Optimal Coherent Control to Counteract Dissipation

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We study to what extent the detrimental impact of dissipation on quantum properties can be compensated by suitable coherent dynamics. To this end, we develop a general method to determine the control Hamiltonian that optimally counteracts a given dissipation mechanism, in order to sustain the desired property, and apply it to two exemplary target properties: the coherence of a decaying two-level system and the entanglement of two qubits in the presence of local dissipation.

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Genuine quantum features such as entanglement or coherence are resources as precious as they are fragile, and their uncovering usually requires strong efforts in isolating and controlling quantum systems. Without thorough measures, decoherence efficiently shields the quantum world from our access and hides it behind its classical guise. While there has been unprecedented progress in the quantum control of various model systems, e.g., ions [1], quantum dots [2], or cold atoms [3], it is impossible to completely decouple these systems from their environment and thus to fully suppress the detrimental effect of decoherence. Standard optimal control techniques therefore focus on accessing quantum features in the transient regime, and the exploration and exploitation of quantum properties is consequently confined to a finite, generically short time window.

There are, however, ways to keep the window to the quantum world enduringly open, e.g., by encoding quantum features in topological properties of a system [4] or by engineering a dominant environment that drives the system into a highly nonclassical stationary state [5,6]. While these approaches in principle permit one to prepare arbitrary nonclassical quantum states, they require in general an exceedingly large overhead of resources.

In this Letter, we therefore ask to what extent already standard Hamiltonian control can enduringly counteract the detrimental effect of decoherence. Explicitly, we seek Hamiltonians that optimally uphold, on *asymptotic* time scales, a given control objective (e.g., coherence, entanglement, or fidelity with respect to a target state) in the presence of dissipation. Such an asymptotic time behavior can be meaningfully formulated for static and, more generally, periodically time-dependent Hamiltonians. In the latter case, the asymptotic dynamics are periodic cycles in state space, reducing to stationary states in the static case.

For our goal to single out the optimal among all conceivable control Hamiltonians, it is not advisable to directly scan the space of Hamiltonians, as the latter cannot efficiently be parametrized. We therefore approach the problem from a different perspective and determine the optimal stationary state or asymptotic cycle directly, i.e., independently of the Hamiltonian. The crucial insight behind this is that physically admissible trajectories in state space are strongly constrained by the dissipative part of the dynamics. It thus turns out that one can characterize all possible stationary states or asymptotic cycles from the dissipative dynamics alone.

While our approach can be applied to the optimization of arbitrary control objectives, we demonstrate its viability with two physically relevant examples: the coherence between the ground and excited states of a decaying two-level system and entanglement of two qubits in the presence of local dissipation.

Static control Hamiltonians.—We consider an open quantum system evolving under a Lindblad master equation [7]

$$\dot{\rho}(t) = i[\rho(t), H(t)] + \mathcal{D}[\rho(t)] \qquad (\hbar = 1), \tag{1}$$

with a dissipator $\mathcal{D}(\rho) = \sum_k \gamma_k [L_k \rho L_k^\dagger - (1/2)\{L_k^\dagger L_k, \rho\}_+]$ composed of Lindblad operators L_k and rates γ_k. For an arbitrary but fixed dissipator $\mathcal{D}(\rho)$, our goal is to optimize the stationary state ρ_{ss} (for static H) or asymptotic cycle $\rho_{ac}(t)$ [for periodic $H(t)$] of Eq. (1) with respect to an arbitrary objective function $\mathcal{O}(\rho)$. We emphasize that this definition of optimality differs from quantum control scenarios that aim at rapidly preparing a given target state on time scales when decoherence is negligible. The target states of such time-optimal protocols are reached quickly but persist only on transient time scales, whereas the optimal cycles in our approach may take a long time to emerge but then persist for arbitrarily long times.

It is instructive to investigate static control Hamiltonians first. Direct optimization over all conceivable Hamiltonians H involves the stationarity condition

$$0 = i[\rho_{ss}, H] + \mathcal{D}(\rho_{ss}), \tag{2}$$

in order to infer the stationary state ρ_{ss} for a given H. Its inversion typically requires numerical means and must be repeated for each sample Hamiltonian, rendering this approach impractical already in low-dimensional systems. Therefore, we develop a different strategy here. Instead of starting from Hamiltonians, we base the optimization on the set of *stabilizable states* $\mathcal{S}$ [8]:

$$\mathcal{S} = \{\rho: \exists H \, \text{s.t.} \, 0 = i[\rho, H] + \mathcal{D}(\rho)\}. \tag{3}$$

It comprises all quantum states that become stationary under a suitable Hamiltonian. As shown below, this set can be characterized *independently* of the Hamiltonian. Optimization of an objective function $\mathcal{O}(\rho)$ can then be done in $\mathcal{S}$ directly.

