# Universal Features of Counting Statistics of Thermal and Quantum Phase Slips in Nanosize Superconducting Circuits

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(Received 1 March 2013; published 11 June 2013)

We perform measurements of phase-slip-induced switching current events on different types of superconducting weak links and systematically study statistical properties of the switching current distributions. We employ two types of devices in which a weak link is formed either by a superconducting nanowire or by a graphene flake subject to proximity effect. We demonstrate that independently of the nature of the weak link, higher moments of the distribution take universal values. In particular, the third moment (skewness) of the distribution is close to $-1$ both in thermal and quantum regimes. The fourth moment (kurtosis) also takes a universal value close to 5. The discovered universality of skewness and kurtosis is confirmed by an analytical model. Our numerical analysis shows that introduction of extraneous noise into the system leads to significant deviations from the universal values. We suggest using the discovered universality of higher moments as a robust tool for checking against undesirable effects on noise in various types of measurements.

DOI: 10.1103/PhysRevLett.110.247001 PACS numbers: 74.78.Na, 74.25.Sv, 74.40.−n, 74.50.+r

*Introduction.*—The field of quantum noise has recently seen rapid development caused by both its growing significance in many areas of condensed matter physics as well as the constant improvement in the capabilities of high-precision measurements [1]. Perhaps the most intensively studied question to date is related to the statistics of charge transport in mesoscopic conductors. In such systems probability distribution of current fluctuations, the so-called full counting statistics, was rigorously derived for various normal and superconducting circuits [2] and tested in the state-of-the-art measurements of the third moment (skewness) of current fluctuations [3]. Given that charge is a quantum mechanical conjugate variable to the phase, it is of fundamental interest to study corresponding statistics of phase fluctuations. Superconducting nanowires and related proximity devices offer a natural platform for this purpose, which we explore in the present work.

The macroscopic quantum tunneling of the phase across a current-biased Josephson junction [4] or a superconducting nanowire [5–7] is arguably the most profound and well-known manifestation of quantum fluctuations at the macroscopic level. This phenomenon is observed by registering Little's phase slip events [8], which proliferate at currents close to the critical and drive transitions between supercurrent-carrying and dissipative branches of current-voltage characteristics [6,7,9,10]. Macroscopic quantum tunneling is usually described in the framework that treats quantum or thermally activated transitions of the phase between neighboring minima of a tilted washboard potential in the presence of a dissipative environment [11–13]. Complimentary approaches employ an effective action for BCS superconductors [14–16].

Unlike experiments associated with the charge transfer where measurements of each moment of the full counting statistics is beyond current experimental capabilities, experiments on switching current allow one to reconstruct full distribution of phase fluctuations since a single phase slip is sufficient to drive the system into resistive state [7,17] by creating a hot spot [18,19]. Thus there exists a one-to-one correspondence between phase fluctuations—Little's phase slips—and switching events.

In this Letter we report a systematic study of higher moments of the switching current distribution as a function of temperature and other parameters of our devices. The higher moments under investigation include skewness $S$ that quantifies an asymmetry of the distribution, and kurtosis $K$ that is a measure of its peakedness (for definitions see below). We present evidence, both experimental and theoretical, that these higher moments are in fact universal constants: $S \approx -1$ and $K \approx 5$. Surprisingly, the observed crossover from a classical escape mechanism (i.e., the thermal activation) to a quantum one (i.e., quantum tunneling from a metastable energy minimum) does not lead to any noticeable changes in these moments. We evince this universality using two types of samples, namely graphene junctions under the proximity effect as well as ultrathin superconducting nanowires. Apparent universality of $S$ and $K$ has to be contrasted with the behavior of the standard deviation of the switching current (the second moment $\sigma$) that exhibits nontrivial temperature dependence: the Kurkijärvi power low [20], $\sigma \propto T^{2/3}$, in the thermal regime and $\sigma \propto$ const in the quantum regime [7,9,10,17].

*Devices.*—Nanowire samples were prepared [10,21] by depositing carbon nanotubes across a 100-nm wide trench on a silicon chip, coated by a film of $SiO_2$ and a film of SiN. A film of 10–20 nm of $Mo_{76}Ge_{24}$ was sputtered onto the chip, covering the top SiN surface and the nanotubes crossing the trench. Thus the suspended segments of nanotubes were converted into nanowires. Uniform wires were selected using SEM, and the MoGe film was patterned by photolithography, to define contact pads (electrodes). In such devices the selected nanowire serves as the only conducting link connecting the superconducting thin-film electrodes, positioned on the opposite sides of the trench. Importantly, there is no additional contact resistance between the nanowire and the contact pad since the wire transforms seamlessly into the pad while both are made in the same sputtering run.

