Theory of Carrier-Mediated Magnonic Superlattices

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We present a minimal one-dimensional model of collective spin excitations in itinerant ferromagnetic superlattices within the regime of parabolic spin-carrier dispersion. We discuss the cases of weakly and strongly modulated magnetic profiles finding evidence of antiferromagnetic correlations for long-wave magnons (especially significant in layered systems), with an insight into the ground state properties. In addition, the presence of local minima in the magnonic dispersion suggests the possibility of (thermal) excitation of spin waves with a relatively well controlled wavelength. Some of these features could be experimentally tested in diluted magnetic semiconductor superlattices based on thin doped magnetic layers, acting as natural interfaces between (spin)electronic and magnonic degrees of freedom.

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Introduction.—During the last decade, there has been an increasing interest on the dynamics of collective spin excitations (magnons or spin waves) in magnetic materials at the nanoscale. Recent efforts demonstrated that magnons can exhibit most of the characteristic signatures of wave phenomena including the excitation and propagation along wave guides, interference, reflection, refraction, diffraction, focusing, and tunneling, showing both classical and quantum properties. These findings established the basis of the emerging field of *magnonics* [1–3], the aim of which is to exploit magnons to carry and process information, among others. The building blocks of magnonics are magnetic superlattices for the design of bands that determine the transport properties of magnons (and, specifically, band gaps blocking magnon propagation). Most developments have been implemented in magnetic materials as ferrites and ferromagnetic alloys, where local magnetic moments interact through direct exchange or dipolar coupling depending on the length scale. Surprisingly, little attention has been paid in this respect to itinerant ferromagnets as diluted magnetic semiconductors (DMS) [4]. These are magnetic materials where local magnetic moments are coupled through itinerant spin carriers in the absence of direct coupling, allowing the electrical control of ferromagnetism [5]. An example is GaMnAs, where magnetic superlattices can be built by layering GaAs with modulated Mn-impurity densities along the growth direction at the nanometer scale. These structures were studied mainly in the context of spin-carrier transport for applications in the field of *spintronics* [6] as giant magnetoresistances (GMR) by manipulating the (anti)ferromagnetic coupling between Mn layers [7–10]. To our knowledge, spin waves in magnetic semiconductor superlattices were only studied in the narrow band limit under the action of direct impurity exchange, where carriers motion plays no role [11]. This family of hybrid materials present a natural advantage, acting as interfaces between (spin)electronic and magnonic degrees of freedom. Certainly, this emerging aspect deserves more attention.

Here, we present a first step towards the theoretical study of collective excitations in one-dimensional (1D) magnetic superlattices mediated by itinerant carriers. It is based on a previous theory originally developed for the modeling of ferromagnetism in uniformly doped DMS [12,13], here extended for the study of periodically modulated magnetic doping. The model accounts for dynamic correlations between localized magnetic impurities and itinerant carriers in small excitations from an ordered state, well beyond mean-field and Ruderman-Kittel-Kasuya-Yosida (RKKY) theories. We implement a path-integral formulation where carriers are integrated out, obtaining an effective action for the impurity spins expanded up to the second order in the excitations amplitude. We find a number of spin wave modes with distinguishing features as a consequence of the periodic magnetic profile, showing signatures of antiferromagnetic correlations absent in uniformly doped systems. In combination with an external magnetic field, these could allow the thermal excitation of spin waves of definite wavelength. We discuss the cases of weakly and strongly modulated magnetic profiles in the parabolic regime for carrier dispersion. This makes our study also valid for 1D magnonic superlattices built upon the layering of 3D systems (as, e.g., DMS superlattices).

