

Minimal Self-Contained Quantum Refrigeration Machine Based on Four Quantum Dots

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We present a theoretical study of an electronic quantum refrigerator based on four quantum dots arranged in a square configuration, in contact with as many thermal reservoirs. We show that the system implements the minimal mechanism for acting as a self-contained quantum refrigerator, by demonstrating heat extraction from the coldest reservoir and the cooling of the nearby quantum dot.

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The increasing interest in quantum thermal machines has its roots in the need to understand the relations between thermodynamics and quantum mechanics [1,2]. The progress in this field may as well have important applications in the control of heat transport in nanodevices [3]. In a series of recent works [4–6], the fundamental limits to the dimensions of a quantum refrigerator have been found. It has been further demonstrated that these machines could still attain Carnot efficiency [5], thus launching the call for the implementation of the smallest possible quantum refrigerator. References [4–6] considered self-contained thermal machines defined as those that perform a cycle without the supply of external work, their action being grounded on the steady-state heat transfer from thermal reservoirs at different temperatures. The major difficulty in the realization [7,8] of self-contained refrigerators (SCRs) is the engineering of the crucial three-body interaction enabling the coherent transition between a doubly excited state in contact with a hot (H) and cold (C) reservoir, and a singly excited state coupled to an intermediate (or “room,” R) temperature bath. We get around this problem by proposing an experimentally feasible implementation of a minimal SCR with semiconducting quantum dots (QDs) operating in the Coulomb blockade regime. We are thus able to establish a connection between the general theory of quantum machines and the heat transport in nanoelectronics [3].

QDs contacted by leads were proposed as ideal systems for achieving high thermopower [9–11] or anomalous thermal effects [12]. Here, we study a four-QD planar array (hereafter named a “quadridot” for simplicity) coupled to independent electron reservoirs as shown in Fig. 1; with proper (but realistic) tuning of the parameters, we will show that the quadridot acts as a SCR, which pumps energy from the high temperature reservoir H and the low temperature reservoir C to the intermediate temperature reservoirs R_1 and R_2 . Furthermore, we will analyze the conditions under which the quadridot is able to cool the dot QD_2 which is directly connected to the bath C at an effective temperature that is lower than the one it would have had in the absence of the other reservoirs. This will lead us to introduce an operative definition of the local

effective temperature, depending on the measurement setup, and to predict the existence of working regimes where, for instance, the refrigeration is not accompanied by the cooling of QD_2 . We start analyzing the system Hamiltonian, identifying the conditions that allow us to mimic the behavior of the SCR of Ref. [4].

In the absence of the coupling to the leads, the quadridot shown in Fig. 1 is described by the Hamiltonian

$$\mathcal{H}_{\text{QD}} = \sum_{i=1,\dots,4} \epsilon_i n_i + \sum_{i \neq j} \frac{U_{ij}}{2} n_i n_j - t(c_1^\dagger c_4 + c_2^\dagger c_3 + \text{H.c.}),$$

where for $i = 1, \dots, 4$, c_i^\dagger , c_i , and $n_i = c_i^\dagger c_i$ represent respectively the creation, annihilation, and number operators associated with the i th QD. In this expression, the quantities ϵ_i gauge the single-particle energy levels, t defines the tunneling coupling between the dots, and U_{ij} describes the finite-range contribution of the Coulomb repulsion. To reduce the maximum occupancy in each QD to one electron, we will assume the on-site repulsion terms U_{ii} to be the largest energy scale in the problem. Furthermore, in order to mimic the dynamics of Ref. [4], we will take $U_{12} = U_{34} = U_\perp$ and $U_{23} = U_{14} = U_\parallel$, both much larger than the “diagonal” terms $U_{24} = U_{13} = U_d$, and tune the single-electron energy level of the