Graphene flakes were deposited onto a $SiO_2$ surface of a Si chip by mechanical exfoliation [22]. Electron-beam lithography was utilized to pattern the electrodes into a comb shape. After the resist was exposed and developed, we deposit, using thermal evaporation, a 4-nm Pd film (so-called sticking layer) and a 100-nm Pb film on the top. Lift-off was performed by placing the sample in an acetone bath for five minutes, sonicating it for ten seconds every other minute. The 100-nm Pb layer induces superconductivity in the graphene through the proximity effect. The samples were measured in a $^3$He cryostat. Electromagnetic noise was filtered from the system using $\pi$ filters at room temperature and a copper powder and silver-paste radio-frequency noise filters at low temperatures.

A sinusoidal bias current, having an amplitude greater than the critical current of the device, was applied across each sample. As the current increased from zero to its maximum, the voltage across the sample demonstrated a sudden jump from zero to some large, nonzero value, indicating the system switched from a superconducting state to a normal, resistive state. The value of bias current at which the jump took place was recorded as the switching current. Then the bias current returned to zero, and the system once again became superconducting. This process was repeated $N = 10^4$ times (or 5000 in some cases) for each set of parameters. Each measurement gave slightly different value of the switching current, due to inherent stochasticity of the phase slips, thus producing switching current distributions. The skewness and kurtosis of each distribution was calculated from the recorded data by using standard expressions, $S = N^{-1}\sum_{i=1}^{N}(I_{\mathrm{sw},i} - \langle I_{\mathrm{sw}}\rangle)^3/\sigma^3$ and $K = N^{-1}\sum_{i=1}^{N}(I_{\mathrm{sw},i} - \langle I_{\mathrm{sw}}\rangle)^4/\sigma^4$, where each $I_{\mathrm{sw},i}$ represents an applied bias current at which a switching event took place, $\langle I_{\mathrm{sw}}\rangle$ is the mean switching current, and $\sigma$ is the standard deviation of the switching distribution.

*Experimental results.*—We first discuss the effect of temperature on the skewness and kurtosis of the switching current distributions. We find that in both types of samples—superconducting nanowires [Figs. 1(c) and 1(d)], and graphene proximity junctions [Figs. 2(c) and 2(d)]—the skewness and kurtosis are constant with temperature. Surprisingly, these moments are identical within experimental uncertainty for the two qualitatively different systems. The value of the skewness in both cases is near −1, and the value of kurtosis is near 5. In nanowire samples, these moments remain constant even as the system experiences a crossover from the high temperature regime, at which phase slips are predominantly caused by thermal activation, to low temperatures, at which quantum tunneling of phase slips is responsible for the premature switching. This crossover is evident in Fig. 1(a) as the standard deviation changes from the power law at high temperatures to a constant value at low temperatures. The classical-to-quantum crossover temperature is typically in the range 0.6–0.8 K for the studied samples (Fig. 1).

In superconductor-graphene-superconductor (SGS) samples in addition to the temperature dependence, we also study the effect of gate voltage $V_g$ on the skewness and kurtosis. Both moments remain constant within the experimental uncertainty over a wide range of $T$ and $V_g$ (see Figs. 2(c) and 2(d) and [23]). It should be noted that unlike nanowire samples, SGS junctions do not show crossover to the quantum tunneling dominated regime within experimentally tested temperatures. However, we do expect that such crossover might occur at lower temperatures, as recently reported [24].