To derive this Hamiltonian-independent characterization of $\mathcal{S}$, we exploit that the coherent dynamics induced by H and the dissipative dynamics induced by $\mathcal{D}(\rho)$ must compensate each other for a stationary state. Since the coherent part of the master equation (1) $i[\rho, H]$ necessarily leaves the spectrum of ρ invariant, this must also hold for the dissipator $\mathcal{D}(\rho)$ at a stationary state. In other words, $\mathcal{D}(\rho)$ does not modify the purity $p = \text{Tr}[\rho^2]$ or any higher moment $\text{Tr}[\rho^n]$ with $n > 2$. This implies $\partial_t \text{Tr}[\rho^n]|_{H=0} \overset{(1)}{=} n \, \text{Tr}[\rho^{n-1}\mathcal{D}(\rho)]$, if ρ is in $\mathcal{S}$. Since the moments of a d-dimensional quantum state are independent only up to $n = d$, this leads to $d - 1$ necessary conditions for ρ to be stabilizable:

$$\rho \in \mathcal{S} \Rightarrow \forall n \in \{2, \ldots, d\}: \text{Tr}[\rho^{n-1}\mathcal{D}(\rho)] = 0. \tag{4}$$

Denoting by $\mathcal{S}_n$ the set of states that fulfill Eq. (4) for fixed n, we have $\mathcal{S} \subset \bigcap_n \mathcal{S}_n$. For states with nondegenerate eigenvalues, criterion (4) is also sufficient [9], and hence $\mathcal{S} = \bigcap_n \mathcal{S}_n$. Note that this hierarchical characterization of $\mathcal{S}$ does not require reference to the stabilizing Hamiltonian H, in contrast to definition (3). Given a stabilizable state $\rho \in \mathcal{S}$, however, it is straightforward to derive the corresponding H from Eq. (2), based on the spectral decomposition $\rho = \sum_\alpha \lambda_\alpha |\alpha\rangle\langle\alpha|$:

$$H = \sum_{\alpha,\beta:\lambda_\alpha \neq \lambda_\beta} \frac{i\langle\alpha|\mathcal{D}(\rho)|\beta\rangle}{\lambda_\alpha - \lambda_\beta} |\alpha\rangle\langle\beta|. \tag{5}$$

To demonstrate the viability of our method, we first discuss the case of a single qubit. There, one finds an intuitive geometric representation of $\mathcal{S}$. Since $d = 2$, Eq. (4) imposes merely a single constraint ($n = 2$). In terms of the Bloch vector $\vec{r} = \text{Tr}[\rho\vec{\sigma}]$, this constraint defines a quadric hypersurface in the Bloch ball:

$$\vec{r} \in \mathcal{S} \Rightarrow \vec{r}(D\vec{r} + \vec{d}) = 0. \tag{6}$$

Here, the 3×3 matrix $(D)_{ij} = \text{Tr}[\sigma_i \mathcal{D}(\sigma_j)]$ and the vector $(\vec{d})_i = \text{Tr}[\sigma_i \mathcal{D}(\mathbb{1})]$ characterize the dissipator in Bloch notation. According to Eq. (6), a state $\vec{r}$ is stabilizable, if (and only if [10]) the dissipative flux $D\vec{r} + \vec{d}$ is orthogonal to $\vec{r}$, i.e., if it has no radial component.

Specifically, we consider a qubit exposed to the three most common incoherent processes: decay of the excited state at rate γ_-, absorption from the ground state at rate γ_+, and dephasing between the ground and excited states at rate γ_d. Experimental realizations of this scenario include both atomic and solid-state two-level systems, such as trapped ions [11], superconducting qubits [12], or color centers in diamond [13]. Specifically, in the latter case, the incoherent processes are triggered by a nuclear spin bath, resulting in typical incoherent rates in the kHz regime [14].

