# Thermal Convection and Temperature Inhomogeneity in a Vibrofluidized Granular Bed: The Influence of Sidewall Dissipation

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Using a vertically vibrated, fully three-dimensional granular system, we investigate the impact of dissipative interactions between the particles in the system and the vertical sidewalls bounding it. We find that sidewall dissipation influences various properties of the bed including, but not limited to, the spatial distribution of granular temperatures, the functional form of velocity distributions, and the strength of convection. Simple, monotonic relationships are observed for all the aforementioned properties, including a striking linear relationship between convection strength and wall dissipation. We conclude that sidewall effects are not limited to the vicinity of the walls themselves, but extend into the bulk of the system and hence must be considered even in relatively wide, three-dimensional systems. We also propose the possibility of using the alteration of sidewall material as a method of ''tuning'' certain system parameters in situations where changing the bulk properties or driving parameters of a granular system may be undesirable.

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*Granular materials*—large conglomerations of discrete, macroscopic particles—can, when exposed to an external energy source, exhibit a variety of behaviors both similar and dissimilar to classical materials [1]. They can display solid-, liquid-, and gaslike behavior, but also exhibit features not observed in molecular materials, such as spatially inhomogeneous density and temperature distributions [2]. These phenomena arise from the inherently dissipative nature of granular interactions, and can strongly affect the behavior of granular systems. For instance, dissipative interactions between grains and the side boundaries of a fluidized granular system can lead to the formation of convection rolls reminiscent of those observed in conventional fluids [3]; an increased level of dissipation at the walls of the system creates a local region of increased density and reduced granular temperature [4] which will ''sink'' to the bottom of the container. This sinkage is balanced by a net upward motion in the central region of the bed, creating convection [5].

Although thermal convection was first observed over a decade ago [8], systematic study of the effect of sidewall dissipation on convection strength (as well as other system parameters) is surprisingly lacking considering the ubiquity of granular materials in nature and industry [1], and the significant impact of convection within these materials [9]. Consider, as an example, industrial processes where granular segregation may be an unwanted consequence of convection within a system. Trying to limit the effect of convection by changing the material properties of the product in question is unlikely to be viable. Altering the driving parameters may also be near impossible, for instance, if the excitation is due to vibrations during transport of the product. Moreover, even in situations where this is possible, the relationships between vibrational driving parameters and the various properties of granular fluids are often found to be complex and nonuniversal [10]. The ability to ''tune'' the behavior of a system simply by altering the container in which it is housed would surely be a boon.

This Letter explores a fully 3D, vertically vibrated system in which the material of the walls containing a granular bed is varied, altering the magnitude of particle-wall dissipation. The system comprises a cuboid container 200 mm high with a 100 mm square base, containing a granular bed of $107 \leq N \leq 1070$ spherical glass beads of diameter $d = 5$ mm, giving a dimensionless width in particle diameters of $W^* = W/d = 20$. Considering the dilute nature of our system, however, the particle mean free path, $\lambda$, is perhaps a more appropriate choice of length scale. The variation of $N$ in the system thus allows the estimation of a range of dimensionless widths $\tilde{W} = (\pi N d^2/WH) = (W/\lambda) \in (0.67, 12.7)$. Here, $H$ corresponds to the height of the main bulk of the system, defined as the point at which the system density decays below 10% of its maximal value. The use of a 3D system allows insight into the importance of sidewall interactions in systems more typical of ''real-world'' scenarios, as opposed to the constrained systems used in previous experiments [11]. The container is driven sinusoidally in the vertical direction at a frequency of 70 Hz with amplitude 1.93 mm, producing a fully fluidised bed for all $N$. The frequency is adequately high that the system's center of mass motion has no phase dependence [12]. The height of the container ensures an ''open'' system, allowing temperature gradients to evolve freely. The sidewalls of the system can be replaced with various different materials, allowing the influence of sidewall dissipation to be investigated—see Table I. It is

 

TABLE I. Effective inelasticities for the various materials used as system sidewalls.