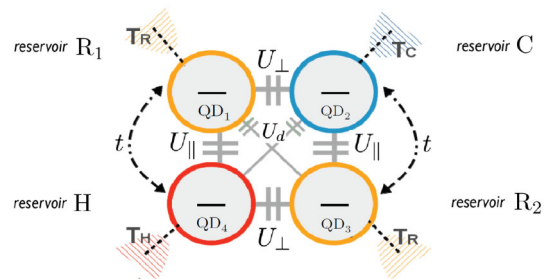


FIG. 1 (color online). The quadridot. The four quantum dots QD_1 , QD_2 , QD_3 , and QD_4 are weakly coupled to the reservoirs R_1 , C , R_2 , and H , respectively, which are all grounded and maintained at temperatures $T_H > (T_{R_1} = T_{R_2} = T_R) > T_C$. Tunneling is allowed only between QD_1 and QD_4 , and between QD_2 and QD_3 (t being the gauging parameter).

upper-right dot (which will be coupled to the cool reservoir C) so that $\epsilon_2 = \epsilon_1 + \epsilon_3 - \epsilon_4$. These choices ensure that in the absence of tunneling ($t = 0$), the “diagonal” two-particle states $|d\rangle = |1, 0, 1, 0\rangle$ and $|\bar{d}\rangle = |0, 1, 0, 1\rangle$ shown in Fig. 2 are degenerate (the charge states are labeled according to the occupation of the four dots $|n_1, n_2, n_3, n_4\rangle$). These are the only states of the two-electron sector which play an active role in the system evolution, mimicking the role of the vectors $|01\rangle$ and $|10\rangle$ of Ref. [4]. Because of the presence of U_\perp or U_\parallel , the other configurations are indeed much higher in energy to get permanently excited in the process. Still, the states $|u\rangle = |1, 1, 0, 0\rangle$ and $|l\rangle = |0, 0, 1, 1\rangle$ play a fundamental role in the SCR as their presence generates (via a Schrieffer-Wolff transformation [13,14] and the nonzero hoppings t) an effective coupling term between $|d\rangle$ and $|\bar{d}\rangle$ of the form $H_{\text{eff}} = g_{d\bar{d}}(|d\rangle\langle\bar{d}| + |\bar{d}\rangle\langle d|)$ with

$$g_{d\bar{d}} \simeq \frac{2t^2(U_d - U_\perp)}{(U_d - U_\perp)^2 - (\epsilon_4 - \epsilon_1)^2} \ll t. \quad (1)$$

In our model, $g_{d\bar{d}}$ is analogous to the perturbative parameter g of Ref. [4]. Its role is to open a devoted channel which favors energy exchanges between the couple H - C and the couple R_1 - R_2 by allowing two electrons to pass from the first to the second through the mediation of the quadridot states $|d\rangle$ (which is in contact with H and C) and $|\bar{d}\rangle$ (which is connected to R_1 and R_2). For proper temperature imbalances, this is sufficient to establish a positive heat flux from C to QD_2 even if T_C is the lowest of all bath temperatures. The mechanisms can be heuristically explained as follows: if T_H is sufficiently higher than the other bath temperatures, then the dot which has more chances of getting populated by its local reservoir is QD_4 . When this happens, the large values of U_\perp and U_\parallel will prevent QD_1 and QD_3 from acquiring electrons, too. On the contrary, while QD_4 is populated, QD_2 is allowed to accept an electron from its reservoir C (U_d being much smaller than U_\perp and U_\parallel), creating $|\bar{d}\rangle$. The coupling provided by H_{eff} will then rotate the latter to $|d\rangle$, giving the two electrons (the one from H

and the one from C) a chance of being absorbed by R_1 - R_2 . The opposite process (creation of $|\bar{d}\rangle$ by absorption of a couple of electrons from R_1 - R_2 , rotation to $|d\rangle$, and final emission toward H - C) is statistically suppressed due to the (relatively) low probability that QD_1 or QD_3 will get an electron from their reservoir before QD_4 gets its own from H : the net result is a positive energy flux from H - C to R_1 - R_2 .