With these particular incoherent processes, $\mathcal{S}$ is the surface of a spheroid [8], with the polar axis pointing in the z direction; cf. Fig. 1. The polar and equatorial diameter depend on the incoherent rates. The optimal stationary state with respect to an arbitrary objective $\mathcal{O}(\vec{r})$ is now conveniently determined by maximizing over the surface of this spheroid; e.g., one may consider the coherence $\mathcal{C} = 2|\langle 0|\rho|1\rangle|$ between the ground and excited states. In Bloch notation, this objective corresponds to the distance to the z axis; see Fig. 1. Hence, the optimal coherence equals the equatorial semiaxis of the spheroid, yielding $\Gamma_-/2\Omega$, with $\Omega = \sqrt{\Gamma_+(\Gamma_+/2 + \gamma_d)}$ and $\Gamma_\pm = \gamma_- \pm \gamma_+$. According to Eq. (5), the corresponding Hamiltonian reads $H^* = -(\Omega/2)\sigma_y$. It can be realized, e.g., by resonantly driving the qubit with a Rabi frequency Ω [15].

As a higher-dimensional example, we consider two qubits ($d = 4$). The stabilizable states $\mathcal{S}$ then lie in the intersection of three hypersurfaces $\mathcal{S}_2$, $\mathcal{S}_3$, and $\mathcal{S}_4$. Similarly to a single qubit, one can represent ρ by a fifteen-dimensional Bloch vector $\vec{r}$ [16,17]. The lowest order constraint ($n = 2$) can then be cast into the same form as Eq. (6), defining again a quadric surface. The higher order constraints for $n = 3$, 4, however, lead to polynomial expressions of the third and fourth degree in $\vec{r}$. Therefore, instead of determining the optimal state in $\mathcal{S}$ directly, it is favorable to determine the optimal state $\vec{r}_*$ in $\mathcal{S}_2$ first and then to verify that $\vec{r}_*$ lies also in $\mathcal{S}$. If so, it must be the optimum in $\mathcal{S}$, since $\mathcal{S} \subset \mathcal{S}_2$. If not, the procedure provides an upper bound for the optimal value in $\mathcal{S}$.

Relevant target properties for two qubits address, e.g., their entanglement. The most detrimental situation is then

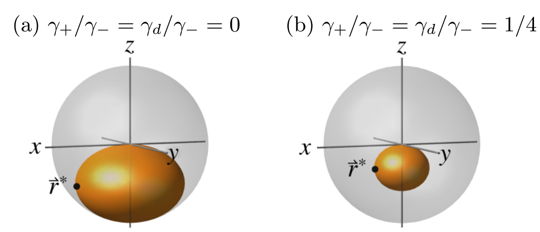

FIG. 1 (color online). Bloch representation of the set $\mathcal{S}$ of stabilizable states [surface of the orange (dark shaded) spheroid] for a single qubit, subject to spontaneous decay, excitation, and dephasing. The respective rates γ_-, γ_+, and γ_d determine the shape of $\mathcal{S}$. (a) For spontaneous decay only, the optimal stationary state $\vec{r}_*$ with respect to the coherence $\mathcal{C}$ (i.e., the distance to the z axis) reaches $\mathcal{C} = 1/\sqrt{2}$. (b) With finite γ_+ and γ_d, the ellipsoid is contracted, reducing the optimal value of $\mathcal{C}$.

certainly encountered when the incoherent processes act locally on both qubits. Therefore, we exclusively consider qubits undergoing (individual) spontaneous decay at rate γ_-, as encountered in the experimental scenarios mentioned above for a single qubit.

As a specific entanglement objective, we study the fidelity $\mathcal{F}(\rho) = \langle\Psi_+|\rho|\Psi_+\rangle$ with respect to the maximally entangled Bell state $|\Psi_+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$. It is relevant, e.g., in teleportation protocols [18]. Its optimization over $\mathcal{S}_2$ can be carried out analytically, yielding the optimal stationary state

$$\rho^* = (1/2)|00\rangle\langle 00| + (1/2)|\Psi_+\rangle\langle\Psi_+|, \tag{7}$$

with $\mathcal{F}(\rho^*) = 1/2$. The Hamiltonian $H(\alpha, \beta) = \mathbb{1} \otimes (\alpha\sigma_z + \beta\sigma_x) + (\alpha\sigma_z + \beta\sigma_x) \otimes \mathbb{1} - 2\alpha(\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+)$ stabilizes ρ^* in the limit of $\beta/\gamma_- \to \infty$ and $\alpha/\beta \to \infty$ [19]. This Hamiltonian is readily realized in various experimental setups; in particular, the interaction term describes an excitation hopping mechanism, realizable with trapped ions [1], superconducting circuits [20], dipole-dipole interactions between excitons [21,22], color centers in diamond [23], and Rydberg atoms [24,25].

