classical domain I and J coincide as there is an underlying joint probability distribution that can be used to express the joint state of the two systems in terms of the local states of individual subsystems. In that case, Bayes' rule holds, and I is identically equal to J [8]. However, in quantum physics obtaining conditional information requires a measurement, and that generally alters the measured state, so the information obtained by local measurements is less than the mutual information present in the joint premeasurement state. The difference between the mutual information I and the information accessible via the measurement apparatus J is known as quantum discord $\delta(\mathcal{S}:\mathcal{A})_{\{E_i^{\mathcal{A}}\}} = I(\mathcal{S}:\mathcal{A}) - J(\mathcal{S}:\mathcal{A})_{\{E_i^{\mathcal{A}}\}}$ [9–12]. Quantum discord is then at least as large as its minimum $\delta(\mathcal{S}:\mathcal{A}) = \min_{\{E_i^{\mathcal{A}}\}} \delta(\mathcal{S}:\mathcal{A})_{\{E_i^{\mathcal{A}}\}}$.

In recent years, various applications for quantum discord and related quantum correlations have been discovered. They range from interpreting the difference between quantum and classical Maxwell's demons [13], to the creation of entanglement in the measurement process [14,15], and the use of entanglement in the task of quantum state merging [16,17]. An alternative thermodynamical approach has also been presented in [18]. Recent results also provide evidence that quantum discord is a resource in the tasks of entanglement distribution [19,20], remote state preparation [21], and information encoding [22]. The role of quantum discord in the historical debate between EPR and Bohr was also subjected to scrutiny, showing that quantum discord is closely related to Bohr's criterion for disturbance [23]. Experimentally friendly measures of quantum discord have also been considered in [24,25]. Moreover, quantum discord was identified as the resource for the quantum computing protocol known as DQC1 [26,27]. Remarkably, the algorithm does not require any entanglement, and it was shown in [27] that a typical instance of DQC1 has nonzero quantum discord.

We revisit Bohr's original idea that a quantum measurement outcome is classical when it can be communicated by classical means in light of recent information-theoretic results. We consider the scenario illustrated in Fig. 1. Initially, information about the system $\mathcal{S}$ manifests itself in a joint quantum state between $\mathcal{S}$ and a measurement apparatus $\mathcal{A}$ (left part of Fig. 1). Suppose that the state of the apparatus is communicated classically to the recipient $\mathcal{R}$. As is customary in quantum information theory, we allow classical communication and arbitrary quantum operations to be performed locally on $\mathcal{A}$ and $\mathcal{R}$. The total procedure is known as "local operations and classical communication" (LOCC) [28]. The aim of this process is to give the recipient $\mathcal{R}$ all the information about the system $\mathcal{S}$, which was initially present in the apparatus $\mathcal{A}$ (right part of Fig. 1). If this procedure was possible for some LOCC protocol, we would say that the measurement results can be perfectly communicated by classical means. Then, according to Bohr's postulate, the apparatus would carry solely classical information about the system $\mathcal{S}$. If, on the other hand, the process is not possible for any LOCC protocol, some part of the information about the system is unavoidably lost on the way to the recipient. In this case, the apparatus must have contained information about the system, which could not be communicated by classical means, and thus (according to Bohr) must have been quantum.

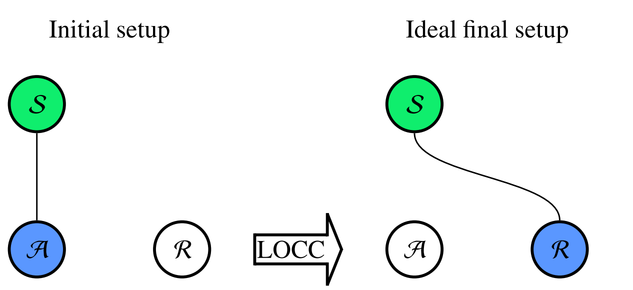

FIG. 1 (color online). Ideal classical communication to a distant recipient. Left part of the figure shows the initial situation: the system $\mathcal{S}$ is correlated with the apparatus $\mathcal{A}$. The recipient $\mathcal{R}$ is initially not correlated with $\mathcal{SA}$. Right part of the figure shows the ideal final setup after the application of LOCC protocol between $\mathcal{A}$ and $\mathcal{R}$. In the ideal case the recipient $\mathcal{R}$ obtains the full information which was initially present in the apparatus $\mathcal{A}$.