To verify this picture, we explicitly solve the open dynamics of the quadridot and study its asymptotic behavior. Specifically, we model our four local baths H , C , R_1 , and R_2 as independent electron reservoirs (leads) characterized by their own chemical potential μ_i and their own temperature [both quantities entering in the Fermi-Dirac occupation functions $f_i(\epsilon)$ associated with the reservoir]. For the sake of the simplest correspondence with the model of Ref. [4], in this study, all μ_i will be set to be identical and fixed to a value that will be used as reference for the single-particle energies of the system (e.g., setting $\epsilon_i = 0$ in the Hamiltonian corresponds to have the i th quadridot level at resonance with the Fermi energy of the reservoirs). Furthermore, as detailed in the caption of Fig. 1, the temperatures will be chosen to satisfy the relation $T_C < T_{R_1} = T_{R_2} < T_H$ [15]. Within these assumptions, the only required external action is exerted in maintaining the local equilibrium temperatures and chemical potential, in accordance to the standard definition of self-containment. The quadridot-bath couplings (parametrized by the amplitudes $\Gamma_i^{(k)}$) are hence expressed as tunneling terms of the form

$$H_T = \sum_{i=1,\dots,4} \sum_k \Gamma_i^{(k)} c_i^\dagger a_{i,k} + \text{H.c.}, \quad (2)$$

with $a_{i,k}$ being an annihilation operator which destroys an electron of momentum k in the lead i . In the Born-Markov-Secular limit [16,17], these extra terms give rise to a Lindblad equation for the reduced density matrix ρ of the quadridot. The presence of t plus the rotation into the low-energy sector eliminates degeneracies among all possible energy transitions between the eigenstates of the quadridot. The evolution of ρ is then determined by

$$\dot{\rho} = \sum_i \mathcal{D}_i[\rho], \quad (3)$$

where for each reservoir \mathcal{D}_i represents an associated Lindblad dissipator. This equation can be solved in the steady-state regime ($\dot{\rho} = 0$) yielding the asymptotic configuration ρ^∞ , from which the heat currents $\langle \mathbf{J}_{Q,i} \rangle$ flowing through the i th reservoir are then computed as [16]

$$\langle \mathbf{J}_{Q,i} \rangle = \text{Tr}(\mathcal{H}_{\text{QD}} \mathcal{D}_i[\rho^\infty]). \quad (4)$$

If our implementation of the SCR is correct, we should see a direct heat flow from the hot H and cold C reservoirs to the reservoirs R_1 and R_2 , while the dot QD_2 should reach an occupation probability corresponding to an effective temperature which is lower than the one dictated by its

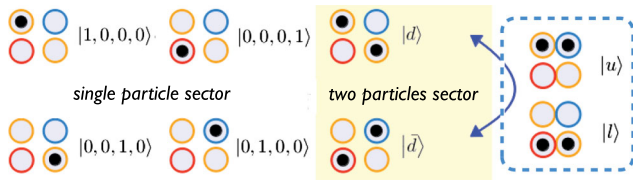


FIG. 2 (color online). Pictorial view of the low-energy electronic charged states (a black circle indicates occupation by an electron). Because of the hoppings terms t and $g_{d\bar{d}}$, the eigenstates of the low-energy Hamiltonian are bonding-antibonding states $|d\rangle \pm |\bar{d}\rangle$ and the four bonding-antibonding delocalized single-particle states (the completely empty state is not shown). For $t = 0$, the two electron states $|d\rangle$ and $|\bar{d}\rangle$ are resonant, while $|u\rangle$ and $|l\rangle$ are the high-energy virtual states responsible for the effective interaction $g_{d\bar{d}}$ coupling $|d\rangle$ and $|\bar{d}\rangle$.

