Cancellation of Internal Quantum Noise of an Amplifier by Quantum Correlation

Jia Kong,[1] F. Hudelist,[1] Z. Y. Ou,[1,3,*] and Weiping Zhang[1,2,†]

[1]*Department of Physics, Quantum Institute for Light and Atoms, East China Normal University, Shanghai 200062, People's Republic of China*

[2]*State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai 200062, People's Republic of China*

[3]*Department of Physics, Indiana University-Purdue University Indianapolis, 402 North Blackford Street, Indianapolis, Indiana 46202, USA*

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Quantum noise is usually added in an amplification process through the internal degrees of the amplifier. Coupling the squeezed state to the internal degree can suppress the extra noise. Here, we demonstrate another method: when the internal degree of the amplifier is correlated with the input signal via quantum entanglement, quantum destructive interference between the input and the internal degree may result in noise reduction at the amplified output. We achieve a quantum noise reduction of 2.3 dB at the output and an improvement of 4.0 ± 0.2 dB in signal-to-noise ratio during the amplification process with a quantum noise gain of 4.5 dB.

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It is generally believed that quantum noise must be added in a phase-insensitive amplifier through the internal degrees of the amplifier to preserve the quantum commutation relationship for the output of the amplifier [1], as shown in Fig. 1(a). It is this noise that makes it impossible to create a macroscopic Schrödinger catlike state by amplifying a microscopic quantum superposition state [2]. The added noise through the internal degrees of the amplifier is the key in the decoherence process from the microscopic quantum world to the macroscopic classical world. On the other hand, quantum amplification has already played an important role in quantum information and communication [3,4]. A noiseless amplifier may also be a quantum optical cloning amplifier [5]. Recently, there has been a renewed interest in the amplification of an entangled state [6] for quantum imaging [7,8].

With the availability of a squeezed state for quantum noise reduction, a strategy was proposed [9] and demonstrated [10] to place the internal degrees in squeezed states in order to reduce the extra noise, as shown in Fig. 1(b). Notice that this scheme only suppresses the excess noise thus at best preserves the signal-to-noise ratio (SNR) during the amplification process. This is true if the input and the internal degrees of the amplifier are independent of each other.

As is well known, however, quantum mechanics also allows entanglement of distinct systems. One property of quantum entanglement is the correlation of quantum fluctuations, which in some cases can be larger than what classical physics allows [11], and in other cases can be used to subtract out quantum noise [12,13]. This correlation is the basis for a number of quantum information protocols such as teleportation of a quantum state [14,15]. Therefore, if we make the internal mode of the amplifier entangled with the input mode, as shown in Fig. 1(c), the correlation in their quantum noise may lead to the noise cancellation in the output. The noise reduction here is not limited to the excess noise of the internal degrees but also applied to the input noise. So, the amplified output may have better SNR than the input. Here in this Letter, we wish to report on the first experimental implementation of the cancellation of the internal noise of an amplifier by quantum entanglement. We study the quantum noise performance of a parametric amplifier from a four-wave mixing process with correlated quantum noise between the input signal field and the amplifier's internal idler field. We find that the output noise can be reduced due to destructive quantum interference between the input and the internal mode of the amplifier in an entangled state and is 2.3 dB below the output noise level when the amplifier is

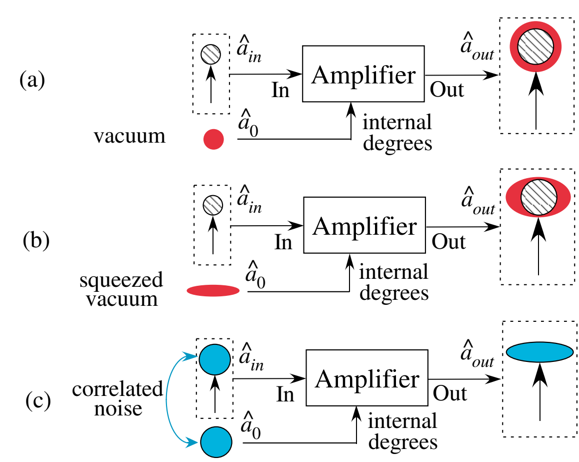

FIG. 1 (color online). Schematic diagram for an amplifier with its internal degrees in (a) vacuum; (b) squeezed states; (c) entangled states with the input. The dotted boxes indicate signal (arrow) with its noise (circles and ellipses).

in vacuum. This leads to an improvement of 4.0 ± 0.2 dB in the signal-to-noise ratio while the signal is amplified by 4.5 ± 0.2 dB.