Model.—We study a Kondo-like 1D model describing a system of localized spins distributed periodically along the z axis with spin density $\mathbf{S}(z)$ coupled ferromagnetically to conduction-band electrons. This is described by the Hamiltonian $H = H_{\rm kin} + H_{\rm Z} + H_{\rm ex}$, with contributions [14]

$$H_{\rm kin} = \int dz \sum_{\sigma=\pm 1} \hat{\psi}^{\dagger}_{\sigma}(z)\left(\frac{p^2}{2m} - \mu\right)\hat{\psi}_{\sigma}(z), \qquad (1)$$

$$H_{\rm Z} = \int dz\,[g\mu_{\rm B}\mathbf{B}\cdot\mathbf{s}(z) + g^{*}\mu_{\rm B}\mathbf{B}\cdot\mathbf{S}(z)], \qquad (2)$$

$$H_{\mathrm{ex}} = J\int dz\, \mathbf{S}(z)\cdot \mathbf{s}(z). \tag{3}$$

Here, H_{kin} accounts for the kinetic Hamiltonian of conduction-band carriers [spinorial fields $\hat{\psi}_\sigma(z)$] with effective mass m, introducing the chemical potential μ as a reference. An external magnetic field $\mathbf{B}$ contributes with a Zeeman energy H_Z, Eq. (2), where we introduce the itinerant-carrier spin density $\mathbf{s}(z) = (1/2)\sum_{\sigma\sigma'}\hat{\Psi}^\dagger_\sigma(z)\times \boldsymbol{\tau}_{\sigma\sigma'}\hat{\Psi}_{\sigma'}(z)$ with $\boldsymbol{\tau}$ the vector of Pauli matrices. The coupling between itinerant carriers and local spin is modeled by H_{ex}, Eq. (3), with ferromagnetic coupling constant $J < 0$ [15]. This model simplifies considerably when the itinerant-carrier density $n(z)$ is much smaller than the local-spin density $N(z)$ [16], in which case, we can treat $N(z)$ [and, consequently, $\mathbf{S}(z)$] as a continuous distribution where disorder is neglected by coarse graining. This regime applies, among others, to bulk and nanostructured DMS [12,13].

By assuming positive g and g^* factors, a field $\mathbf{B} = -B\hat{z}$ tends to organize all spins parallel to the z axis. This serves as a mean-field reference state from which small spin fluctuations are defined. By resorting to Holstein-Primakoff (HP) bosonic fields $b(z)$ and $b^\dagger(z)$, the spin density $\mathbf{S}(z)$ can be approximated in the small-fluctuation regime by $S^+(z) \approx b(z)\sqrt{2N(z)S}$, $S^-(z) \approx b^\dagger(z)\sqrt{2N(z)S}$, and $S^z(z) = N(z)S - b^\dagger(z)b(z)$. This allows the introduction of a coherent-state path integral representation for the partition function in imaginary time ξ:

$$\mathcal{Z} = \int \mathcal{D}[\bar{\Psi}\Psi]\mathcal{D}[\bar{\omega}\omega]\, e^{-\int_0^\beta d\xi\, \mathcal{L}[\bar{\Psi}\Psi,\bar{\omega}\omega]} \tag{4}$$

with Lagrangian $\mathcal{L} = \int dz[\bar{\omega}\partial_\xi\omega + \sum_\sigma \bar{\Psi}_\sigma\partial_\xi\Psi_\sigma] + H[\bar{\Psi}\Psi,\bar{\omega}\omega]$. Here, we replace in H the fermionic-field (carrier) operators by Grassmann numbers $\bar{\Psi}_\sigma$, Ψ_σ, and the HP bosonic (local-spin fluctuation) operators by complex variables $\bar{\omega}$, ω (where we omit the arguments z, ξ for simplicity). By integrating out the fermionic fields in Eq. (4)—thanks to the bilinear dependence of $\mathcal{L}[\bar{\Psi}\Psi]$—, we end up with an effective picture in terms of local-spin degrees of freedom $\mathcal{Z} = \int \mathcal{D}[\bar{\omega}\omega]\exp(-\mathcal{S}_{\mathrm{eff}}[\bar{\omega}\omega])$ with an action

$$\mathcal{S}_{\mathrm{eff}} = \int_0^\beta d\xi \int dz\,[\bar{\omega}\partial_\xi\omega - g^*\mu_B B(N(z)S - \bar{\omega}\omega)] - \ln\det(G_{\mathrm{MF}}^{-1} + \delta G^{-1}). \tag{5}$$