| Material | Effective inelasticity, $\varepsilon_w$ |
|---|---|
| Mild steel | $0.70 \pm 0.006$ |
| Stainless steel | $0.68 \pm 0.024$ |
| Copper | $0.58 \pm 0.008$ |
| Brass | $0.52 \pm 0.010$ |
| Tufnol | $0.39 \pm 0.012$ |
| Clear perspex | $0.33 \pm 0.014$ |
| Lead | $<0.01$ |

ensured that the surfaces of all sidewalls used are smooth in order to minimize differences in friction. $\varepsilon_w$ is defined as the average fraction of energy lost by a single particle undergoing a collision with a wall, determined experimentally using high speed photography. $\varepsilon_w$ is inclusive of energy lost due to inelasticity as well as that lost to rotational motion [13]. $\varepsilon_w$ is taken as the mean of values obtained over a range of particle velocities chosen based on the experimentally observed velocity range. The errors quoted are representative of the standard deviation over the range of readings taken.

Data were acquired using positron emission particle tracking (PEPT), a noninvasive technique capable of recording the 3D motion of a single particle within a granular system to millimeter precision and millisecond time resolution [14]. For steady-state, ergodic systems such as the one described here, analysis of the time-averaged behavior of this single particle can be used to determine a variety of relevant quantities pertaining to the system as a whole [15,16]. A minimum run length of 45 min was used [17], at least 2 orders of magnitude greater than the time scale of convection for each case, allowing the tracer to fully explore the system and its motion. For further details of the PEPT technique, please refer to Refs. [14,15].

*Convection strength.*—In order to quantify convection strength for the various sidewall materials, the convective flow rate, $J$, for each data set was determined; the height of the center of convection, across which there is no horizontal component of velocity, was determined from velocity data. $J$ could then be calculated from the average of $|v_y|$ for data points falling within a horizontal "slice" through the data at this height, i.e., $J = (\sum_i^n |v_y^i|)/(2n)$, where $v_y^i$ is the vertical velocity corresponding to the $i$th data point and $n$ is the number of data points in the slice [18].

Figure 1 shows the variation of $J$ with $\varepsilon_w$ for $N = 863$. $J$ is observed to decrease monotonically with $\varepsilon_w$ in what appears to be a linear relationship. While a monotonic decrease is to be expected, the linear nature of the relationship is yet to be explained theoretically. Also notable is the graph's $x$ intercept at $\varepsilon_w \approx 0.9$. For this experiment, the inter-particle coefficient of restitution is $\varepsilon_p \approx 0.91$. Thus, our results suggest that wall-driven convection may be suppressed for $\varepsilon_w \gtrsim \varepsilon_p$. Finally, it is worth noting the

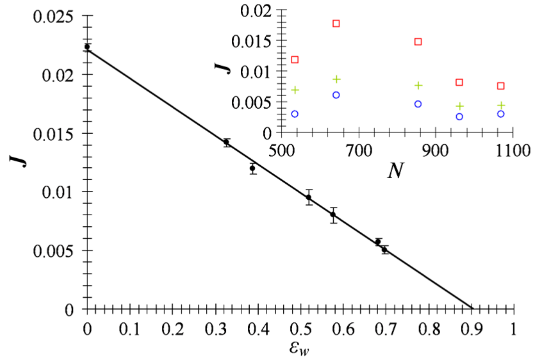


FIG. 1 (color online). The convective flow rate ($J$) in $\mathrm{ms}^{-1}$ as a function of $\varepsilon_w$. Inset: $J$ as a function of the number of particles in the system, $N$, for steel (open circle), copper (plusses), and perspex (open square) sidewalls.

degree to which $\varepsilon_w$ affects the convective behavior of the system—for the example shown in Fig. 1, $J$ varies by more than a factor of 4 over the range of $\varepsilon_w$ tested. For relatively dilute systems, simply altering the wall material while keeping all other parameters constant can even induce convection in a previously nonconvective system or, conversely, suppress convection in a previously convective system. The inset of Fig. 1 shows the variation of $J$ with $N$, demonstrating a roughly consistent trend for all $\varepsilon_w$. Furthermore, for all $N$ values where convection is observable, the relationship between $J$ and $\varepsilon_w$ is found to be linear with intercept $\varepsilon_w \approx 0.9$, the only difference being the gradient of the relation.

