Manipulating Polarization and Impedance Signature: A Reciprocal Field Transformation Approach

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We introduce a field transformation method for wave manipulation based on completely reciprocal and passive materials. While coordinate transformations in transformation optics (TO) change the size and shape of an object, field transformations give us direct control on the impedance and polarization signature of an object. Using our approach, a new type of perfect conductor can be realized to completely convert between transverse electric and transverse magnetic polarizations at any incidence angles and a perfect magnetic conductor of arbitrary shape can be mimicked by using anisotropic materials. The approach can be further combined with TO to enhance existing TO devices. For example, a dielectric cylinder can become completely transparent for both polarizations using bianisotropic materials.

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Transformation optics (TO) is becoming an emerging framework in designing optical components with unusual optical properties. Prominent examples include invisibility cloaks [1–13], optical illusion devices [14,15], static exclusion [16–20], and other transformation optical devices such as special classes of lenses and cavities [21,22]. The essential ingredient behind the scene is a coordinate transformation. Once a coordinate transformation between the virtual system (the perceived one) and the physical system is specified, it induces also a transformation on the material parameters, ε and μ tensors. These parameters are in turn realized by metamaterials. There are also some recent approaches in extending TO. For example, one can use the covariant nature of TO. If we start from a virtual configuration which is already bianisotropic, the induced medium from a coordinate transformation in the physical space is bianisotropic too [23–27]. These have been applied in transforming chiral and gyrotropic media in order to create different types of active, nonreciprocal and switching devices [28,29]. There are also approaches to consider general space-time transformations to generate an event cloak or complex coordinate transformations to have unidirectional reflection [30–34].

In this Letter, we propose a field transformation method in manipulating waves, which is complementary to the mentioned TO approaches about coordinate transformations. Figure 1(a) shows the action for a coordinate transformation in TO in stretching or compressing a plane wave traveling in two dimensions (the on-plane arrows). It modulates the phase of the plane wave on a 2D fabric. When there is an object at the center, it effectively changes the size and the shape of the object. On the other hand, the field transformation (FT) transforms the fields by multiplying the field vector with a matrix at each location. In its scalar version, its action is about stretching or compressing the above plane wave in the out-of-plane direction, as shown in Fig. 1(b). It modulates the amplitude of the plane wave (the out-of-plane arrows). As we shall see later, it is equivalent to changing the impedance or the polarization characteristics of the object in our scheme. In this perspective, FT complements the coordinate transformation in TO. Here, we scale the field amplitudes directly without the need of a coordinate transformation. The scheme we propose should not be confused with the amplitude variation inside a gain or loss medium, which is induced from complex coordinate transformation proposed in Refs. [33,34]. The transformation in our scheme can in fact be fulfilled by reciprocal and lossless materials. We would explore how FT can manipulate waves in terms of polarization, which will need anisotropic materials, and in terms of impedance, which will need bianisotropic materials.

Let us confine to 2D in-plane wave propagations on the x-y plane with both material parameters and fields invariant in the z direction. We assume the virtual space before FT

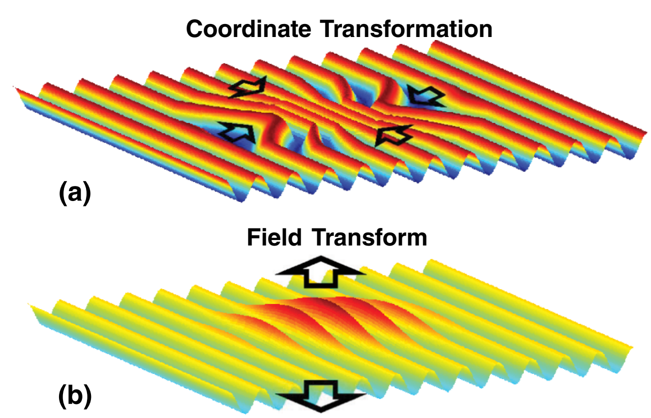

FIG. 1 (color online). (a) Coordinate transformation gives phase control, changing size and shape of object. (b) Field transformation manipulates the wave amplitudes, changing impedance and polarization signature of object.