Note that the final state between the system $\mathcal{S}$ and the recipient $\mathcal{R}$ is never entangled. This means that the procedure described above cannot be implemented perfectly when the system is initially entangled with the apparatus. This observation underscores the quantum nature of entanglement: all entangled states contain information that is nonclassical. One might expect that this quantum feature disappears for all *separable* system-apparatus states, since these states can be produced using solely classical means. However, this is not the case: we will see below that even separable states can contain information which cannot be communicated classically.

In the following, we quantify the information in the apparatus $\mathcal{A}$ about the system $\mathcal{S}$ by the quantum mutual information $I(\mathcal{S}:\mathcal{A})$. The information gained by the recipient $\mathcal{R}$ about the system $\mathcal{S}$ after applying the LOCC protocol is given by their mutual information $I(\mathcal{S}:\mathcal{R})$. Our main question can then be stated as follows: How much information can a recipient gain about a system by classical means? That is, we are interested in the maximal mutual information between $\mathcal{S}$ and $\mathcal{R}$, maximized over all possible LOCC protocols between the apparatus $\mathcal{A}$ and the recipient $\mathcal{R}$. The corresponding quantity will be called I^c, where the superscript c tells us that the communication is classical. Our main result is the following closed expression for I^c:

$$I^c = I(\mathcal{S}:\mathcal{A}) - \delta(\mathcal{S}:\mathcal{A}), \quad (1)$$

which is also equal to the measure of classical correlations introduced in [11].

The key idea behind the proof of this result is the fact, that the final state $\rho_{\mathrm{final}}^{\mathcal{SR}}$ is never entangled, and that the total initial state has the product form $\rho^{\mathcal{SA}} \otimes \rho^{\mathcal{R}}$ [29]. If we consider the map Λ from the initial state $\rho^{\mathcal{SA}}$ onto the separable final state $\rho_{\mathrm{final}}^{\mathcal{SR}} = \Lambda(\rho^{\mathcal{SA}})$, this map must be entanglement breaking [30]. This implies that the final state has the form $\rho_{\mathrm{final}}^{\mathcal{SR}} = \sum_i \mathrm{Tr}_{\mathcal{A}}[E_i^{\mathcal{A}} \rho^{\mathcal{SA}}] \otimes \sigma_i^{\mathcal{R}}$. From this result the structure of the optimal LOCC protocol becomes evident: the apparatus $\mathcal{A}$ is measured with a measurement $\{E_i^{\mathcal{A}}\}$, and the outcome i is communicated to the recipient $\mathcal{R}$, who prepares the state $\sigma_i^{\mathcal{R}}$ locally. The best choice for states $\sigma_i^{\mathcal{R}}$ is to take them pure and orthogonal [31]. In this case the mutual information becomes $I(\mathcal{S}:\mathcal{R}) = J(\mathcal{S}:\mathcal{A})_{\{E_i^{\mathcal{A}}\}}$. Equation (1) is obtained by taking the maximum over all measurements.

When the quantum discord between the system $\mathcal{S}$ and the apparatus $\mathcal{A}$ is nonzero, I^c will always be below the initial mutual information $I(\mathcal{S}:\mathcal{A})$. This tells us that *any* state with nonzero quantum discord contains nonclassical information, i.e., information which cannot be communicated by classical means. Interestingly, this statement also includes system-apparatus states that are separable, i.e., not entangled. This follows from the fact that quantum discord can be nonzero even when the system is not entangled with the apparatus [10].