local reservoir C [see Fig. 3(b)]. We have verified this by setting the system parameters to be consistent with those presented in Ref. [4]—making sure, however, that for such a choice no additional degeneracies are introduced into the system due to the larger dimension of our physical model. While the performances of the device do not change qualitatively when varying the parameters according to the above prescriptions, in the following we focus on a specific scenario where we fixed $\epsilon_1 = 2.1$, $\epsilon_3 = 2.9$, $\epsilon_4 = 4.0$, and $U_\perp = 12.0$ (U_\parallel instead is taken to be infinitely large for simplicity as its effect could be absorbed in the energy level renormalization after the Schrieffer-Wolff transformation). The value of $g_{d\bar{d}}$ is finally taken to be -0.001 , determining t (≤ 0.1) through Eq. (1), while the couplings terms $\Gamma_i^{(k)}$ which link the quadridot to the reservoirs via Eq. (2) are chosen to provide effective dissipation rates of order ~ 0.0001 [18,19]. Solving numerically the steady-state equation (3), we observe that for each $T_C < T_R$, there

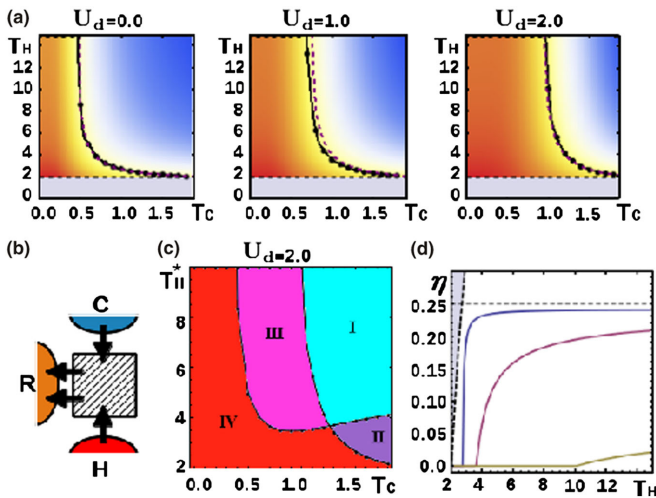


FIG. 3 (color online). (a) The panels refer to $U_d = 0$ (left), $U_d = 1$ (center), and $U_d = 2$ (right). When T_H approaches the black curve, i.e., at values very close to the analytical value of Eq. (5) (dashed line), a change of sign in the heat flow occurs. For T_H above the threshold (blue region), the machine works as pictured in (b), extracting heat from the C and H reservoirs and pumping it into R . In the opposite regime (under the threshold, red region), heat cannot be extracted from C . The dashed black line above the gray region indicates $T_H = T_R = 2$. The background color intensity is proportional to the actual heat pumped to (extracted from) C . (c) Comparison between heat extraction and single-particle occupation for $U_d = 2$. In regions I and II, the SCR is working (heat is extracted from C), while in regions III and IV, the C bath receives heat. In regions I and III, we have an effective decrease of the occupation number of QD_2 (i.e., $\langle n_2 \rangle < \langle n_2^0 \rangle$). (d) Efficiency of the SCR compared to the Carnot efficiency (dashed black line) for $T_C = 1.0$. The curves represent, respectively from top to bottom, $U_d = 0$ (blue), $U_d = 1$ (magenta), and $U_d = 2$ (brown). The dashed horizontal line indicates the maximum limit of efficiency for the quadridot computed (for $U_d = 0$) as in Ref. [4].