*Temperature inhomogeneity.*—Spatially heterogeneous temperatures and densities are a distinctive feature of granular gases. Figure 2 shows the variation of the normalised horizontal component of granular temperature, $T_x$, with height above the base, $y$, for 3 distinct values of $\varepsilon_w$ in a relatively dense system. $T$ is calculated by dividing the experimental system into a series of equally sized 3D

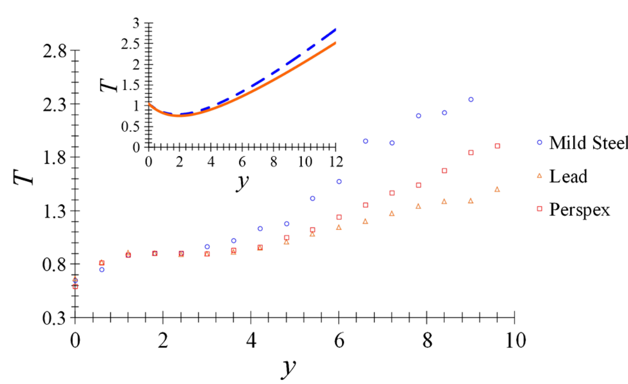


FIG. 2 (color online). Normalized vertical temperature profiles for $N = 963$. Profiles are shown for various sidewall materials, with all other variables held constant. Inset: theoretical temperature profiles for $\varepsilon_w = 0.7$ (dashed curve) and $\varepsilon_w = 0.33$ (solid curve).

"pixels." For each pixel, the average velocity corresponding to the data points lying within its domain is calculated. $T$ is then calculated for each pixel from the fluctuation of the individual particle velocities about the average [19], thus ensuring that the resulting value is not affected by the net convective motion of the system. Each point in Fig. 2 corresponds to the average temperature of a horizontal row of pixels through the central region of the bed, where $T$ is approximately constant (i.e., does not vary significantly with horizontal position). For all data sets, it is ensured that the pixel dimensions, and hence the height of this row, are considerably smaller than the mean free path of the system. The profiles follow the expected general trend [20], with $T$ passing through a minimum before increasing with height for large $y$. The increase in $T_x$ at low $y$ corresponds to the initial transfer of energy from the vertical to the horizontal direction through interparticle collisions.

At small $y$, where density is greatest, the profiles follow a similar trend. As $y$ increases, however, and the local density decreases, the profiles diverge significantly for differing $\varepsilon_w$, with more dissipative sidewalls producing shallower gradients in $T$. This considerable disparity at high altitudes, due entirely to wall effects, illustrates the importance of considering $\varepsilon_w$ when theoretically modeling granular $T$ profiles—a typical model considering only interparticle dissipation would produce the same curve for all cases shown. The fact that the data shown correspond to the *central region* of the bed, ignoring the boundary layer near the system edge, is another clear demonstration that sidewall effects can influence an entire system, not merely particles in the direct vicinity of the walls. The ability demonstrated here to alter $T$ gradients simply by adjusting sidewall dissipation is a potentially valuable concept. In order to validate the above observations, a theoretical continuum model of a similar form to that of [20] was used to produce $T$ profiles such as those seen in Fig. 2. The theory was modified to include wall effects through the use of a density-dependent "effective" coefficient of restitution comprising both $\varepsilon_p$ and $\varepsilon_w$. Although this simplistic formulation is not intended to exactly reproduce experimental results, a good qualitative agreement is found between theory and experiment.

As $N$—and hence the system density—is increased, the form of the temperature profiles undergoes a transition, for low $N$, $T$ is observed to *decrease* monotonically with height at large altitudes. For higher-density systems, however, one observes a monotonic *increase*. More specifically, as $N$ increases, the gradient of the decreasing large-$y$ profile observed in the dilute limit becomes progressively shallower. After the transition to the increasing large-$y$ profile, a further increase in density is observed to give a steeper gradient. In both high- and low-density cases, increasing $\varepsilon_w$ is observed to decrease the large-$y$ gradient—i.e., in the low-density case the gradient is made *more negative*, in the high density case the gradient is made *less positive*.