Here, the kernel G^{-1} splits into mean-field (G_{MF}^{-1}) and fluctuating (δG^{-1}) contributions:

$$G_{\mathrm{MF}}^{-1} = \left(\partial_\xi + \frac{p^2}{2m} - \mu\right)\mathbb{1} + \frac{\Delta(z)}{2}\tau_z, \tag{6}$$

$$\delta G^{-1} = \frac{J}{2}[-\bar{\omega}\omega\tau_z + \sqrt{2N(z)S}(\omega\tau^- + \bar{\omega}\tau^+)], \tag{7}$$

where $\Delta(z) = JN(z)S - g\mu_B B < 0$ is the (local) spin splitting of the itinerant carriers. Notice that the dynamics of the itinerant carriers is contained in the effective action, accounting for the retarded interaction between local spins.

A noninteracting spin wave theory for the local spins is derived from Eq. (5) by expanding $\mathcal{S}_{\mathrm{eff}}$ up to the 2nd order in the bosonic variables $\bar{\omega}$, ω. After dropping an irrelevant energy offset, we find

$$\mathcal{S}_{\mathrm{eff}}[\bar{\omega},\omega] = \int_0^\beta d\xi \int dz \Big\{\bar{\omega}(z,\xi)\partial_\xi\omega(z,\xi) + g^*\mu_B B\bar{\omega}(z,\xi)\omega(z,\xi) - \frac{J}{2}[n^\uparrow_{\mathrm{MF}}(z) - n^\downarrow_{\mathrm{MF}}(z)]\bar{\omega}(z,\xi)\omega(z,\xi) + \frac{J^2S}{2}\int_0^\beta d\xi' \int dz' \sqrt{N(z)N(z')}G^\uparrow_{\mathrm{MF}}(z,\xi;z',\xi')G^\downarrow_{\mathrm{MF}}(z',\xi';z,\xi)\bar{\omega}(z',\xi')\omega(z,\xi)\Big\}. \tag{8}$$

Here, we introduced the mean-field spin density $n^\sigma_{\mathrm{MF}}(z)$ and Green's function $G^\sigma_{\mathrm{MF}}(z,\xi;z',\xi')$ of the itinerant carriers ($\sigma = \uparrow,\downarrow$) under the action of an effective potential $U^\sigma(z) = \sigma\Delta(z)/2$ produced by the combined action of the local-spin distribution $N(z)$ and the magnetic field $\mathbf{B}$ [17]. The 1st and 2nd terms of Eq. (8) are local in space, representing the mean-field exchange field experienced by the local spins. The 3rd term is nonlocal, instead, accounting for correlation effects due to the response of itinerant carriers to local-spin reorientations.

The collective dynamics of the local spins is best understood by studying the action $\mathcal{S}_{\mathrm{eff}}$ in Fourier representation

$$\mathcal{S}_{\mathrm{eff}}[\bar{\omega},\omega] = \frac{1}{\beta}\sum_{j,n,m}\int\frac{dk}{2\pi}\bar{\omega}(k+K_n,\nu_j) \times[D^{-1}(k,\nu_j)]_{nm}\,\omega(k+K_m,\nu_j), \tag{9}$$

where the action's kernel is the inverse spin wave propagator with matrix elements $[D^{-1}]_{nm}$. These are ultimately determined by the spectral decomposition of the periodic local-spin distribution $N(z) = \sum_n N_n\exp(iK_nz)$, where $K_n = 2n\pi/z_0$ with z_0 the magnetic superlattice constant. In the absence of disorder, a periodic potential $U^\sigma(z)$ leads to the development of a carrier band structure with the 1st Brillouin zone defined by $-K_1/2 \le k \le K_1/2$, with k the carriers wave number, and lowest-band width $E_1 \approx (\hbar^2/2m)(K_1/2)^2$ for almost-free carriers motion. Besides, the expressions for $n^\sigma_{\mathrm{MF}}(z)$ and $G^\sigma_{\mathrm{MF}}(z,\xi;z',\xi')$ simplify conveniently by working within the parabolic-band regime for a small majority-spin carrier Fermi energy $E_F = \mu + |\Delta_0|/2 \ll E_1$ (with $\Delta_0 = JN_0S - g\mu_B B$, the carriers mean spin splitting), implying that the carriers Fermi wavelength is much larger than z_0.