Quantum theory of amplification was well established in the 1980s [1,16] and experimentally tested [5,10,17]. A phase insensitive amplifier can be described quantum mechanically by [1,16]

$$\hat{a}_{\rm out} = G\hat{a}_{\rm in} + \hat{F}, \tag{1}$$

where G is the amplitude gain of the amplifier. In Eq. (1), the commutation relation for $\hat{a}_{\rm out}$ requires the existence of $\hat{F}$, which is related to the "internal degrees" of the amplifier. These degrees are usually unattended and left in vacuum state, as shown in the schematic diagram in Fig. 1(a). $\hat{F}$ is responsible for spontaneous emission, which is a source of extra noise added to the signal from the amplifier. Even though $\hat{F}$ may be related to many modes of the internal degrees of the amplifier, we can define a new operator $\hat{a}_0 \equiv \hat{F}^\dagger/g$ with $g \equiv \sqrt{G^2-1}$. From Eq. (1), we see that $[\hat{a}_0, \hat{a}_0^\dagger] = 1$ so that $\hat{a}_0$ corresponds to a single mode annihilation operator. When $\hat{F}$ (or $\hat{a}_0$) is in vacuum, the photon numbers of the input and output are related by

$$N_{\rm out} = G^2 N_{\rm in} + |g|^2. \tag{2}$$

Obviously, $|g|^2$ is the spontaneous emission of the amplifier and acts as the noise from the vacuum contribution of the internal degrees. For the quantum noise reduction scheme with squeezed states, we rewrite Eq. (1) with the new operator for internal degrees in the form of quadrature-phase amplitudes:

$$\hat{X}_{\rm out} = G\hat{X}_{\rm in} + |g|\hat{X}_0(\varphi_g), \tag{3}$$

where $\hat{X}_s \equiv \hat{a}_s + \hat{a}_s^\dagger (s = {\rm out, in})$ and $\hat{X}_0(\varphi_g) \equiv \hat{a}_0 e^{-j\varphi_g} + \hat{a}_0^\dagger e^{j\varphi_g} (e^{j\varphi_g} \equiv g/|g|)$. So, the excess noise in $\hat{X}_{\rm out}$ is suppressed if the internal mode $\hat{X}_0$ is in a squeezed state.

Furthermore, we find that the output is a superposition of the input and the internal field. So if we place the input $\hat{X}_{\rm in}$ and the internal degree $\hat{X}_0(\varphi_g)$ in an entangled state, quantum interference between $\hat{a}_{\rm in}$ and $\hat{a}_0$ will occur and destructive interference may lead to the cancellation of the quantum noise of the input and internal fields and quantum noise reduction at the output [18].

The detailed experimental layout is shown in Fig. 2. Here both the amplifier and the entangled source are based on a nondegenerate four-wave mixing process in hot Rb-85 atomic vapor cells [19]. The four-wave mixing process without any signal injection will produce an entangled source [20]. But in principle, the entangled source may be generated by other means. The atomic cell, when pumped by a strong beam with about 1.4 GHz blue detuned above the transition line of Rb-85 $F = 2 \rightarrow F'$ at 795 nm, produces two fields at an angle of 0.4 degrees from the pump at two sides near symmetrically (see inset of Fig. 2 for the energy diagram of relevant components). The two fields, denoted as "signal" and "idler," are entangled with each other in their quadrature-phase amplitudes [20]. Therefore, their noise are correlated quantum mechanically. The degree of correlation depends on the pump power and is related to the noise level of each individual field [21]. In Fig. 2, the first Rb cell produces the fields with quantum noise correlation. The fields are shown as dashed blue and red lines (dashed dark and gray lines in printed version).

On the other hand, such a device can also act as an amplifier for a seed injected into the input "signal" port [6], with the amplifier's gain depending on the pump power. In Fig. 2, the second Rb cell (labeled as "Amplifier") serves as the amplifier. In this case, the idler field corresponds to the internal degree of the amplifier and is usually in vacuum as shown in Fig. 1(a). This occurs when we block the input to the idler mode (dashed red line). But to achieve noise reduction in amplification, the internal mode or the idler mode of the amplifier is coupled to one of entangled fields produced from the first Rb cell (labeled as "Entangled Source") while the other one of the entangled fields is injected into the signal mode of the amplifier after encoded with a small signal (see later for more). We employ a 4-F system ($L1$, $L2$) to match the spatial modes between the entangled source and the inputs of the amplifier.

