Tetraquark Mesons in Large-N Quantum Chromodynamics

Steven Weinberg*

Department of Physics, Theory Group, University of Texas Austin, Texas 78712, USA

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It is argued that exotic mesons consisting of two quarks and two antiquarks are not ruled out in quantum chromodynamics with a large number N of colors, as generally thought. Tetraquarks of one class are typically long-lived, with decay rates proportional to $1/N$.

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The suggestion [1] to consider quantum chromodynamics in the limit of a large number N of colors, with the gauge coupling g vanishing in this limit as $1/\sqrt{N}$, has had some impressive success in reproducing qualitative features of strong interaction phenomena. In his classic Erice lectures [2] describing these results, Coleman concluded that, for large N, quantum chromodynamics should not admit tetraquark mesons—exotic mesons that are formed from a pair of quarks and a pair of antiquarks—a result that has been widely accepted [3]. This Letter will reach a different conclusion. The large N approximation not only does not rule out tetraquark mesons, it helps to understand their properties.

Coleman's reasoning was as follows. By Fierz rearrangements of fermion fields, any color-neutral operator formed from two quark and two antiquark fields (aside from terms involving just one quark and one antiquark field) can be put in the form

$$\mathcal{Q}(x) = \sum_{ij} C_{ij}\mathcal{B}_i(x)\mathcal{B}_j(x), \tag{1}$$

where the $\mathcal{B}_i(x)$ are various color-neutral quark bilinears:

$$\mathcal{B}_i(x) = \sum_a \overline{q^a(x)}\,\Gamma_i q^a(x) - \sum_a \langle\overline{q^a(x)}\Gamma_i q^a(x)\rangle_0. \tag{2}$$

Here, q^a is a column of canonically normalized quark fields, with a an N-component SU(N) color index and with spin and flavor indices suppressed, the Γ_i are various N-independent spin and flavor matrices, $\langle\cdots\rangle_0$ denotes a vacuum expectation value, and the C_{ij} are some symmetric numerical coefficients, which we will take as N independent. [In his article, Coleman used bilinears $\mathcal{B}'_i(x)$ defined to contain an extra factor of $g^2N^{1/2} \propto N^{-1/2}$. This makes no difference to results for observables.] Coleman considered the vacuum expectation value of the product of two of these operators, given by a decomposition into disconnected and connected parts:

$$\langle\mathcal{Q}(x)\mathcal{Q}^\dagger(y)\rangle_0 = \sum_{ijkl} C_{ij}C^*_{kl}[\langle\mathcal{B}_i(x)\mathcal{B}^\dagger_k(y)\rangle_0\langle\mathcal{B}_j(x)\mathcal{B}^\dagger_l(y)\rangle_0 + \langle\mathcal{B}_i(x)\mathcal{B}_j(x)\mathcal{B}^\dagger_l(y)\mathcal{B}^\dagger_k(y)\rangle_{0,\mathrm{conn}}]. \tag{3}$$

A one-tetraquark pole can only appear in the Fourier transform of the final, connected, term, but according to the usual rules for counting powers of N, the first term is of order N^2, while the final term is only of order N, and so any one-tetraquark pole would make a contribution in (3) that is relatively suppressed by a factor $1/N$.

So far, so good, but what does this really show? Coleman concluded "In the large-N limit, quadrilinears make meson pairs and nothing else." But is this justified? If there is a tetraquark meson pole in the connected part of the propagator, what difference does it make if its residue is small compared with the disconnected part? To make an analogy, the amplitude for ordinary meson-meson scattering is proportional to the connected part of a four-point function involving four quark-antiquark bilinear operators, which is of order N, while the disconnected parts of the same four-point function are of order N^2. Does this mean that ordinary mesons do not scatter in the large N limit?