exists a minimal threshold value for T_H above which the SCR indeed extracts heat from the cold reservoir C . This is shown in Fig. 3(a) for $T_R = 2$ and different values of U_d ; the quadridot works as a SCR in the region above the threshold. Consistently with the second principle of thermodynamics, the threshold value of T_H (black curve in the plot) is always greater than $T_R = 2$ (for T_H below T_R , the machine cannot produce work from H to pump heat from C), and the region above this threshold gets larger as U_d gets smaller. The existence of a threshold for T_H also implies that for given $T_H > T_R$, there is a minimal temperature T_C^* for the cold reservoir under which the SCR cannot work. Interestingly, for $T_H/T_R \rightarrow \infty$, the value of T_C^* appears to asymptotically converge toward a finite nonzero temperature which depends upon the engine microscopic parameters and which can be interpreted as the emergent absolute zero of the model. An approximate analytical expression for T_C^* can be derived exploiting the recent general theory of genuine, maximally efficient self-contained quantum thermal machines [6]. This is done by interpreting the quadridot as a composite system, consisting of an “effective” virtual qubit formed by the states $|0, 0, 0, 1\rangle$ and $|d\rangle$, which (through $g_{d\bar{d}}$) mediates the interaction between QD_2 and the reservoirs H , R_1 , and R_2 . The average occupations of the virtual qubit levels (determined by the coupling with the reservoirs H , R_1 , and R_2) define the effective (average) temperature of H , R_1 , and R_2 , which is perceived by QD_2 : such a temperature competes with T_C in cooling down the dot and can be identified with the value of T_C^* of our model. Observing that the energy levels of $|0, 0, 0, 1\rangle$ and $|d\rangle$ are ϵ_4 and $\epsilon_1 + \epsilon_3 + U_d$, respectively, from Ref. [6] we get

$$T_C^* \simeq T_R T_H \frac{\epsilon_1 + \epsilon_3 + U_d - \epsilon_4}{T_H(\epsilon_1 + \epsilon_3 + U_d) - T_R \epsilon_4}, \quad (5)$$

which fits pretty well our numerical results [see Fig. 3(a)] and which for $T_H \rightarrow \infty$ yields $T_R(1 - \epsilon_4/[\epsilon_1 + \epsilon_3 + U_d])$ as the emergent absolute zero of the model.

Following Ref. [5], we evaluate the ratio $\eta = \langle \mathbf{J}_{Q,2} \rangle / \langle \mathbf{J}_{Q,4} \rangle$ between the heat current through the cold and hot reservoirs, comparing it with the upper bound $[1 - (T_R/T_H)] / [(T_R/T_C) - 1]$ posed by the Carnot limit, and with the theoretical value $\eta_{th} = (\epsilon_1 + \epsilon_3 - \epsilon_4)/\epsilon_4$ of Ref. [4] applied to the quadridot for $U_d = 0$ [20]. The dependence of η upon T_H is plotted in Fig. 3(d) for different values of U_d . We noticed that in the case of $U_d = 0$, the efficiency of the quadridot converges indeed toward the theoretical value η_{th} of Ref. [4] at least for large enough T_H .

Measurements and effective local temperatures.—An important question is whether this refrigeration effect is accompanied with a cooling of QD_2 , namely, whether its effective local temperature $T_C^{(eff)}$ decreases as T_H increases, for sufficiently high T_H , in analogy with the qubit cooling described in Ref. [4]. While for such an idealized qubit model the definition of the local temperature is relatively

straightforward, in nanoscale systems out of equilibrium, local temperatures must be *operationally* defined [21]. The most common way to proceed is to introduce a probe reservoir P (a “thermometer”), which is weakly coupled to that part of the system we are interested in (the dot QD₂ in our case), and identify the effective temperature of the latter with the value of the temperature T_P of the probe which nullifies the heat flow through P . This procedure yields a natural way of measuring the effect we are describing and can be implemented easily in our model by adding an extra term in (2) that connects the new reservoir P to QD₂ with a tunnel amplitude Γ_P which is much smaller than those associated with the other reservoirs of the system. (In the calculation, we set the ratio between Γ_P and Γ_i of the other reservoirs to be of the order 10^{-3} : this makes sure that the presence of P does not perturb the system.) The obtained values of $T_C^{(\text{eff})}$ are presented in Fig. 3(a), where it is shown that, according to this definition of the local temperature, the conditions for cooling of QD₂ (i.e., $T_C^{(\text{eff})} < T_C$) are the same for the SCR to work (implying incidentally that in this case $T_C^{(\text{eff})}$ is always greater than the emergent zero temperature of the system \bar{T}_C).