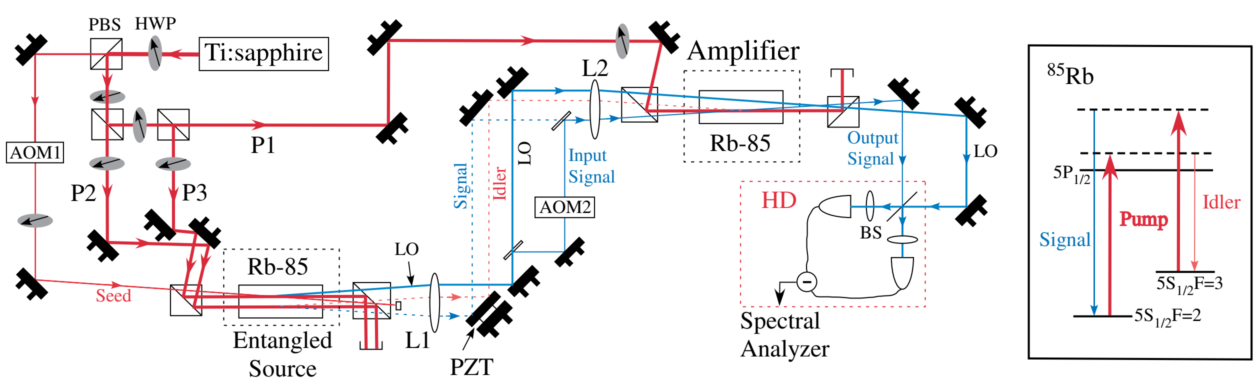

FIG. 2 (color online). Detailed experimental arrangement.

During the experiment, the Rb-85 vapor cells are temperature stabilized at 117 °C. They are, respectively, pumped by a vertically polarized beam ($P1$, $P2$) at a maximum power of 400 mW with a waist of 500 μm. The pump beams are from a Ti:sapphire laser frequency stabilized to a stable reference cavity. The pump power can be adjusted with a combination of polarization beam splitters (PBSs) and half wave plates (HWPs).

The quantum noise of the amplifier and the entangled source is measured by a balanced homodyne detection (HD) ensemble. To produce a local oscillator mode matched to the signal, we use another four-wave mixing process side by side in the same cell as the entangled source but pumped by $P3$. The difference is that we inject to the process a seed at the ‘‘idler’’ frequency and use the generated ‘‘signal’’ field as the LO (shown as a solid blue line in Fig. 2). The seed at ‘‘idler’’ frequency is 3.04 GHz redshifted from the pump beam by an acousto-optic modulator (AOM1). At the output of the atomic cell, the amplified seed is discarded but the accompanying conjugate beam at the other side of the pump beam serves as the LO for the homodyne detection. Its frequency is right at the ‘‘signal’’ frequency that is 3.04 GHz blueshifted from the pump.

We first measure the noise performance of the amplifier under various conditions by blocking the coherent signal. Figure 3(a) shows the typical results in logarithmic scale. Trace (i) is the vacuum noise or the shot noise level (SNL), which is the reference for all the noise levels. It is measured

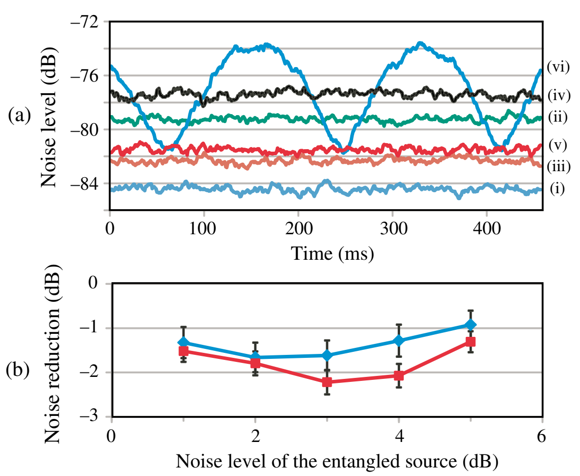

FIG. 3 (color online). (a) Noise levels of the amplifier under various inputs: (i) shot noise level; (ii) amplified noise level with the input and idler in vacuum; (iii) input noise level for a thermal state; (iv) amplified noise level with the input in a thermal state and the idler in vacuum; (v) noise level of the amplifier with the input and the idler entangled; (vi) noise level of the amplifier with the phase between the two fields of the entangled source scanned. (b) Noise reduction of the amplifier with the entangled input versus the noise level of the entangled source for the noise gain of the amplifier at 4 dB (blue diamond), and 6 dB (red square), respectively.

by blocking the field to the HD ensemble. Trace (ii) shows the amplified vacuum noise level when we block all the inputs to the amplifier. This trace relative to the SNL gives the amplifier's quantum noise gain G_q [$G_q = 2G^2 - 1$ from Eq. (1)]. This corresponds to Fig. 1(a).