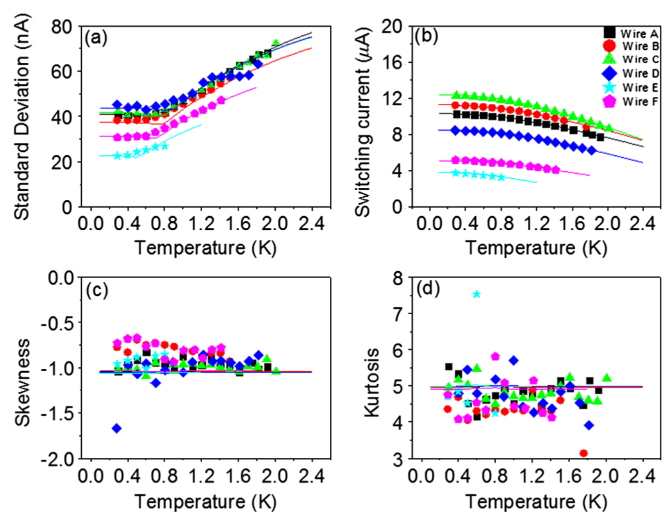


FIG. 1 (color online). (a) standard deviation, (b) mean switching current, (c) skewness, and (d) kurtosis of the switching current distributions in nanowire samples A, B, C, D, E, and F vs temperature $T$. The experimental values are represented by symbols. Simulation curves are shown by solid lines. One point in (c) and two points in (d) lie outside the ranges shown. Fitting parameters used in the simulation are summarized in Table I of the Supplemental Material [23].

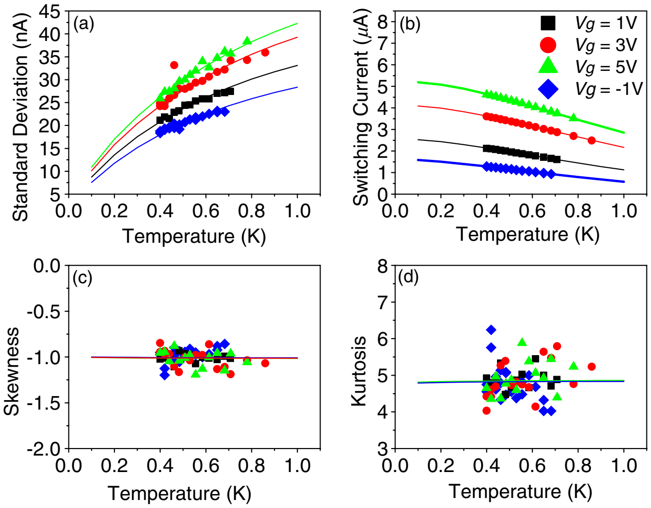


FIG. 2 (color online). (a) standard deviation, (b) mean switching current, (c) skewness, and (d) kurtosis of the switching current distributions in SGS sample 111s vs temperature at a gate voltages of 1V, 3V, 5V, and $-1$V. We use the same convention as in the previous figure. One point in (c) lies outside the range shown. Fitting parameters used in the simulation are summarized in Table II of the Supplemental Material [23]. Data for SGS sample 105s are shown in [23].

We also demonstrate numerically [23] that the presence of extraneous noise leads to a substantial reduction of the universal moments. This observation provides an independent tool for assessing the relevance of noise to the interpretation of experimental data.

*Numerical simulations and fitting.*—The fitting curves presented in Figs. 1 and 2 were obtained using the Arrhenius-type activation formula for the rate of phase slips (hereafter $\hbar = k_B = 1$) [10,25],

$$\Gamma(I,T) = \Omega(I,T)[e^{-U(I,T)/T} + e^{-U(I,T)/T_q}], \tag{1}$$

which accounts for both thermal and quantum escape processes. Here $\Omega$ is the attempt frequency, $U$ is the energy barrier for a phase slip, $T$ is the base temperature and $T_q$ is the quantum temperature used to model the regime of macroscopic quantum tunneling observed in nanowire samples at low temperatures [26]. For both systems activation energy has power-law functional dependence on the applied bias current

$$U(I,T) = \frac{\kappa I_c(T)}{e}[1 - I/I_c(T)]^{\eta}. \tag{2}$$

For SGS devices we took $\kappa = \sqrt{8}/3$ and $\eta = 3/2$ [27,28] and used Zaikin-Zharkov critical current

$$I_c(T) = \frac{64\pi T}{eR_N}\sum_{n=0}^{\infty}\frac{\Delta^2(L/L_n)\exp(-L/L_n)}{[\omega_n + W_n + \sqrt{2(W_n^2 + \omega_n W_n)}]^2}, \tag{3}$$