Another relevant two-qubit objective is the entanglement measure concurrence $\mathcal{E}(\rho)$ [26]. In contrast to the fidelity, it does not favor a specific state, assigning full concurrence $\mathcal{E} = 1$ to all maximally entangled states. Since $\mathcal{E}(\rho)$ is not linear in ρ, however, its optimization cannot be treated analytically. Numerical optimization in $\mathcal{S}_2$ reveals that the optimal state coincides with the fidelity-optimal state ρ^* of Eq. (7), yielding $\mathcal{E}(\rho^*) = 1/2$. This is plausible, since a high Bell state fidelity typically implies strong entanglement. With respect to concurrence, however, ρ^* is not the unique optimum; e.g., Eq. (7) with Ψ_+ replaced by Ψ_- is stabilizable with concurrence $1/2$, as well.

These results clearly indicate the possibilities and limitations of coherent control of open systems: While we proved the impossibility of exceeding the fifty-fifty mixture (7) of the deexcited state and a maximally entangled Bell state, $\mathcal{E} = 1/2$ is still significantly higher than the average concurrence $\mathcal{E} = 0.18$ of typical stationary states, as we verified by a statistical sampling of Hamiltonians.

Periodic control Hamiltonians.—So far, we have developed a method to determine the static Hamiltonian H that upholds the optimal amount of an objective $\mathcal{O}$ in the stationary state. We now extend these concepts to the envisaged, substantially more general case of periodic control Hamiltonians $H(t) = H(t + T)$. Since this comprises static Hamiltonians as a special case, one generally expects this additional freedom in control to improve the optimization results.

Specifically, our aim is to determine periodic Hamiltonians $H(t)$ which optimize $\bar{\mathcal{O}} = 1/T \int_0^T \mathcal{O}(\rho_{\mathrm{ac}}(t))dt$, i.e., the time average of the objective $\mathcal{O}$ in the asymptotic cycle $\rho_{\mathrm{ac}}(t)$. To this end, we optimize $\bar{\mathcal{O}}$ in the set $\mathcal{A}$ of *stabilizable cycles*, comprising all periodic trajectories $\rho(t) = \rho(t + T)$ (with arbitrary period T) for which a periodic $H(t)$ exists such that $\rho(t)$ solves the master equation (1). Criterion (4) is then generalized to

$$\forall t\, \forall n\colon \mathrm{Tr}\{\rho^{n-1}(t)\mathcal{D}[\rho(t)]\} = \frac{1}{n}\partial_t \mathrm{Tr}[\rho(t)^n], \tag{8}$$

which holds for any $\{\rho(t)\} \in \mathcal{A}$. Equation (8) reflects the fact that only the dissipative term $\mathcal{D}(\rho)$ can change the spectrum of ρ and thus its spectral moments $\mathrm{Tr}[\rho^n]$. As before, criterion (8) is also sufficient for $\{\rho(t)\} \in \mathcal{A}$, if $\rho(t)$ has nondegenerate eigenvalues for all t. The Hamiltonian $H(t)$ stabilizing a given cycle in $\mathcal{A}$ is found analogously to Eq. (5).

E.g., for the purity p (i.e., $n = 2$), Eq. (8) implies that a consistent cycle must equally probe regions of the state space where the *purity flux* $f(\rho) \equiv \mathrm{Tr}[\rho\mathcal{D}(\rho)]$ of the dissipator is positive and regions where it is negative. The regions of positive and negative purity flux are separated by the hyperplane of vanishing flux, i.e., by the set $\mathcal{S}_2$ introduced in the discussion of the static case. Hence, any cycle must intersect $\mathcal{S}_2$ an even number of times (at least twice). Moreover, no cycle can explore regions where the purity is larger than the maximal purity in $\mathcal{S}_2$ [27].