Next, we inject one of the entangled fields into the input port of the amplifier. It is well known that one of the entangled fields alone has thermal excess noise [12] (it is impossible to create correlated quantum noise at vacuum level). The other one of the entangled fields is discarded. So the internal mode of the amplifier is still in vacuum. The noise level of the input field is shown as trace (iii) in Fig. 3(a) and is 2.1 dB above the shot noise level. Trace (iv) shows the noise level of the output of the amplifier in this case. This still corresponds to Fig. 1(a) but with a noisy input.

Most interesting are traces (v) and (vi), which are the output noise level of the amplifier when its internal mode (idler) is entangled with the input (signal), as sketched in Fig. 1(c). Trace (vi) is for the case with the phase between the two fields scanned while trace (v) is for the phase held at minimum noise level. Notice that trace (v) is 4 dB below the output noise level of the amplifier with vacuum at its internal mode [trace (iv)], indicating noise reduction of the internal noise of the amplifier. It is even 2.3 dB lower than the noise level when both the input and the idler are in vacuum [trace (ii)], indicating reduction of vacuum input noise as well.

Let us characterize this noise reduction. We use the amplified vacuum noise level of the amplifier [trace (ii)] as a reference. The noise reduction due to entanglement is the amount of the output noise level [trace (v)] below this reference [trace (ii)]. In Fig. 3(b), we plot the noise reduction due to entanglement as a function of the noise level of the entangled source at two different gains of the amplifier. The noise level of the entangled source is measured as the noise level of its individual field, as trace (iii) shown in Fig. 3(a). The figure shows that there is an optimum noise level of entanglement at each gain of the amplifier for the minimum noise reduction. This is because the input and the internal modes of the amplifier are superimposed with different weights of G and g, respectively, as seen from Eq. (3). Thus, less than perfect correlation is needed for the optimum noise cancellation [22]. The best noise reduction occurs at the noise level of 3 dB for the entangled source when the quantum noise gain of the amplifier is 5 dB [not shown in Fig. 3(b)]. This will be our operating point for the following study.

Next, we encode a coherent signal in the input to the amplifier. It is split from the LO beam and frequency-shifted up by 2 MHz with two AOMs (AOM2) and is combined with the signal field of the entangled source with a 10/90 beam splitter before being injected into the input port of the amplifier. Figure 4 shows the signal level and the noise level at both the input (lower traces) and the

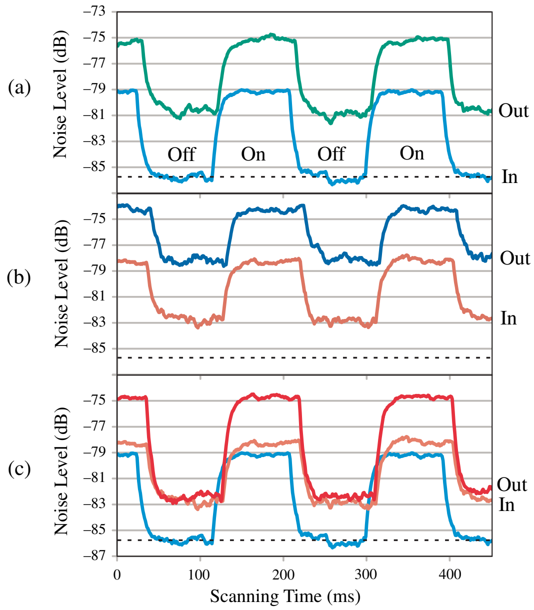

FIG. 4 (color online). Signal and noise levels of the input and output of an amplifier under different situations: (a) input signal at vacuum level and the internal degree of the amplifier is in vacuum; (b) input signal with extra noise and the amplifier is in vacuum; (c) input is the same as in (b) but the internal degree of the amplifier is entangled with the input.