where $R_N$ is the normal state resistance of a junction, $\Delta$ is the superconducting gap in the leads, $\omega_n = (2n+1)\pi T$, $W_n = \sqrt{\Delta^2 + \omega_n^2}$, $L_n = \sqrt{D/2\omega_n}$. The sum over $n$ was taken until convergence (roughly 10 terms). Expression (3) follows from the theory of disordered superconductor–normal metal–superconductor junctions [29–31]. It has to be stressed that ballistic theory of the proximity effect in SGS junctions [32] fails to account for the temperature and gate voltage dependencies of the critical current for our devices [see Fig. 2(b)]. This observation is also consistent with the previous reports on the proximity effect in SGS systems [24,27,33–38]. From the normal state resistance of our samples, we deduce a typical mean free path $l \sim 20$ nm, which corresponds to the diffusion coefficient $D \sim 50\ \mathrm{cm}^2/\mathrm{s}$. Because the mean free path and the Thouless energy $E_{\mathrm{Th}} = D/L^2 \sim 80\ \mu\mathrm{eV}$ are much smaller than the junction spacing of $L \sim 300$ nm and the energy gap $\Delta \sim 1$ meV, respectively, our SGS junctions correspond to a long diffusive junction limit.

For superconducting nanowires there are two known models for $U$ in Eq. (2). If a wire forms a phase slip junction then $\kappa = \sqrt{6}/2$ and $\eta = 5/4$ [14,39–41]. The corresponding expressions for $\kappa$ and $\eta$ for the more thoroughly studied case of a Josephson junction have the same values as above for the SGS devices. It is worth noting that qualitatively the two models are very similar. Following the previous work [42], we model the critical current of nanowire devices by the phenomenological Bardeen's formula [43]

$$I_c(T) = I_c(0)(1 - T^2/T_c^2)^{3/2}. \tag{4}$$

Finally, for both SGS and nanowire systems, the escape attempt frequency in Eq. (1) was described by

$$\Omega(I,T) = \Omega_0(T)[1 - I/I_c(T)]^{\nu}, \tag{5}$$

with $\nu = 1/4$ for JJ model, and $\nu = 5/8$ for phase slip junction model.

Equations (1)–(5) were combined to determine the rate of phase slips. For a given set of parameters this rate was used to predict the switching distribution as a function of bias current and then calculate its mean, standard deviation, skewness and kurtosis. Such procedure was repeated at different temperatures to produce the temperature dependence of the moments. In the case of SGS samples the above scheme was also repeated at different gate voltages. Parameters ($\Omega_0$, $I_c(0)$, $T_c$, $T_q$) for nanowire samples and ($\Omega_0$, $R_N$, $T_c$ and $D$) for SGS samples were then adjusted within the expected range of values until the predicted switching current and standard deviation vs temperature curves matched the data. These, along with the resulting skewness and kurtosis curves, were used as fits to the data and are plotted as solid lines in Figs. 1 and 2 [23,44].

*Analytical model.*—In this section we compute skewness and kurtosis by using an approach developed for the problem of escape from a metastable potential well subject to a steadily increasing bias field [20,45]. We consider a general situation in which the phase slip rate of the system—either thermal or quantum—can be written in terms of the reduced current variable $\epsilon = 1 - I/I_c$ as

$$\Gamma(\epsilon) = A\epsilon^{a+b-1}\exp(-B\epsilon^b). \tag{6}$$

This form is general enough to cover all range of parameters relevant for our experiment on both types of devices. The powers $a$ and $b$ depend on whether the escape is quantum or thermally activated, while parameters $A$ and $B$ depend on the degree and type of damping (in particular, we estimate that our SGS junctions are moderately underdamped with the quality factor $Q \simeq 4$). The distribution function for phase slips can be expressed in terms of the rate as

$$P(\epsilon) = \frac{1}{|\dot{\epsilon}|}\Gamma(\epsilon)\exp\left[-\frac{1}{|\dot{\epsilon}|}\int_{\epsilon}^{\infty}\Gamma(\epsilon')d\epsilon'\right], \tag{7}$$

where $|\dot{\epsilon}|$ is a constant ramp speed. We are interested in central moments $m_n$ of variable $\epsilon$, i.e., moments defined around its mean value $\bar{\epsilon}$,

$$m_n \equiv \langle(\epsilon - \bar{\epsilon})^n\rangle = \int_0^{\infty} d\epsilon(\epsilon - \bar{\epsilon})^n P(\epsilon), \tag{8}$$

where $\bar{\epsilon} = \int_0^{\infty} d\epsilon\epsilon P(\epsilon)$. Dispersion, skewness, and kurtosis can be expressed in terms of central moments. To this end, it is convenient to introduce a dimensionless parameter

$$Z = \ln\left[\frac{A/|\dot{\epsilon}|}{bB^{1+a/b}}\right], \tag{9}$$

which only weakly depends on the characteristics of the system in question. For self-consistency of the description, this parameter should be large which can be achieved by tuning the ramp speed $|\dot{\epsilon}|$.