It is again instructive to consider a single qubit first. A cycle $\{\rho(t) : t \in [0, T)\}$ is then represented by a closed trajectory $\{\vec{r}_t\}$ in the Bloch ball, and stabilizable cycles are characterized by criterion (8) with $n = 2$, reading

$$\underbrace{\vec{r}_t(D\vec{r}_t + \vec{d})}_{\equiv f(\vec{r}_t)} = \underbrace{\frac{1}{2}\partial_t|\vec{r}_t|^2}_{\equiv \dot{p}(\vec{r}_t)}. \tag{9}$$

It reflects the fact that the time evolution of the purity $p = \mathrm{Tr}[\rho^2] \equiv (|\vec{r}|^2 + 1)/2$ is exclusively governed by the radial part $f(\vec{r})$ of the dissipator.

While an optimization over all stabilizable cycles is certainly unfeasible, the problem can be reduced to the tractable class of *two-point cycles* (TPCs), based on the following argument: Any cycle undergoes subsequent stages of strictly monotonic purity gain and loss. Without loss of generality, we consider cycles that consist of two stages, intersecting $\mathcal{S}_2$ twice; general cycles reduce to this case. To each point $\vec{r}_p^{\,+}$ in the purity-increasing stage $[f(\vec{r}_p^{\,+}) > 0]$, one can assign a point $\vec{r}_p^{\,-}$ of equal purity p in the purity-decreasing stage $[f(\vec{r}_p^{\,-}) < 0]$. Hence, the cycle can be parametrized by p; see Fig. 2. Denoting by p_0 (p_1) its minimal (maximal) purity and using Eq. (9), we find that the objective $\bar{\mathcal{O}}$ for an arbitrary cycle is majorized by a TPC

$$\bar{\mathcal{O}} = \frac{\int_{p_0}^{p_1}\left(\frac{\mathcal{O}(\vec{r}_p^{\,+})}{|f(\vec{r}_p^{\,+})|} + \frac{\mathcal{O}(\vec{r}_p^{\,-})}{|f(\vec{r}_p^{\,-})|}\right)dp}{\int_{p_0}^{p_1}\left(\frac{1}{|f(\vec{r}_p^{\,+})|} + \frac{1}{|f(\vec{r}_p^{\,-})|}\right)dp} \le \max_p \bar{\mathcal{O}}_{\mathrm{TPC}}(\vec{r}_p^{\,+}, \vec{r}_p^{\,-}). \tag{10}$$

Here, $\bar{\mathcal{O}}_{\mathrm{TPC}}$ defines the time-averaged objective of a TPC:

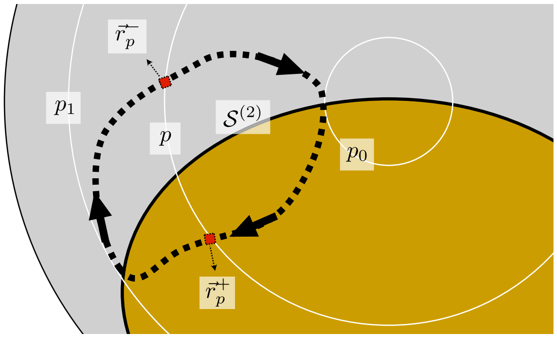

FIG. 2 (color online). Illustration of a general asymptotic cycle $\{\vec{r}_t\}$ (dotted loop) and a two-point cycle $\{(\vec{r}_p^+, \vec{r}_p^-)\}$ (red boxes). White rings connect points of equal purity p. The region of positive purity flux is orange (dark gray). It is separated from the region of negative flux (light gray) by the hypersurface $\mathcal{S}_2$ (solid black line) of states that can be stabilized by static Hamiltonians.

$$\bar{\mathcal{O}}_{\rm TPC}(\vec{r}_p^+, \vec{r}_p^-) = \frac{\mathcal{O}(\vec{r}_p^+)|f(\vec{r}_p^-)| + \mathcal{O}(\vec{r}_p^-)|f(\vec{r}_p^+)|}{|f(\vec{r}_p^+)| + |f(\vec{r}_p^-)|}. \quad (11)$$