The results presented so far fully support Bohr's seminal idea: a measurement outcome can be regarded as classical when it can be classically communicated to a distant recipient. From this point of view, the distinction between the classical and quantum world arises from constraints on the way information is communicated. We could also paraphrase Wheeler's summary of Bohr's views and say that "No phenomenon is a phenomenon until it is a classically communicable phenomenon" [32]. On the other hand, when *quantum communication* between the apparatus $\mathcal{A}$ and the recipient $\mathcal{R}$ is allowed, the process always succeeds, even if the measurement apparatus is nonclassical [33]. Thus, our main result in Eq. (1) provides a physical interpretation for quantum discord: it measures the advantage of quantum communication for passing on information. In particular, this advantage is maximal for maximally entangled states, since these states also have maximal quantum discord. One might expect that the advantage remains at least to some degree if the information is communicated to more than one recipient. However, as we show in the following, already for two recipients the superiority of quantum communication disappears for all pure system-apparatus states.

We consider the system $\mathcal{S}$ and the apparatus $\mathcal{A}$, initially in a joint state (left part of Fig. 2). Instead of one recipient $\mathcal{R}$ we now introduce two recipients $\mathcal{R}_1$ and $\mathcal{R}_2$. The recipients have access to the apparatus $\mathcal{A}$ via a communication channel Λ, that will be specified below. Aim of the process is to give both recipients the same amount of information about the system, i.e., $I(\mathcal{S}:\mathcal{R}_1) = I(\mathcal{S}:\mathcal{R}_2)$ (right part of Fig. 2). Applying the same line of reasoning as before we arrive at the following question: How much information can each of two recipients gain about a system? We denote the maximal amount of such shared information attainable via LOCC by I_2^c, where the lower index gives the number of recipients. From the previous results it is clear that the number of recipients does not change anything in the case of classical communication: $I_n^c = I^c$ for any number of recipients n.

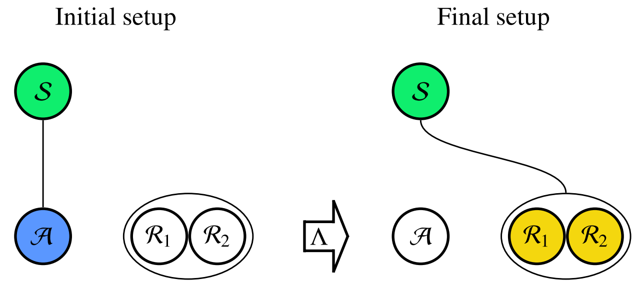

FIG. 2 (color online). Distributing information to two recipients. Left part of the figure shows the initial situation: The apparatus $\mathcal{A}$ is initially correlated with the system $\mathcal{S}$, while two recipients $\mathcal{R}_1$ and $\mathcal{R}_2$ are not correlated with $\mathcal{S}\mathcal{A}$. The final situation after the application of a general quantum channel Λ is shown in the right part of the figure. In the desired final case both recipients have the same information about the system, $I(\mathcal{S}:\mathcal{R}_1) = I(\mathcal{S}:\mathcal{R}_2)$.

We will now compare this quantity to the amount of mutual information attainable via *quantum communication*, which will be denoted by I_2^q. Since quantum communication is more general than any LOCC protocol, it must be that $I_n^q \geq I_n^c$, which is indeed true for any number of recipients n. In the single-recipient scenario this inequality becomes strict for all states with nonzero quantum discord: in all these cases quantum communication provides an advantage. In particular, for all pure system-apparatus states we have $I_1^q = 2I_1^c$; i.e., quantum communication can outperform the best LOCC protocol by a factor of 2. However, as we show in the following, quantum communication does not provide any advantage for any number of recipients $n \geq 2$, if the initial system-apparatus state is pure:

$$I_n^q = I_n^c. \tag{2}$$

For proving this result we first consider the two-recipients scenario, i.e., $n = 2$. The amount of information attainable by classical means is $I_2^c = S(\rho^{\mathcal{S}})$. We will now show that $I_2^q = I_2^c$. For this we can assume that each recipient initially has the state $|0\rangle^{\mathcal{R}_i}$; i.e., the total initial state is $|\psi_{\text{tot}}\rangle = |\psi\rangle^{\mathcal{S}\mathcal{A}}|0\rangle^{\mathcal{R}_1}|0\rangle^{\mathcal{R}_2}$. After the application of a quantum channel, the final state can be written as $\rho_{\text{final}}^{\mathcal{S}\mathcal{A}\mathcal{R}_1\mathcal{R}_2} = \text{Tr}_{\mathcal{B}}[U\rho_{\text{tot}}U^\dagger]$, where $\mathcal{B}$ is an ancilla, $\rho_{\text{tot}} = |\psi_{\text{tot}}\rangle\langle\psi_{\text{tot}}| \otimes |0\rangle\langle 0|^{\mathcal{B}}$, and U is a unitary acting on $\mathcal{A}\mathcal{R}_1\mathcal{R}_2\mathcal{B}$. Since mutual information does not increase under partial trace [31], it follows that $I(\mathcal{S}:\mathcal{R}_1) + I(\mathcal{S}:\mathcal{R}_2) \leq I(\mathcal{S}:\mathcal{A}\mathcal{R}_1) + I(\mathcal{S}:\mathcal{R}_2\mathcal{B})$, where $\rho^{\mathcal{S}\mathcal{R}_2\mathcal{B}} = \text{Tr}_{\mathcal{A}\mathcal{R}_1}[U\rho_{\text{tot}}U^\dagger]$. On the other hand, since the total system $\mathcal{S}\mathcal{A}\mathcal{R}_1\mathcal{R}_2\mathcal{B}$ is in a pure state, we get $S(\rho^{\mathcal{A}\mathcal{R}_1}) = S(\rho^{\mathcal{S}\mathcal{R}_2\mathcal{B}})$ and $S(\rho^{\mathcal{R}_2\mathcal{B}}) = S(\rho^{\mathcal{S}\mathcal{A}\mathcal{R}_1})$ so that $I(\mathcal{S}:\mathcal{A}\mathcal{R}_1) + I(\mathcal{S}:\mathcal{R}_2\mathcal{B}) = 2S(\rho^{\mathcal{S}})$, leading to the inequality $I(\mathcal{S}:\mathcal{R}_1) + I(\mathcal{S}:\mathcal{R}_2) \leq 2I_2^c$. Since we demand that $I(\mathcal{S}:\mathcal{R}_1) = I(\mathcal{S}:\mathcal{R}_2)$, both quantities cannot be larger than I_2^c, which implies $I_2^q = I_2^c$. Note that this reasoning also implies the equality $I_n^q = I_n^c$ for any number of recipients $n \geq 2$, since it shows that for each pair of recipients $\mathcal{R}_i$ and $\mathcal{R}_j$ with $i \neq j$ the sum $I(\mathcal{S}:\mathcal{R}_i) + I(\mathcal{S}:\mathcal{R}_j)$ never exceeds $2I_2^c$. These results can also be generalized to asymmetric protocols, where the final mutual information between the system and each of n recipients is not necessarily the same. In this case, quantum communication cannot outperform LOCC *on average*: $(1/n)\sum_{i=1}^n I(\mathcal{S}:\mathcal{R}_i) \leq I_n^c$. This can be seen using the same arguments as above, noting that the sum $\sum_{i=1}^n I(\mathcal{S}:\mathcal{R}_i)$ never exceeds $n \cdot I_n^c$.

It is now natural to ask whether Eq. (2) also holds for initially mixed states $\rho^{\mathcal{SA}}$. We will see in the following that in general this is not the case: for mixed system-apparatus states quantum communication can outperform classical communication. This will be demonstrated using the initial state

$$\rho^{\mathcal{SA}} = \frac{1}{2}|0\rangle\langle 0|^{\mathcal{S}} \otimes |\psi\rangle\langle\psi|^{\mathcal{A}} + \frac{1}{2}|1\rangle\langle 1|^{\mathcal{S}} \otimes |\phi\rangle\langle\phi|^{\mathcal{A}} \quad (3)$$

with $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$ and $|\phi\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$. For this initial state, I^c can be evaluated using the Koashi-Winter relation [34]. The result is shown in Fig. 3.