This eventually restricts our analysis to long-wave magnons.

Elementary spin excitations.—Close to uniformly doped DMS [12,13], we find three different sets of excitations. Collective modes with dispersion $\Omega(k)$ are determined by studying the conditions under which $\det[D^{-1}(k, i\nu_j = \Omega)] = 0$. These modes organize in two branches at different energy scales: soft modes $\Omega_{\rm soft} < x_s|\Delta_0|$ and hard modes $\Omega_{\rm stiff} \sim |\Delta_0|$, where $x_s = (\bar{n}^{\uparrow}_{\rm MF} - \bar{n}^{\downarrow}_{\rm MF})/2N_0S \ll 1$ is the ratio between carrier and local-spin mean spin densities. Besides, a continuum of Stoner excitations (corresponding to spin flipping in the carrier subsystem by an electron-hole transition) is determined by finding the $\Omega_S(k)$ satisfying ${\rm Im}D^{-1}(k, i\nu_j) \neq 0$ after analytical continuation $i\nu_j \to \Omega + i0$. In the following, we focus our discussion on two study cases: weakly and strongly modulated superlattices in the limit of vanishing magnetic field ($B = 0$). Moreover, we limit ourselves to a symmetric magnetic profile $N(z) = N(-z)$, so that $N_n = N_{-n} = N_n^*$.

Weak modulation.—We first consider the case of a finite local-spin density N_0 perturbed by a weak harmonic modulation such that $N(z) = N_0[1 + \alpha\cos(2\pi z/z_0)]$ with $\alpha \ll 1$. The kernel reduces to $D^{-1}(k, i\nu_j) = -i\nu_j\mathbb{1} - x_s\Delta_0\mathbb{1} + x_s\Delta_0 I(k, i\nu_j)\mathbb{M}$, where each term derives from the corresponding 1st, 2nd, and 3rd one of Eq. (8). Here, the integral factor

$$I(k, i\nu_j) = \frac{\Delta_0}{x_s 2N_0S}\int\frac{dq}{2\pi}\frac{f(E_q^{\uparrow}) - f(E_{q+k}^{\downarrow})}{i\nu_j + E_q^{\uparrow} - E_{q+k}^{\downarrow}} \quad (10)$$

accounts for correlation effects due to the carriers response to local spin reorientations, where $f(E_q^{\sigma})$ is the Fermi distribution for carriers of spin σ and energy $E_q^{\sigma} = E_q + \sigma\Delta_0/2 - \mu$ with parabolic dispersion $E_q = (\hbar^2/2m)q^2$. We evaluate Eq. (10) for zero temperature. Regarding $\mathbb{M}$, it is a symmetric diagonal-constant (Toeplitz) matrix with elements 1 and $\alpha/2$ along the 1st and 2nd diagonals, respectively.