It is straightforward to show that moments of distribution (7) can be written as an asymptotic power series in $1/Z \ll 1$ as follows:

$$\langle\epsilon^n\rangle = B^{-n/b}Z^{n/b}\left[1 + \sum_{j=1} Z^{-j}f_j(n, \ln Z)\right]. \tag{10}$$

Definition of the expansion coefficients $f_j$ are relegated to the Supplemental Material [23] because of their cumbersome form. Within the model $f_j$ depend on power exponents $a$ and $b$, and very weakly (as a double logarithm), on temperature-dependent parameters $A$ and $B$, and the ramp speed $|\dot{\epsilon}|$. This implies that temperature scaling of both moments $\langle\epsilon^n\rangle$ and central moments $\langle(\epsilon - \bar{\epsilon})^n\rangle$ is fully dominated by the temperature scaling of $B$, which is proportional to the height of the phase slip barrier $\{\langle\epsilon^n\rangle, \langle(\epsilon - \bar{\epsilon})^n\rangle\} \propto B^{-n/b}(T)$. To determine proportionality coefficients, one needs to use the explicit form of $f_j$. Up to the order $1/Z$, the first two moments are given by

$$\bar{\epsilon} = Z^{1/b}B^{-1/b}\left(1 + \frac{v/b}{Z}\right), \tag{11}$$

$$\begin{aligned}\sigma^2 &\equiv m_2\\ &= Z^{2/b-2}B^{-2/b}\left(\frac{\pi^2}{6b^2} + \frac{1}{Zb^3}\{a\pi^2/3 + (1-b)\right.\\ &\quad\left.\times[\pi^2 v/3 - \psi''(1)]\}\right).\end{aligned} \tag{12}$$

We have defined $v = (a/b)\ln Z + \gamma$, where $\gamma \approx 0.577$ is the Euler-Masheroni constant, and $\psi''(1) \approx -2.404$ is the tetragamma function [46]. Despite the increasing complexity of the calculation, the leading term in the third and fourth central moments are given by simple expressions,

$$m_3 = B^{-3/b}Z^{3/b-3}\left(-\frac{\psi''(1)}{b^3} + \delta_3\right), \tag{13}$$

$$m_4 = B^{-4/b}Z^{4/b-4}\left(\frac{3\pi^4}{20b^4} + \delta_4\right). \tag{14}$$

The correction terms are of the order $\{\delta_3, \delta_4\} \propto Z^{-1}$. For example $\delta_3 = (1/60Zb^4)[90a\pi^2 v(v-1) - 11\pi^4(b-1) - 180\psi''(1)(a - v(b-1))]$ [23]. We are now in the position to compute skewness and kurtosis and thus find

$$S = -m_3/m_2^{3/2} = 6^{3/2}\psi''(1)/\pi^3 + O(Z^{-1}), \tag{15}$$

$$K = m_4/m_2^2 = 27/5 + O(Z^{-1}), \tag{16}$$

which are central results of this section. Remarkably, to the leading order in $1/Z \ll 1$, both skewness and kurtosis are given by universal numbers $S \approx -1.139$ and $K \approx 5.4$, which are independent of the parameters of the system and are the same for both thermal and quantum phase slips. The magnitude of the correction terms $\delta_3$, $\delta_4$ is analyzed for different values of the ramp speed and different models of a weak link in [23].

*Conclusion.*—We have experimentally demonstrated the universality of higher moments—skewness and kurtosis—of the switching current distribution in superconducting nanocircuits. Our results are supported both by analytical modeling and by numerical simulations. We have also pointed out that the universality of higher moments is affected by extraneous noise [23] and suggested using this observation to detect the presence of unwanted noise in the data.

The work was supported by DOE Award No. DE-FG02-07ER46453, ONR No. N000140910689, and NSF No. DMR 10-05645. V. V. was supported by the U.S. Department of Energy, Office of Basic Energy Sciences, Division of Materials Sciences and Engineering under Award No. DE-FG02-08ER46544 and by the Theoretical Interdisciplinary Physics and Astrophysics Center at JHU. A. L. acknowledges support from Michigan State University.

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