On the other hand, the maximal amount of information I_2^q attainable via a quantum channel can be bounded below by any imperfect cloning protocol for the two states $|\psi\rangle$ and $|\phi\rangle$. In the following we will use the protocol for "optimal state-dependent cloning" presented in [35]. The corresponding mutual information $I(\mathcal{S}:\mathcal{R}_1) = I(\mathcal{S}:\mathcal{R}_2)$ between the system $\mathcal{S}$ and each of the recipients $\mathcal{R}_1$ and $\mathcal{R}_2$ is shown in Fig. 3. As can be seen from the difference $I^c - I(\mathcal{S}:\mathcal{R}_1)$ shown in the inset of Fig. 3, the cloning procedure outperforms any LOCC protocol in the region $\theta' < \theta < \pi/4$ with $\theta' \approx 0.093\pi$. However, quite surprisingly, we also see that LOCC outperforms the protocol of optimal state-dependent cloning for $0 < \theta < \theta'$. The reason for this counterintuitive behavior is the fact that the protocol considered in [35] was optimized for a different figure of merit known as global fidelity. These results demonstrate the fundamental difference between the new task of distributing correlations considered in this Letter and the task of cloning a quantum state [36,37].

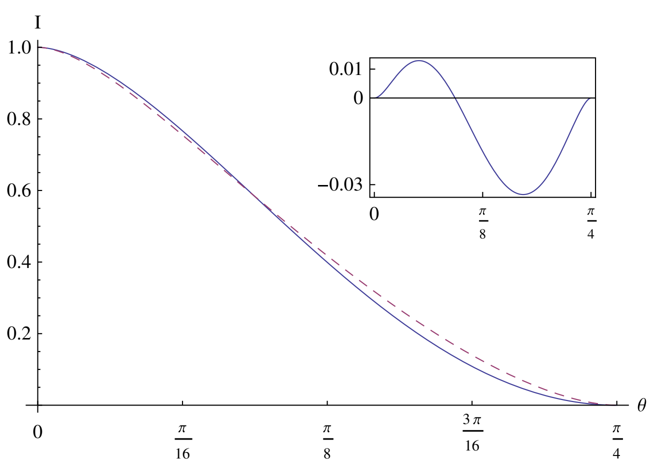

FIG. 3 (color online). Main figure shows the maximal amount of information attainable by classical means I^c (solid line) and the mutual information $I(\mathcal{S}:\mathcal{R}_1) = I(\mathcal{S}:\mathcal{R}_2)$ for the "optimal state-dependent cloning" (dashed line) for the state given in Eq. (3) as a function of θ. The inset shows the difference $I^c - I(\mathcal{S}:\mathcal{R}_1)$. For $\theta' < \theta < \pi/4$ with $\theta' \approx 0.093\pi$ quantum communication outperforms any LOCC protocol, see main text for details.

The results presented so far can be seen as an extension of Bohr's postulate: classicality emerges also whenever the outcome of a measurement is shared by more than one recipient. As sharing of information is a prerequisite for "objective reality" [38–40], our results indicate that objective information is necessarily classical: For pure system-apparatus states classicality arises whenever data are communicated to more than one party, independently from the nature of the communication channel. Moreover, a natural operational definition of discord turns out to be a counterpoint of the operational definition of entanglement: A composite state that cannot be assembled by classical means is entangled. Similarly, a composite state that cannot be disassembled by classical means is still quantum—it has a nonvanishing discord, the quantum information lost in the process of deconstructing it into classical ingredients.