has decoupled wave propagations for TE (transverse electric, E_z-field) and TM (transverse magnetic, H_z-field) polarizations separately. Maxwell's equations (in Heaviside-Lorentz units) can then be written as

$$\begin{aligned} \nabla \times \mu_{TT}^{-1} \cdot \nabla \times \hat{z} E_z &= k_0^2 \varepsilon_{zz} \hat{z} E_z, \\ \nabla \times \varepsilon_{TT}^{-1} \cdot \nabla \times \hat{z} H_z &= k_0^2 \mu_{zz} \hat{z} H_z, \end{aligned} \tag{1}$$

where $\varepsilon_{TT}/\mu_{TT}$ is the 2×2 symmetric tensor (reciprocity assumed) for the transverse permittivity or permeability in the x and y directions and k_0 is the wave number in vacuum. We further assume the permittivity (ε) and permeability (μ) tensors satisfy

$$\varepsilon_{TT} = \mu_{TT}, \qquad \varepsilon_{zz} = \mu_{zz}. \tag{2}$$

Usually we just transform from a simple isotropic homogneous medium, in a similar spirit of TO. We consider such a general case for the ease of discussion later when we combine FT with TO. From this point onwards, we denote the above set of fields and material parameters in the virtual space with a superscript "(0)." Now we apply a form of FT defined by

$$\begin{pmatrix} E_z \\ iH_z \end{pmatrix} = \begin{pmatrix} u\cos\phi & -u\sin\phi \\ v\sin\phi & v\cos\phi \end{pmatrix} \begin{pmatrix} E_z^{(0)} \\ iH_z^{(0)} \end{pmatrix}. \tag{3}$$

The special case with $\phi(x, y) = 0$ returns to the simple scalar FT that Fig. 1(b) shows intuitively for the operation on one of the polarizations. Then Maxwell's equations are proved to be still valid on the transformed fields (E_z, H_z) if we use a more complicated yet reciprocal physical medium with constitutive relation

$$\begin{pmatrix} D_T \\ D_z \\ iB_T \\ iB_z \end{pmatrix} = \begin{pmatrix} \varepsilon_{TT} & \varepsilon_{Tz} & 0 & \kappa_{Tz} \\ \varepsilon_{zT} & \varepsilon_{zz} & \kappa_{zT} & 0 \\ 0 & \eta_{Tz} & \mu_{TT} & \mu_{Tz} \\ \eta_{zT} & 0 & \mu_{zT} & \mu_{zz} \end{pmatrix} \begin{pmatrix} E_T \\ E_z \\ iH_T \\ iH_z \end{pmatrix}, \tag{4}$$

while the transverse fields are transformed according to

$$\begin{pmatrix} E_T \\ iH_T \end{pmatrix} = \begin{pmatrix} v^{-1}\cos\phi & v^{-1}\sin\phi \\ -u^{-1}\sin\phi & u^{-1}\cos\phi \end{pmatrix} \begin{pmatrix} E_T^{(0)} \\ iH_T^{(0)} \end{pmatrix}. \tag{5}$$

In Eq. (4), the ε and μ tensors are transformed according to

$$\varepsilon_{TT}/v^2 = \mu_{TT}/u^2 = \varepsilon_{TT}^{(0)}, \qquad \varepsilon_{zz} u^2 = \mu_{zz} v^2 = \varepsilon_{zz}^{(0)}, \tag{6}$$

with new bianisotropic terms (κ and $\eta = \kappa^T$ for reciprocity) induced by the scaling factor $u(x, y)$ and $v(x, y)$:

$$\begin{aligned} \kappa_{Tz} &= \eta_{zT}^T = -\hat{z} \times \nabla \ln v / k_0, \\ \kappa_{zT}^T &= \eta_{Tz} = -\hat{z} \times \nabla \ln u / k_0. \end{aligned} \tag{7}$$

The definitions and the derivations of the FT are given in more details in the Supplemental Material [35]. In fact, Eq. (6) can be regarded as the "reduced parameter approximation" often employed to redistribute the electric and magnetic response of metamaterials by keeping the local dispersion surface unchanged after a transformation medium is obtained [5]. If we impose the additional bianisotropic terms listed in Eq. (7) on a transformation medium, we can regard the reduced parameters become rigorous without approximation. It gives FT a rigorous framework for impedance control as we shall see. In Eq. (3), a nonzero $\phi(x, y)$ induces additional off-diagonal terms between the transverse and the z components in permittivity and permeability tensors as

$$\begin{aligned} \varepsilon_{Tz} u/v = \varepsilon_{zT}^T u/v = -\mu_{Tz} v/u = -\mu_{zT}^T v/u \\ = -\hat{z} \times \nabla\phi/k_0. \end{aligned} \tag{8}$$

This additional anisotropy couples the TE and TM polarizations together, in contrast to the case of TO.