We solve $\det[D^{-1}(k, \Omega)] = 0$ by noticing that the Hermitian D^{-1} recalls a tight-binding Hamiltonian of an homogeneous, infinite chain (in *momentum* space) where Bloch's theorem applies: the solution of the eigenvalue equation $D^{-1}(k, \Omega)|\theta\rangle = \lambda(\theta)|\theta\rangle$ reads $\{|\theta\rangle = \sum_n \exp(in\theta)|K_n\rangle, \lambda(\theta) = -\Omega - x_s\Delta_0 + x_s\Delta_0(1 + \alpha\cos\theta)\times I(k, \Omega)\}$, where $0 \leq \theta = 2\pi(z/z_0) \leq 2\pi$ for $0 \leq z \leq z_0$ and $|K_n\rangle$ is a spin wave state of wave number K_n. The spin wave spectrum $\Omega(k)$ is then found by solving the equation $\lambda(\theta) = 0$. For each θ (indicating the location of the excitation within each unit cell), we find two solutions corresponding to soft and hard modes. A scanning over all values of θ yields a continuum of excitations bounded by the curves defined by $\theta_{\mp} = 0, \pi$. In Fig. 1, we depict the low-energy modes $\Omega_{\rm soft}$ together with the Stoner continuum Ω_S for fully ($E_F \leq |\Delta_0|$) and partly ($E_F > |\Delta_0|$) polarized carriers. For half metallic carriers, we find a small-momentum (long-wave) dispersion

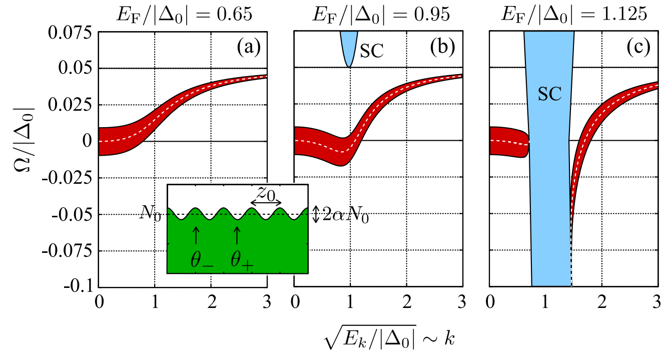

FIG. 1 (color online). Continuum of soft modes for fully [panels (a) and (b)] and partly [panel (c)] polarized carriers corresponding to *weakly* modulated local-spin density as sketched in the inset ($\alpha = 0.1$). In all cases, the lower and upper curves limiting the continuum are defined by the extreme values $\theta_{\mp} = 0, \pi$, respectively. Note the development of minima as $E_F/|\Delta_0|$ increases, occurring even in the case of uniform magnetic profile (dashed curve). The mean-field limit corresponds to $\Omega/|\Delta_0| = x_s = 0.05$. In panels (b) and (c), the Stoner continuum (SC) lies between the curves $-\Delta_0 + E_k \pm 2\sqrt{E_FE_k}$ for $E_F \leq |\Delta_0|$ and also between $-\Delta_0 - E_k \pm 2\sqrt{(E_F + \Delta_0)E_k}$ for $E_F > |\Delta_0|$.

$$\Omega_{\rm soft} = -\alpha\cos\theta x_s|\Delta_0| + (1 + \alpha\cos\theta)x_s\left(1 - \frac{4E_F}{3|\Delta_0|}\right)E_k + \mathcal{O}(E_k^2). \quad (11)$$

For large momenta (short wavelengths), we obtain the mean-field limit $\Omega_{\rm soft} \to x_s|\Delta_0|$. This is a consequence of the parabolic approximation for carriers dispersion: otherwise, periodic magnon dispersion is obtained. The results shown in Fig. 1 (and Fig. 2, as well) are then valid in the central region of the 1st Brillouin zone.