The quantity $T_C^{(\text{eff})}$ introduced above has a clear operational meaning and, according to the literature, it is a good candidate to define the effective temperature of QD₂. Still, it is important to acknowledge that in experiments, the cooling of QD₂ can also be detected by using the noninvasive techniques of, e.g., Ref. [22] to look at the decrease of the mean asymptotic occupation number of QD₂ ($\langle n_2 \rangle = \langle 0, 1, 0, 0 | \rho^\infty | 0, 1, 0, 0 \rangle + \langle d | \rho^\infty | d \rangle$), with respect to the same quantity computed when the SCR is “turned off” (e.g., $\langle n_2^0 \rangle = \langle 0, 1, 0, 0 | \rho_0^\infty | 0, 1, 0, 0 \rangle + \langle d | \rho_0^\infty | d \rangle$), where now ρ_0^∞ is the asymptotic stationary state of the system reached when all the reservoirs but C are disconnected, i.e., $\Gamma_{i \neq C} = 0$). We notice, however, that the cooling condition hereby defined does not coincide with the same pictured in Fig. 3(a). We indeed exemplify in Fig. 3(c) for $U_d = 3$ that, according to this new definition, different operating regimes are possible for the SCR. The QD₂ might be either colder ($n_2 < n_2^0$ in zone I) or hotter ($n_2 > n_2^0$ in region II) when the device extracts heat from the C reservoir. Conversely, we might also achieve a colder QD₂ when the quadridot pumps heat into the colder bath (III). In region IV, none of the refrigeration effects are active. Similar regimes emerge with other activation prescriptions, such as defining $\langle n_2^0 \rangle$ as the occupation for $T_H = T_R = T_C$ while maintaining all tunnel couplings as constant.

Conclusions.—We conclude with experimental considerations. Quadridots in GaAs/AlGaAs heterostructures have been implemented for cellular-automata computation [23] and for single-electron manipulation [24]. Strongly capacitively coupled QDs with interdot capacitance energy (U_\perp and U_\parallel) up to 1/3 of the intradot charging energy

(taken to be infinite in our model) can be fabricated with current lithographic techniques [25]. The diagonal interdot term U_d is expected to be at most $U_\parallel/\sqrt{2} \approx U_\perp/\sqrt{2}$ from geometrical considerations, but practically it is expected to be much smaller [24]. The local charging energy can be as big as 1 meV and usually represents about 20% of the confinement energy [26], which is the typical tunable value of the single-particle levels ϵ_i . Charging effects are expected to be further enhanced by the presence of a significant magnetic field, due to the emergence of the incompressible antidot regime in the dots [27], possibly allowing the working conditions to be achieved even more easily. In this high-field regime, the spin or orbital Kondo effect [28,29] is suppressed [30], as the transport becomes spin polarized, so our effective description is expected to be valid. A final ingredient for the quadridot to act as a SCR is quantum coherence. In QDs, it is known that the main source of decoherence comes from $1/f$ noise arising from background charge fluctuations [31] (however, coherent manipulation of QDs has been reported in several experiments; see, e.g., Ref. [32]). Accordingly, Eq. (3) acquires an extra contribution whose effect (see the Supplemental Material [33]) is to modify the steady-state populations. In our setup, as long as the new rates are of the same order of the ones due to the coupling to the leads, the quadridot will still work as a SCR. [Note, indeed, that the boundary between the regions in Fig. 3(a) does not depend on these rates.] Possibly the only serious challenge is posed by the need that the induced broadening should not be too large with respect to t . For the sake of simplicity, we adopted small values of this parameter; however, it is very much possible that higher values will help the efficiency of the SCR by speeding up the $|d\rangle$ and $|\bar{d}\rangle$ rotations. We finally observe that the maximum thermal energies involved should not exceed the large charging energies (i.e., ≤ 10 K).

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