As the first example, the new framework enables us to implement a special kind of perfect conductor defined with boundary condition

$$\begin{aligned} E_z^{(0)} - iH_z^{(0)} &= 0, \\ E_x^{(0)} + iH_x^{(0)} &= \partial_y (E_z^{(0)} + iH_z^{(0)}) = 0, \end{aligned} \tag{9}$$

where $\hat{\mathbf{y}}$ is the surface normal of the conductor and the region below (above) $y = 0$ is the conductor (air). Here, we refer it as a type-2 perfect conductor (PC2) for its difference in the reflection characteristics from conventional perfect conductors [36]. In particular, Eq. (9) implies that the TE and TM polarizations can be completely reflected into each other independent of the incident angle on the x-y plane. Here, we put an anisotropic coating on top of a perfect electric conductor (PEC) to mimic such a fictional boundary [Fig. 2(a)]. By putting $u = v = 1$ and $\phi = \pi/4$

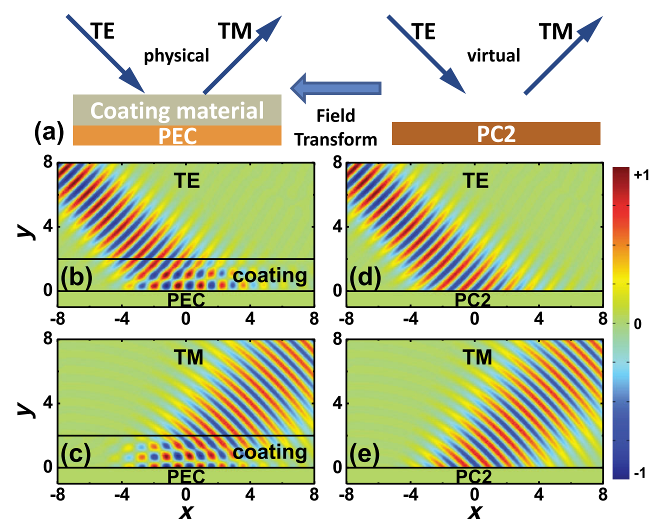

FIG. 2 (color online). (a) Anisotropic coating on PEC to mimic a type-2 perfect conductor (PC2). (b), (c) Simulation results for TE beam impinging on the coated PEC. The beam is completely converted to a reflected TM wave. (d), (e) Results directly simulating the fictional PC2 boundary condition Eq. (9).

into Eqs. (3) and (9) is then transformed to the PEC boundary condition $E_z = E_x = 0$. We design the coating such that ϕ varies linearly from $\pi/4$ (at $y = 0$, PC2) to 0 (at $y = h = 2$, air). By putting $u = v = 1$ and the function ϕ into Eqs. (6) to (8), the transformed medium is homogeneous and is just the same as vacuum except the additional off-diagonal terms in permittivity and permeability:

$$\varepsilon_{xz} = \varepsilon_{zx} = -\mu_{xz} = -\mu_{zx} = -\Delta\phi/(k_0 h), \qquad (10)$$

with $\Delta\phi = \phi(\text{object}) - \phi(\text{air}) = \pi/4$. Figures 2(b) and 2(c) show the E_z- and H_z-field patterns when a TE polarized beam with wavelength $\lambda = 1$ (the same wavelength is used for all simulations in this work) impinges on the surface at 45 deg. Above the coating, there is no reflected TE wave and the incident wave is completely converted into a TM polarized beam reflected at the same angle. This is equivalent to the boundary condition Eq. (9), which is also simulated directly with results shown in Figs. 2(d) and 2(e) for comparison. We note that TM can be also completely converted to TE waves and the perceived PC2 boundary is at $y = 0$, not on the upper surface of coating. The numerical calculations here are implemented by simulating Maxwell's equations with the most general 6×6 constitutive matrix with COMSOL MULTIPHYSICS (with more details given in the Supplemental Material [35]). The polarization conversion ratio, defined as the intensity of the cross-polarization over the total intensity of reflected waves in Ref. [37], is 100% and is independent of the incident angle here.