Several features stand out in Eq. (11). The most important one is the presence of negative-energy excitations ($\Omega < 0$). This means that the reference state with all spins pointing along the z axis is actually *not the ground state* [18]. The latter must be a complex state with lower magnetization, instead, developed by long-range antiferromagnetic (AF) correlations present in the system. Some AF signatures are already expected in *uniform* 1D systems [19]. The introduction of a weak modulation represented by a finite α in Eq. (11) provides an additional source of AF correlations. This is illustrated by the negative-energy excitations found in the neighborhood of $k = 0$ for $0 \leq \theta < \pi/2$ (doping *hills*). Besides, we find that positive dispersions as a function of E_k for a small $E_F/|\Delta_0|$ in Eq. (11) turn into negative ones as $E_F > (3/4)|\Delta_0|$ (reversing the sign of the spin wave stiffness and velocity), eventually leading to the development of a minimum with negative energy (see Fig. 1). The latter is a purely 1D characteristic: the position of the minima is independent of the superlattice constant z_0 and only weakly dependent on the modulation amplitude α, persisting after setting $\alpha = 0$ in Eq. (11) (uniform magnetic profile, depicted as the dashed curve in Fig. 1, which also corresponds to $\theta = \pi/2$). In addition, we find a set of finite (positive) energy excitations around $k = 0$ for $\pi/2 < \theta \leq \pi$, indicating the development of local magnetic anisotropies at the doping *valleys*. We further notice that the spectrum can be lifted by simply applying a magnetic field, turning negative-energy excitations into positive ones by restoring fully aligned spins as the ground state. Interestingly, this opens a possibility to control thermal excitation of magnons with relatively well-defined wave number around the minimum by a magnetic tuning of the gap.

Magnetic layering.—We now consider the case of strongly modulated local-spin density $N(z) = N_L\Gamma(z)$ by alternating magnetically doped and undoped layers, leading to a periodic stepwise profile given by

$$\Gamma(z) = \begin{cases} 1 & -\frac{z_1}{2} + nz_0 < z < \frac{z_1}{2} + nz_0 \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

where z_1 and N_L are the width and local spin density of the magnetic layer, respectively, while z_0 is the superlattice constant. We notice that a direct substitution of this profile on Eq. (8) could lead to unphysical situations as the presence of local-spin excitations within undoped layers. To avoid this problem, we redefine de HP parametrization of the local-spin density as $S^+(z) \approx b(z)\sqrt{2N_L S}\Gamma(z)$, $S^-(z) \approx b^\dagger(z)\sqrt{2N_L S}\Gamma(z)$, and $S^z(z) = [N_L S - b^\dagger(z)b(z)]\Gamma(z)$ [20]. Let $\Gamma_n = (1/n\pi)\sin(n\pi\Gamma_0)$ be the Fourier components of $\Gamma(z)$ with $0 < \Gamma_0 = z_1/z_0 < 1$. The kernel then reads $D^{-1}(k, i\nu_j) = -i\nu_j\mathbb{1} - x_s\Delta_0\mathbb{M}_1 + x_s\Delta_0 I(k, i\nu_j)\mathbb{M}_2$, with $\Delta_0 = JN_L\Gamma_0 S$ (where $N_L\Gamma_0 = N_0$ is the mean local-spin density) and $I(k, i\nu_j)$ defined in Eq. (10). $\mathbb{M}_1$ and $\mathbb{M}_2$ are Toeplitz matrices with elements Γ_n and Γ_n/Γ_0 along the nth diagonal, respectively.

To find the corresponding low-energy modes, we proceed as in the weakly modulated case by studying the vanishing eigenvalues $\lambda(\theta) = -\Omega + x_s\Delta_0[-1 + I(k, \Omega)/\Gamma_0]\sum_n \Gamma_n \cos(n\theta) = 0$ of the Hermitian D^{-1}. Here, we find that the factor $\sum_n \Gamma_n \cos(n\theta)$ in $\lambda(\theta)$ is nothing but $\Gamma(z_0\theta/2\pi) = \Gamma(z)$ [by noticing that $\theta = 2\pi(z/z_0)$ and $\Gamma_n = \Gamma_{-n}$]. This leads to three different kind of solutions. The first one is determined by those θs satisfying $\Gamma(z_0\theta/2\pi) = 1$. These correspond to a set of *degenerate inlaid modes* showing, in the half-metallic case, a long-wave dispersion (see solid line in Fig. 2 for the full dispersion)

$$\Omega_{\text{soft}} = \left(1 - \frac{1}{\Gamma_0}\right)x_s|\Delta_0| + \frac{x_s}{\Gamma_0}\left(1 - \frac{4E_F}{3|\Delta_0|}\right)E_k + \mathcal{O}(E_k^2). \tag{13}$$