Finally, we point out that the single-recipient scenario, as shown in Fig. 1, can also be regarded as a modified version of the quantum state merging [41]. There are however two essential differences to the situation considered in [41]. We do not demand that the total system $\mathcal{SAR}$ is in a pure state and, in contrast to [41], we do not allow any entanglement to be shared between $\mathcal{A}$ and $\mathcal{R}$. Moreover, the discussion in [41] concentrates on the scenario where many copies of the same state are available. We expect that a fruitful comparison of the two approaches can be made, if the results of our work are also extended to the many-copy case. On the other hand, the situation of many recipients, as illustrated in Fig. 2, can be related to the protocol known as local broadcasting [42,43]. While in Ref. [43] the authors conclude that perfect local broadcasting is only possible for states with zero quantum discord, our result in Eq. (2) can be regarded as a generalization to the case of all pure states. From this point of view, our results imply that local broadcasting of a pure state never requires quantum communication; the best performance is always achievable with LOCC.

We conclude with Bohr's statement [44]: "No observation is an observation unless we can communicate the results of that observation to others in plain language." In this work we have shown that this statement is not only philosophical: if measurement outcomes are to be communicated to distant recipients by classical means, the measurement apparatus must be in a classical state. In this sense, classicality naturally arises when communication is restricted to plain language. Moreover, we have applied our results to the case when measurement outcomes are communicated to more than one recipient. We found that for *pure* system-apparatus states quantum communication does not provide any advantage, showing that classicality arises in this case regardless of the nature of the communication. Although quantum communication is superior to classical communication on *mixed* system-apparatus states, there is strong evidence [45–47] that in the limit $n \to \infty$, i.e., for a large number of recipients, the result in Eq. (2) becomes valid for all mixed states. A rigorous proof of this statement is left open for future research.

We acknowledge discussion with Gerardo Adesso, Dagmar Bruß, Otfried Gühne, Hermann Kampermann, Matthias Kleinmann, Tobias Moroder, Marco Piani, Volkher Scholz, and Michael Zwolak. This work was supported by DOE LDRD program at Los Alamos, and in part by John Templeton Foundation, Deutsche Forschungsgemeinschaft (DFG) and ELES.