In the second example, we transform a PEC object into a perfect magnetic conductor (PMC). A PMC can be mimicked by using a PEC with an air gap or by using a high-impedance surface for plane waves for a limited range of incidence angles [38,39]. However, it is generally more difficult to have a PMC of arbitrary shape or working for more general excitations. Here, by having an anisotropic coating on the surface of a PEC arrow [in Fig. 3(a)], we make an optical illusion that the observer outside perceives the object as a PMC of the same shape and size in air. The coating consists of different pieces (number 1 to 7) of the same material with thickness 0.4. For example, the second piece is the medium similar to vacuum and with additional anisotropy in Eq. (10) with $\Delta\phi = \pi/2$ this time (i.e., the boundary condition is transformed from PEC to $H_z = \partial_y E_z = 0$). All the other pieces are just the same anisotropic material but with the positive y axis oriented to the local surface normal. Figure 3(c) shows the full wave simulation (E_z field) when the PEC arrow with coating is impinged by a plane wave of TE polarization from the left. The scattered field outside the coating resembles that of a PMC arrow in air [Fig. 3(d)]. In contrast, the PEC arrow without coating will cut the field sharply, as shown in Fig. 3(e). The object appears as PMC for other kinds of excitation, e.g., a point source, and also for TM polarization under similar arguments as well.

In the final example, we would like to demonstrate the additional control over impedance. We further combine FT with TO to make a purely dielectric cylinder completely transparent in air for both TE and TM polarizations. For TO, it requires us to compress an air cylinder to have a refractive index of the target cylinder. It is called the field concentrator [40], which has a radial map between the virtual (r') and the physical (r) coordinates, e.g., by a linear map [first "arrow" in Fig. 4(a)]:

$$r' = a' + (b - a')\frac{r - a}{b - a}, \qquad (11)$$

for the physical shell of radius r from $a = 1.5$ to $b = 4$, and

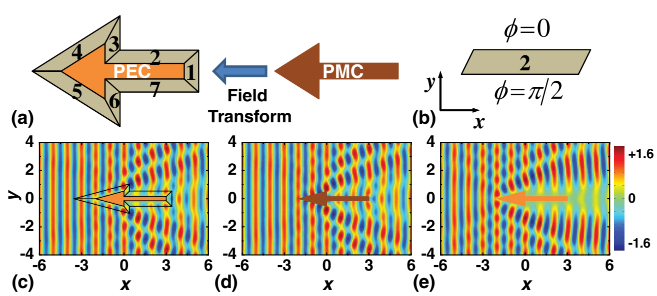

FIG. 3 (color online). (a) Coating an arrow-shaped PEC with anisotropic materials mimicking a PMC object of the same shape. (b) One piece of the anisotropic material with ϕ varying from 0 (outside) to $\pi/2$ (boundary of PEC). (c)–(e) Full wave simulations for a TE wave impinging from the left on (c) the coated PEC arrow, (d) a PMC arrow, and (e) a PEC arrow of the same shape.

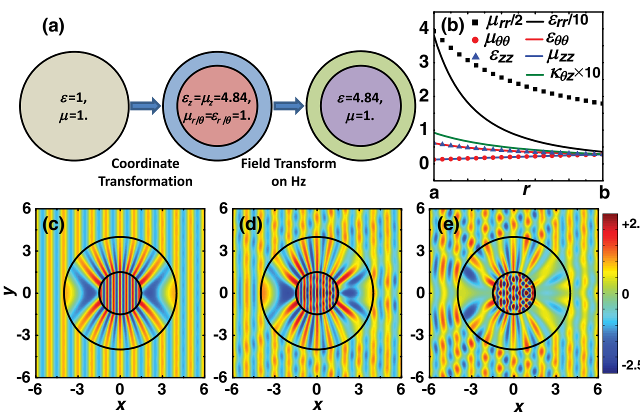

FIG. 4 (color online). (a) A radial coordinate transformation and a subsequent scalar field transformation on H_z field to make a purely dielectric cylinder transparent for both TE and TM polarizations with material parameters of cylinder listed along the transformation. (b) Material parameters of the final shell from radius a to b. (c)–(e): Simulated H_z-field profile for TM plane wave incident from the left towards a dielectric cylinder ($\varepsilon = 4.84$, $\mu = 1$) covered by (c) the final shell, (d) the reduced-parameter approximated shell, and (e) the conventional concentrator with only coordinate transformation.