Here, we find some features similar to those discussed in the weakly modulated case, including the presence of negative-energy excitations and the switch to negative dispersion (with the eventual development of a minimum) as $E_F/|\Delta_0|$ increases. Interestingly, we also find a long-wave limit $\Omega_{\text{soft}}(k = 0) = (1 - 1/\Gamma_0)x_s|\Delta_0|$, which decreases as Γ_0 approaches zero. This means that AF correlations become *stronger* for thinner or widely separated layers, taking the ground state away from that one with fully aligned spin configuration: reestablishing the reference state as the ground state would require larger magnetic fields. We, secondly, find a set of spurious modes with vanishing energy for those θs satisfying $\Gamma(z_0\theta/2\pi) = 0$, corresponding to undoped regions. These solutions are of no physical relevance. Finally, we notice that the series $\sum_n \Gamma_n \cos(n\theta)$ actually converges to $1/2$ right at the interface between doped and undoped layers. This leads to a new branch of modes, the dispersion of which is obtained by replacing x_s by $x_s' \equiv x_s/2$ in Eq. (13). This means that spins placed at the interface feel a local environment (here represented by x_s) different from those inlaid, lifting the long-wave excitation energy and lowering the mean-field short-wave limit (eventually halving the band width with respect to inlaid modes). These features are a consequence of the particular mathematical properties of the stepwise $\Gamma(z)$. However, more realistic profiles presenting gradual interfaces shall develop a continuum of modes with similar characteristics.

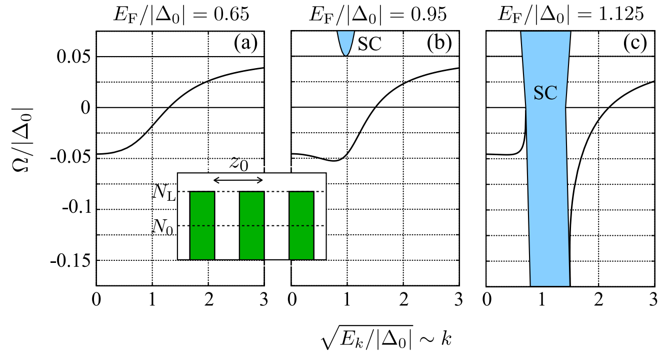

FIG. 2 (color online). Soft mode for fully [panels (a) and (b)] and partly [panel (c)] polarized carriers corresponding to *strongly* modulated local-spin density as sketched in the inset ($\Gamma_0 = 0.5$). Note the negative excitation energy for small k and the development of a minimum as $E_F/|\Delta_0|$ increases. The mean-field limit corresponds to $\Omega/|\Delta_0| = x_s = 0.05$. The Stoner continuum (SC) in panels (b) and (c) coincides with that of Fig. 1.

Additionally, we notice the development of Kohn-like anomalies (divergence of spin wave group velocity $\sim\partial\Omega/\partial k$) in the dispersion of low-energy modes for partly polarized carriers, close to the points where meeting the Stoner continuum (see Fig. 2).

Conclusions.—We present a minimal 1D model as a "proof of concept" for the study of magnonic superlattices in itinerant systems, accounting for backaction and correlation effects in the regime of parabolic spin-carrier dispersion. We discuss the cases of weakly and strongly modulated magnetic profiles. The latter could be achieved in DMS superlattices based on few nanometer thick doped magnetic layers [7–10]. This would allow us to reproduce some 1D characteristics of interest as negative magnon dispersion and local minima in 3D systems, opening the door to thermal excitation of spin waves with a relatively well-defined wave number in a controlled way. The finding of negative excitation energies for long-wave magnons indicate the presence of AF correlations, especially strong for thin magnetic layers with $\Gamma_0 \ll 1$. These AF signatures may arise from the effective coupling between distant layers, and not necessarily between neighboring ones. The control of spin wave excitations in this context may require the application of magnetic fields.

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