The real question is the decay rate of a supposed tetraquark meson. If the width of the tetraquark grows as some power of N, while its mass is independent of N, then for very large N it may not be observable as a distinct particle. Although Coleman did not address this issue, his discussion may suggest that the rate for a tetraquark meson to decay into two ordinary mesons must grow with N. As we will now see, this is not correct.

To calculate decay rates, we need to represent particle states with operators that are properly normalized to be used as Lehmann, Symanzik, and Zimmerman interpolating fields. The propagator for a quark bilinear operator $\mathcal{B}_n(x)$ representing an ordinary meson is proportional to N, but the residue of the pole in the propagator of a properly normalized operator should be N independent; so as noted by Coleman, the properly normalized operators for creating and destroying ordinary mesons are $N^{-1/2}\mathcal{B}_n(x)$. Similarly, if there is a one-tetraquark pole in the leading part of the connected term in (3), which is of order N, then the correctly normalized operator for creating or destroying a tetraquark meson of this type is $N^{-1/2}\mathcal{Q}(x)$. The amplitude for the decay of such a tetraquark meson into ordinary mesons of type n and m is then proportional to a suitable Fourier transform of the three-point function

$$N^{-3/2}\langle T\{\mathcal{Q}(x)\mathcal{B}_n(y)\mathcal{B}_m(z)\}\rangle_0 = N^{-3/2}\sum_{ij}C_{ij}\langle T\{\mathcal{B}_i(x)\mathcal{B}_n(y)\}\rangle_0\langle T\{\mathcal{B}_j(x)\mathcal{B}_m(z)\}\rangle_0 + N^{-3/2}\langle T\{\mathcal{Q}(x)\mathcal{B}_n(y)\mathcal{B}_m(z)\}\rangle_{0,\mathrm{conn}}. \tag{4}$$

The first term on the right-hand side is of order $N^{-3/2}N^2 = N^{1/2}$, which would give a decay rate proportional to N if the Fourier transform of this term contained a tetraquark pole, but it cannot contain such a pole, since it is just the convolution of two meson propagators. The connected second term on the right-hand side is of order $N^{-3/2}N = N^{-1/2}$, giving a decay rate of the tetraquark into two light ordinary mesons proportional to $1/N$, just as in the decay of ordinary mesons. Numerous authors [4] have identified the $f_0(980)$ and other narrow states as tetraquarks, though not in the context of the large-N approximation.

There may be tetraquarks whose poles only appear in subleading terms in the unrenormalized tetraquark propagator and decay amplitude, terms of lower than first order in N [5,6]. Such a tetraquark would have a decay rate of higher order in N than $1/N$, though of course it would still be long-lived if its mass were only a little larger than the total mass of the mesons into which it could decay.

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*weinberg@physics.utexas.edu

[1] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974).
[2] S. Coleman, *Aspects of Symmetry* (Cambridge University Press, Cambridge, England, 1985), pp. 377–378.
[3] For instance, see P. R. Page, in *Proceedings of the 8th Conference on Intersections of Particle and Nuclear Physics*, edited by Z. Parsa (American Institute of Physics, New York, 2003), p. 513; J. R. Peláez, Phys. Rev. Lett. **92**, 102001 (2004).
[4] R. J. Jaffe, Phys. Rev. D **15**, 267 (1977); F. E. Close and N. A. Törnqvist, J. Phys. G **28**, R249 (2002); E. Braaten and M. Kusunoki, Phys. Rev. D **69**, 074005 (2004); F. E. Close and P. R. Page, Phys. Lett. B **578**, 119 (2004); L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. Lett. **93**, 212002 (2004); C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa, and C. Sabelli, Phys. Rev. Lett. **103**, 162001 (2009); A. Ali, C. Hambrock, and W. Wang, Phys. Rev. D **85**, 054011 (2012); N. N. Achasov and A. V. Kiselev, Phys. Rev. D **86**, 114010 (2012); T. Friedmann, Eur. Phys. J. C **73**, 2298 (2013).
[5] M. Knecht and S. Peris (private communication).
[6] J. Peláez (private communication).