Litvinenko and Derbenev Reply: We thank Stupakov and Zolotarev [1] for pointing out a deficiency in our Letter wherein we listed a maximum amplification of a short-range (aka δ function) density modulation as $\sim 1,000$ without specifying its dependence on the free electron laser (FEL) and electron beam parameters.

Their observation has some similarity with findings in the coherent electron cooling (CeC) theory [2–4] that the FEL amplification is reduced compared with a simple exponential estimate. But the Comment [1] also has a serious deficiency, stating that maximum FEL amplification is proportional to the FEL bandwidth [Eq. (4)], $\delta\omega/\omega$. Our formula, derived below and tested by accurate 3D FEL simulations [5], shows the maximum gain scaling as $\sqrt{\delta\omega/\omega}$. With $\delta\omega/\omega \sim 10^{-2}$–$10^{-3}$ this scaling makes a significant difference.

We are taking this Reply as an opportunity to rectify the problem and rigorously define a model-independent limitation imposed by saturation in the CeC amplifier. We present a derivation for a generic instability including a FEL (details are given in [6]). We rely on the well-established fact that a self-consistent set of the Maxwell and Vlasov equations describes beam instabilities. While Maxwell's equations are linear, Vlasov's equation is not, and the instability saturation occurs when the density modulation becomes comparable to the initial density:

$$\left|\frac{\delta n}{n}\right| \sim 1. \tag{1}$$

This assumption definitely holds for FEL saturation, wherein the harmonic of the density modulations reaches $\sim$0.6–0.8. In the linear regime, the amplification of density modulation can be represented by a Green function [2], describing the response on the local distortion $\delta n = \delta(z - z_o)$ at $t = 0$:

$$n(\tau) = n_o + \delta(z - z_o) + G_\tau(z - z_o). \tag{2}$$

As we described in the Letter, there are two components of the shot noise entering the amplifier; one is from the electrons, and the other is induced by the hadrons (to which we here assign a relative weight, X). Thus, at the exit of the amplifier we get

$$n(\tau, z) = n_o + \sum_{i=1}^{N_e} G_\tau(z - z_i) + X \sum_{j=1}^{N_i} G_\tau(z - z_j), \tag{3}$$

where z_i is the locations of electrons, and z_j are those of the Debye ellipsoids at the amplifier's entrance. For a random uncorrelated Poisson distribution of the initial locations Eq. (1) yields the limit on the maximum gain [6]:

$$g_{\max} \leq \sqrt{\frac{M}{N_c(1 + X^2 \frac{M_I}{M_e})}} \cong 144 \sqrt{\frac{I_{\mathrm{pe}}[\mathrm{A}]\lambda_o[\mu\mathrm{m}]}{N_c(1 + \frac{X^2}{Z}\frac{I_{\mathrm{pI}}}{I_{\mathrm{pe}}})}}, \tag{4}$$

wherein we defined the gain as the ratio between the initial- and the amplified-density modulation at the wave number of the instability, $k_o = 2\pi/\lambda_o$:

$$g(z_i) = \int_{-z_i}^{\lambda_o - z_i} G_\tau(z) e^{ik_o z} dz;$$
$$\int_{-\infty}^{\infty} |g(z)|^2 dz = g_{\max}^2 N_c \lambda_o, \tag{5}$$

where $N_c \sim \omega/\delta\omega$ is the correlation length in units of the resonant wavelength defined in the Letter, and $\Lambda_e = M_e/\lambda_o$; $\Lambda_I = M_I/\lambda_o$, respectively, are linear densities of the electrons and the hadrons. Equation (4) yields a practical estimate for the maximum achievable gain as a function of the electron- and hadron-beams' peak currents I_{pe}, I_{pI}, with Ze being the charge of the hadrons. This model-independent formula applicable to any amplifier, including FELs, states that maximum attainable gain scales as the FEL wavelength, square root of the electron beam peak current, and a square root of the FEL bandwidth:

$$g_{\max} \sim \lambda_o \sqrt{I_{\mathrm{pe}} \delta\omega}.$$

The CeC's performance is still spectacular even with the gain limitation of Eq. (4). Our optimization of the CeC process brought us to conclude that the best cooling is achieved with electron bunches length comparable with that of the hadron's, e.g., $\sigma_{\mathrm{ze}} \sim \sigma_{\mathrm{zh}}$. Our estimate shows CeC providing the proton beam cooling times of under 1 hr for LHC at 7 TeV, and under 10 min for a RHIC at 250 GeV. Main parameters are shown in the following table:

Ring	I_{pe}, A	$\varepsilon_{e\,\mathrm{norm}}$, mm mrad	λ_o, nm	λ_w, cm
LHC	20	1	260	20
RHIC	10	1	422	3

Our analysis clearly shows a significant difference in the scaling of the maximum attainable gain compared with that in the comment, e.g., $\sqrt{\delta\omega/\omega}$ vs $\delta\omega/\omega$ For example, for the RHIC parameters listed above, Eq. (4) in the Comment [1] yields $g_{\max} = 7.7$, while our Eq. (4) gives $g_{\max} = 26.3$. The later is in very good agreement with $g_{\max} = 24.3$, obtained by simulation using the well-tested 3D FEL code GENESIS [4].

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Vladimir N. Litvinenko[1] and Yaroslav S. Derbenev[2]
[1]BNL, Upton, New York 11973, USA
[2]JLab, Newport News, Virginia 23606, USA

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