$$r' = a'r/a, \tag{12}$$

for the core cylinder ($r < a$) with $a' = 3.3$. According to TO, it yields $\varepsilon_{zz} = \mu_{zz} = (a'/a)^2 = 4.84$ and $\mu_{rr/\theta\theta} = \varepsilon_{rr/\theta\theta} = 1$ for the core. For TE polarization, we only need to fulfill ε_{zz}, μ_{rr}, and $\mu_{\theta\theta}$ for the core and therefore the TO device is sufficient to make a dielectric cylinder with isotropic $\varepsilon = 4.84$ and $\mu = 1$ transparent. However, the same TO device does not work for the TM polarization due to the impedance mismatch: $\varepsilon_{\theta\theta}/\mu_{zz} \neq \varepsilon/\mu$. It causes severe scattering shown in the TM full-wave simulation in Fig. 4(e).

Now suppose we apply an additional scalar field transformation, the second arrow in Fig. 4(a), on the H_z field by choosing a continuous function $v(r)$ to vary from a constant $v = 1$ outside the device to another constant $v = a'/a$ within the core. There is no scaling on the E_z field: $u = 1$; and no cross-polarization conversion: $\phi = 0$. According to Eqs. (6)–(8), the core is now transformed to have isotropic permittivity $\varepsilon = (a'/a)^2 = 4.84$ and permeability $\mu = 1$ (target cylinder). As the field transformation is only applied on H_z, μ_{rr}, $\mu_{\theta\theta}$, and ε_{zz} are not changed from the original TO concentrator (so that the device continues to work for TE waves as expected). These material parameters are plotted as different symbols in Fig. 4(b). On the other hand, ε_{rr}, $\varepsilon_{\theta\theta}$, and μ_{zz} are scaled [using Eq. (6)] to

$$\begin{aligned} \varepsilon_{rr} &= v^2 \mu_{rr} = v^2 \frac{r'}{r}\frac{\partial r}{\partial r'}, \\ \varepsilon_{\theta\theta} &= v^2 \mu_{\theta\theta} = v^2 \frac{r}{r'}\frac{\partial r'}{\partial r}, \\ \mu_{zz} &= \frac{1}{v^2}\varepsilon_{zz} = \frac{1}{v^2}\frac{r'}{r}\frac{\partial r'}{\partial r}, \end{aligned} \tag{13}$$

and the additional bianisotropic term [Eq. (7)] is

$$\kappa_{\theta z} = \eta_{z\theta} = -v'(r)/(k_0 v). \tag{14}$$

As an example, we choose $v = r'/r$ and these material parameters are plotted as solid lines in Fig. 4(b). The bianisotropy used here can be achieved by using magnetic resonators (e.g., split-ring) with broken mirror symmetry [41,42] or detuned resonators [43,44]. The atoms should be aligned so that an electric field in the θ direction generates a magnetic dipole in the z direction. In addition, its value can be further reduced by using a thicker shell which has smaller $\nabla \ln v$ in Eq. (7). Figure 4(c) shows the full-wave simulation for the TM wave incident from the left. The cylinder is now transparent without causing scattering. The H_z field is gradually scaled up by 2.2 times ($v = a'/a$ for $r < a$) when it reaches the dielectric core. However, we note that the power concentration is still exactly the same as the case of TE waves. E_T is actually scaled down by the same factor according to Eq. (5) as the medium is reciprocal and passive. For comparison, Fig. 4(d) shows the case when we turn off the bianisotropy manually, corresponding to the reduced parameter approximation, some scattering is introduced. The field transformation thus provides an additional flexibility in matching impedance to make the dielectric cylinder completely transparent for both polarizations. In the above examples, we are transforming a vacuum; the framework can be extended to transforming other media such as a dielectric background (with more details given in the Supplemental Material [35]).

In conclusion, we have employed FT to modify the polarization and the impedance signature of an object. This is complementary to changing size and shape in TO. As a result, we have mimicked a PMC in arbitrary shape and a fictional perfect conductor to do a complete cross-polarization conversion between TE and TM waves (or equivalently reflect circularly polarized light only to its co-polarization). By combining FT and TO, FT can also improve existing functionalities of TO devices, e.g., by making a dielectric cylinder completely transparent for both polarizations. The investigations may also open up new applications on manipulating wave propagations with coupled polarizations. The FT approach introduced here will need metamaterials to fulfill the required flexible optical properties. These metamaterials are still reciprocal and passive; e.g., only anisotropy is needed for polarization control; further investigations are expected for the design of the metamaterials for constructing FT-enabled devices.

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