In Eq. (10), we used the estimate $\int_a^b g(x)dx/\int_a^b w(x)dx \leq \max_x g(x)/w(x)$, holding for any positive $w(x)$ and arbitrary $g(x)$. The cycle that achieves $\bar{\mathcal{O}}_{\rm TPC}(\vec{r}_p^+, \vec{r}_p^-)$ comprises only two points $\vec{r}_p^+$ and $\vec{r}_p^-$ of equal purity p, lying on different sides of the hyperplane $\mathcal{S}_2$. To see this, one realizes that the (infinitesimal) purity δp is lost while residing for a dwell time δt^- at $\vec{r}_p^-$. To close the cycle, this purity δp must be regained during the dwell time δt^+ at $\vec{r}_p^+$. Since the ratio of these dwell times is inverse to the ratio of the respective purity fluxes

$$\frac{\delta t^+}{\delta t^-} \overset{(9)}{=} \frac{\delta p/|f(\vec{r}_p^+)|}{\delta p/|f(\vec{r}_p^-)|} = \frac{|f(\vec{r}_p^-)|}{|f(\vec{r}_p^+)|}, \quad (12)$$

the time average $\bar{\mathcal{O}}$ of the TPC is given by Eq. (11). The TPC rapidly jumps back and forth between $\vec{r}_p^+$ and $\vec{r}_p^-$ via purity-preserving, unitary "kicks" generated by suitable, ideally δ-shaped pulses $H(t)$. In conclusion, Eq. (10) reflects the important result that the search for the optimal asymptotic cycle can be restricted to TPCs, simplifying tremendously the original optimization over *all* closed trajectories that obey Eq. (9).

To exemplify this reduction, we consider again the coherence $\mathcal{C}$ of a single qubit undergoing the same incoherent processes as before (with rates γ_-, γ_+, and γ_d). Because of symmetry, the most general TPC is then parametrized by two azimuthal angles and the common purity p, and it is constrained by $f(\vec{r}_p^+) > 0$, $f(\vec{r}_p^-) < 0$. In a numerical optimization of $\bar{\mathcal{O}}_{\rm TPC}(\vec{r}_p^+, \vec{r}_p^-)$, one finds that for any combination of the incoherent rates, the optimal TPC $\{\vec{r}_*^+, \vec{r}_*^-\}$ degenerates to a single point, namely, the optimal stationary state $\vec{r}_*$ for static Hamiltonians (marked in Fig. 1). Remarkably, this implies that no periodic Hamiltonian $H(t)$ beats the optimal static Hamiltonian H^*. This is, however, a peculiarity of our choices of objective and dissipator and does not hold in general.

The same strategy can be applied beyond a single qubit. Similar to the static case, one obtains an upper bound for the optimum in $\mathcal{A}$ by focusing on the lowest-order set $\mathcal{A}_2$, since $\mathcal{A} \subset \mathcal{A}_2$. One can then again restrict the investigation to TPCs, rendering a numerical optimization feasible. In our two-qubit example with spontaneous decay only, we find that the time-averaged concurrence $\bar{\mathcal{E}} = 1/T \int_0^T \mathcal{E}[\rho(t)]dt$ never exceeds the optimal static result $\mathcal{E}(\rho^*) = 1/2$. This value decreases in the presence of finite absorption ($\gamma_+ > 0$) and/or dephasing ($\gamma_d > 0$), and it never outperforms the static optimum.

Conclusion.—We developed a method to characterize the asymptotic states of open quantum systems in terms of the dynamical constraints imposed by the dissipator. It allows us to access the optimization of arbitrary periodically time-dependent coherent control without resorting to the system Hamiltonian. In the static case, the method leads to the characterization (4) of stabilizable states, reflecting the unitary compensability of the dissipator. Based on that, we showed that in the general periodic case, optimizations can be restricted to the significantly simplified class of two-point cycles. This way, optimization problems with respect to arbitrary objectives can be addressed that were previously prohibited by the vast range of conceivable asymptotic cycles. To demonstrate our method, we determined the maximum asymptotic two-qubit entanglement that can be preserved by periodic coherent control in the presence of a dissipation-inducing environment. Other relevant applications include, e.g., optimal energy transport in quantum networks [28] or the minimization of particle loss in Bose-Einstein condensates [29]. Altogether, our method not only deepens our conceptual understanding of the working principles in open quantum systems but also opens the prospect to treat hitherto intractable optimization problems.

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