output (higher traces) of the amplifier under various conditions as described below. The coherent signal is switched on and off by the AOMs so that we can measure both the noise level (= off) and the signal strength (= on-off). Figure 4(a) corresponds to the situation when we block the entangled source to the amplifier so that both the signal input and the "idler" mode of the amplifier are in vacuum. This is the way in which most amplifiers are operated. The input is measured when we set the gain of the amplifier to unity by blocking the pump to it. From the figure, we find the input SNR is (on-off)/off $= 5.4 \pm 0.2$ dB and the output SNR is 4.2 ± 0.2 dB. There is a degradation of 1.2 dB in SNR due to the noise from the internal mode ("idler") of the amplifier. Figure 4(b) is for the case when the input is in a thermal state with excess noise but the internal mode is still in vacuum. The input SNR is 2.6 ± 0.2 dB while the output SNR is 1.7 ± 0.2 dB with a 0.9 dB degradation in SNR. This degradation is smaller than the case when the input is in vacuum. This is because the added internal noise (vacuum) is smaller than the noisy input and thus makes less influence.

Figure 4(c) shows the case when the internal mode is correlated with the input mode; i.e., the amplifier is in an entangled state [output trace in Fig. 4(c)]. The output has an SNR of 6.6 ± 0.2 dB. The input at the amplifier is still in the thermal state with excess noise [the input traces in Figs. 4(b) and 4(c)] but the internal mode is now coupled to the idler mode of the entangled source and its noise is correlated with the input. There is an improvement of 4.0 dB over the input SNR [input traces in Figs. 4(b) and 4(c) with an SNR of 2.6 ± 0.2 dB] while the coherent signal is amplified by 4.5 ± 0.2 dB. Therefore, Fig. 4(c) demonstrates the advantage with entangled states: when the amplifier is in an entangled state between its input and internal modes, the amplified output has better SNR than the input SNR; i.e., the noise figure can be smaller than one. This seems to violate our traditional belief that a linear amplifier cannot improve SNR during the amplification process. To understand this, we need to recall the fact that when an entangled state is involved, the noise information is encoded in both entangled fields. The true input SNR in this dual-beam case is calculated based on the noise difference between the two entangled fields (dual-beam noise). Under this condition, the internal mode of the amplifier is not independent but has to be considered of as a part of the input. Reference [18] showed that after this consideration, the SNR of the amplified output is at best equal to the dual-beam SNR of the input.

Moreover, Fig. 4(c) shows another surprising effect. Here, we replot the input signal at vacuum noise level (cyan trace with an SNR of 5.4 ± 0.2 dB), which is the same as the one in Fig. 4(a). Notice that the SNR of the amplified output with the amplifier in an entangled state is even larger than the input SNR at vacuum noise level by 1.2 dB. It should be noted that the apparent increase of SNR during amplification process was reported before [8] but it was due to the existence of losses in the detection system: loss will reduce the amplified noise at output but does not change the input vacuum noise level, leading to an apparent increase in SNR. If the loss is taken into account, there is no increase of SNR. For our system, the overall loss is estimated at 14%. So, loss will lead to an apparent increase of 0.7 dB in the SNR but is still short for the observed increase of 1.2 dB.

In fact, the demonstrated increase in SNR of the amplified output is due to entanglement-induced noise reduction. It is known that the dual-beam noise level (noise difference) of an entangled state can be smaller than the vacuum noise level of each individual beam [12,23]. The amplifier with its relation in Eq. (3) provides a way to subtract the quantum noise in the two fields, leading to output noise even below the amplified vacuum noise.

In conclusion, we have demonstrated quantum noise reduction in a phase-insensitive amplifier whose internal mode is entangled with the input. Because entangled beams are used, our amplification scheme cannot be applied directly to amplify an arbitrary signal. It must involve the entangled beams for generating a signal. In one likely scenario, we will first form a coherent beam encoded with the entangled state just like what we did in the experiment and then use it to probe some delicate systems, where low light power is preferred. Therefore, the signal is usually very weak after the probe goes through the samples. Then we can use our scheme to amplify the signal with reduced noise. On the other hand, like any application involving a quantum state, e.g., squeezed states, quantum correlation is sensitive to the loss in the system because of the uncorrelated vacuum noise in the loss. But as shown in the Supplemental Material [24], our scheme can still eliminate the excess noise from the amplifier's internal mode even with a loss of 50%.

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*zou@iupui.edu
†wpzhang@phy.ecnu.edu.cn

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