As well as being spatially inhomogeneous, the temperature within a vertically vibrated granular fluid is also innately *anisotropic*, as energy is injected into the system predominantly in the $y$ direction, and only transferred into the horizontal through dissipative interactions. The degree of anisotropy within the system, defined as $T_y/T_x$ (where $T_y$ and $T_x$ are, respectively, the vertical and horizontal components of the mean system temperature), was measured for each combination of $N$ and $\varepsilon_w$. The system was found to become more isotropic with increasing density, and with increasingly elastic sidewalls. The decrease in $T_y/T_x$ with increasing density can be simply explained by the fact that, at higher densities, the increased collision rate will more rapidly transfer energy from the vertical to the horizontal direction [21]. The increase in $T_y/T_x$ for more dissipative sidewalls can be accounted for by the fact that in highly fluidized systems, where collisional as opposed to frictional interactions dominate [12], sidewall collisions will remove energy primarily in the horizontal direction. Although this result is not particularly surprising, the *extent* to which the sidewalls can affect the system is still noteworthy, in particular for the more dilute cases where altering $\varepsilon_w$ (while keeping all other parameters constant) can alter anisotropy by up to 30%. It is interesting to note that, as the system density increases, the values of $T_y/T_x$ for all $\varepsilon_w$ converge asymptotically to a single, nonunity value, implying that a dissipative granular system will not become fully isotropic for any combination of $N$ and $\varepsilon_w$. Our results agree with the prediction of [22] that $T_y$ will *always exceed* $T_x$ in a vertically vibrated granular system.

*Velocity distributions.*—Another important yet not fully understood feature of granular gases is the presence of non-Gaussian velocity distributions. Reference [11] showed that frictional sidewalls in a quasi-2D system produce velocity distributions with a peak near the zero-velocity region and overpopulated high energy tails. Over the range of parameters investigated, deviations from the Gaussian form are found to increase with increasing $\varepsilon_w$. An example can be seen in Fig. 3. The deviation is perhaps best quantified by analyzing the slope of double-logarithmic plots of the distributions' high-energy tails (see inset Fig. 3). A Gaussian distribution has a slope of 2. Experimentally, we find that as $\varepsilon_w$ is increased from $\approx 0$ to 0.70, the slope increases monotonically from 1.3 to 1.65. The dependence of the population of high-velocity particles on $\varepsilon_w$ was still observed when only data corresponding to the central region of the bed were analyzed. The fact that $\varepsilon_w$ influences particle velocities far from the walls themselves can perhaps be explained as follows: dissipative sidewalls create a localized region of high density in their vicinity. Thus, the density of the remainder of the system will decrease, leading to a reduced collision rate and hence higher particle velocities [23]. Decreasing $\varepsilon_w$ further lowers the density of this region, leading to a larger number of higher-energy particles.

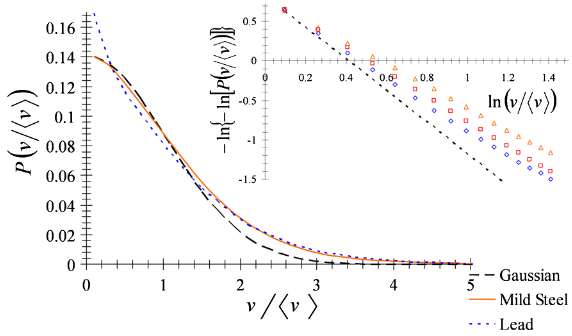


FIG. 3 (color online). The velocity probability distribution function for the two extremal values of $\varepsilon_w$ compared to the Gaussian distribution. *Inset:* double-logarithmic plot of the high energy tails of the velocity distributions for steel (open diamond), tufnol (open square) and lead (open upward triangle) sidewalls compared to the Gaussian.