*streltsov@thphy.uni-duesseldorf.de

[1] E. Schrödinger, Naturwissenschaften **23**, 807 (1935).
[2] W. H. Zurek, Phys. Rev. D **24**, 1516 (1981).
[3] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[4] W. H. Zurek, Phys. Rev. Lett. **90**, 120404 (2003).
[5] W. H. Zurek, Phys. Rev. A **71**, 052105 (2005).
[6] W. H. Zurek, Phys. Rev. Lett. **106**, 250402 (2011).
[7] R. F. Werner, Phys. Rev. A **40**, 4277 (1989).
[8] T. M. Cover and J. A. Thomas, *Elements of Information Theory* (J. Wiley, New York, 1991).
[9] W. H. Zurek, Ann. Phys. (Berlin) **9**, 855 (2000).
[10] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
[11] L. Henderson and V. Vedral, J. Phys. A **34**, 6899 (2001).
[12] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, Rev. Mod. Phys. **84**, 1655 (2012).
[13] W. H. Zurek, Phys. Rev. A **67**, 012320 (2003).
[14] A. Streltsov, H. Kampermann, and D. Bruß, Phys. Rev. Lett. **106**, 160401 (2011).
[15] M. Piani, S. Gharibian, G. Adesso, J. Calsamiglia, P. Horodecki, and A. Winter, Phys. Rev. Lett. **106**, 220403 (2011).
[16] V. Madhok and A. Datta, Phys. Rev. A **83**, 032323 (2011).
[17] D. Cavalcanti, L. Aolita, S. Boixo, K. Modi, M. Piani, and A. Winter, Phys. Rev. A **83**, 032324 (2011).
[18] J. Oppenheim, M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **89**, 180402 (2002).
[19] A. Streltsov, H. Kampermann, and D. Bruß, Phys. Rev. Lett. **108**, 250501 (2012).
[20] T. K. Chuan, J. Maillard, K. Modi, T. Paterek, M. Paternostro, and M. Piani, Phys. Rev. Lett. **109**, 070501 (2012).
[21] B. Dakić, Y. O. Lipp, X. Ma, M. Ringbauer, S. Kropatschek, S. Barz, T. Paterek, V. Vedral, A. Zeilinger, Č. Brukner, and P. Walther, Nat. Phys. **8**, 666 (2012).
[22] M. Gu, H. M. Chrzanowski, S. M. Assad, T. Symul, K. Modi, T. C. Ralph, V. Vedral, and P. K. Lam, Nat. Phys. **8**, 671 (2012).
[23] H. M. Wiseman, arXiv:1208.4964v2.
[24] D. Girolami and G. Adesso, Phys. Rev. Lett. **108**, 150403 (2012).
[25] I. A. Silva, D. Girolami, R. Auccaise, R. S. Sarthour, I. S. Oliveira, T. J. Bonagamba, E. R. deAzevedo, D. O. Soares-Pinto, and G. Adesso, Phys. Rev. Lett. **110**, 140501 (2013).
[26] E. Knill and R. Laflamme, Phys. Rev. Lett. **81**, 5672 (1998).
[27] A. Datta, A. Shaji, and C. M. Caves, Phys. Rev. Lett. **100**, 050502 (2008).
[28] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).
[29] We note that for general initial states $\rho^{\mathcal{SAR}}$ it is possible to create entanglement between $\mathcal{S}$ and $\mathcal{R}$ via LOCC between $\mathcal{A}$ and $\mathcal{R}$. This can be demonstrated using the total initial state $\rho = (1/2)|0\rangle\langle 0|^{\mathcal{A}} \otimes \sigma^{\mathcal{SR}} + (1/2)|1\rangle \times \langle 1|^{\mathcal{A}} \otimes \tau^{\mathcal{SR}}$, which can have arbitrary little entanglement between $\mathcal{S}$ and $\mathcal{R}$, even for highly entangled states σ and τ. By LOCC between $\mathcal{A}$ and $\mathcal{R}$, it is possible to create the final state $\rho' = (1/2)|0\rangle\langle 0|^{\mathcal{R}'} \otimes \sigma^{\mathcal{SR}} + (1/2)|1\rangle\langle 1|^{\mathcal{R}'} \otimes \tau^{\mathcal{SR}}$, where the recipient is now in possession of $\mathcal{R}$ and $\mathcal{R}'$. Here, the entanglement between the system and the recipient can be large, even if they had little or even zero initial entanglement.
[30] M. Horodecki, P. W. Shor, and M. B. Ruskai, Rev. Math. Phys. **15**, 629 (2003).
[31] V. Vedral, Rev. Mod. Phys. **74**, 197 (2002).
[32] *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, NJ, 1983), p. 182.
[33] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[34] M. Koashi and A. Winter, Phys. Rev. A **69**, 022309 (2004).
[35] D. Bruß, D. P. DiVincenzo, A. Ekert, C. A. Fuchs, C. Macchiavello, and J. A. Smolin, Phys. Rev. A **57**, 2368 (1998).
[36] W. K. Wootters and W. H. Zurek, Nature (London) **299**, 802 (1982).
[37] D. Dieks, Phys. Lett. **92A**, 271 (1982).
[38] W. H. Zurek, Nat. Phys. **5**, 181 (2009).
[39] C. J. Riedel and W. H. Zurek, Phys. Rev. Lett. **105**, 020404 (2010).
[40] M. Zwolak and W. H. Zurek, Sci. Rep. **3**, 1729 (2013).
[41] M. Horodecki, J. Oppenheim, and A. Winter, Nature (London) **436**, 673 (2005).
[42] M. Piani, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **100**, 090502 (2008).
[43] S. Luo and W. Sun, Phys. Rev. A **82**, 012338 (2010).
[44] N. Bohr, *Essays 1958-1962 on Atomic Physics and Human Knowledge* (J. Wiley, New York, 1963).
[45] J. Bae and A. Acín, Phys. Rev. Lett. **97**, 030402 (2006).
[46] G. Chiribella and G. M. D'Ariano, Phys. Rev. Lett. **97**, 250503 (2006).
[47] F. G. S. L. Brandão, M. Piani, and P. Horodecki (in preparation).