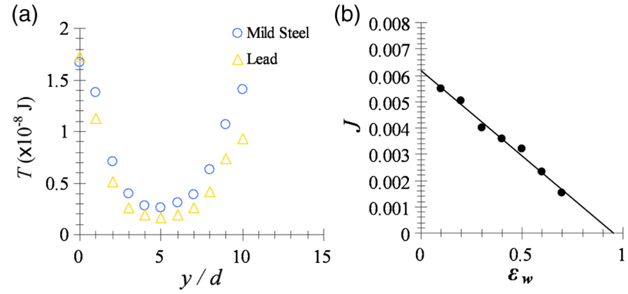


FIG. 4 (color online). (a) Experimental vertical temperature profiles for $W^* = 50$, $\tilde{W} = 20.3$. (b) Convective flow rate as a function of $\varepsilon_w$ for $W^* = 100$, $\tilde{W} = 49.4$ from simulation data.

From these results, we can draw two conclusions. First, it has been demonstrated that interactions between particles and their containers must be considered not only for the special case of a constrained monolayer (as concluded by [11]), but also when dealing with more realistic open, 3D setups. Second, the similarity between the results obtained here, in a highly fluidized system where collisional sidewall interactions dominate, and those of [11], where frictional interactions dominate, support the concept that the specific mechanisms underlying sidewall dissipation are unimportant, and that this dissipation can be characterized by a single effective inelasticity, an idea similar to that proposed in [25].

*Sidewall effects in larger systems.*—In order to test the validity of our main observations in a larger system, supplementary experiments were performed using $\approx 17\,500$ glass particles of 2 mm diameter, thus giving the system a width of $50d$ and a static bed height $H \approx 7$ layers. Despite the significant corresponding increase in $\tilde{W}$, the qualitative effects of $\varepsilon_w$ on both $J$ and $T$ are still clearly observable, as illustrated in Fig. 4. Unlike Fig. 2, this figure has been left un-normalized to demonstrate the fact that, as well as affecting the *form* of the $T$ profiles, $\varepsilon_w$ also modifies the *mean temperature* of the system, including in the dense central region of the bed. The influence is, however, less pronounced at higher densities.

Because of limitations imposed by the resolution of the PEPT technique, a full quantitative analysis of convective motion was not possible for the increased system size. This being the case, simulations were also conducted using an event-driven molecular dynamics algorithm [26] in order to ascertain whether the previously observed relationship between $J$ and $N$ was also present in larger systems. The algorithm considers perfect hard spheres, implying binary collisions and no overlap or long-range forces between particles. Values of the static and dynamic friction coefficients known to accurately reproduce the behavior of similar vibrofluidized systems were chosen [27], and the use of the TC model [28] with a constant value $t_c = 10^{-6}$ ensured that inelastic collapse was avoided. Figure 4(b) shows simulation data of the variation of $J$ with $\varepsilon_w$ for a system of width $100d$ ($\tilde{W} = 49.4$). The observed trend corresponds closely to the original experimental data, lending credence to the idea that the phenomena observed here may be applicable even in larger-scale, real-world scenarios. The previously described qualtitative effects of $\varepsilon_w$ on temperature were also reproduced by the simulation. Noteworthy also is the fact that, for the simulations, the friction coefficient and tangential coefficient of restitution are held constant, the strong agreement between experiment and simulation providing another indication that the behaviours observed in this study are predominantly dependent on $\varepsilon_w$, with little *specific* dependence on the friction coefficient or tangential restitution coefficient individually.

*Conclusions.*—We have shown that, in a fully 3D, vertically vibrated system, the dissipative properties of the sidewalls containing a granular fluid have significant impact on many of its properties, including both the spatial and probabilistic distributions of temperature and density, the functional form of velocity distributions and the presence of and strength of convective motion. Moreover, we have found that this impact is not limited to the vicinity of the walls, but also propagates into the bulk of the system. Our results strongly imply the possibility of controlling various parameters within a fluidized granular bed, such as temperature gradients and convection strength, without altering the material properties of the system, *or* the manner in which it is driven.

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