Dirac-Born-Infeld Genesis: An Improved Violation of the Null Energy Condition

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We show that the Dirac-Born-Infeld conformal galileons, derived from the world-volume theory of a 3-brane moving in an anti–de Sitter bulk, admit a background, stable under quantum corrections, which violates the null energy condition. The perturbations around this background are stable and propagate subluminally. Unlike other known examples of null energy condition violation, such as ghost condensation and conformal galileons, this theory also admits a stable, Poincaré-invariant vacuum. The $2 \to 2$ amplitude satisfies standard analyticity conditions. The full S matrix is likely not analytic, however, since perturbations around deformations of the Poincaré invariant vacuum propagate superluminally.

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The null energy condition (NEC) is the most robust of all energy conditions. It states that, for any null vector n^μ,

$$T_{\mu\nu} n^\mu n^\nu \geq 0. \tag{1}$$

It has proven extremely difficult to violate this condition with well-behaved relativistic quantum field theories. Aside from being of purely theoretical interest, the NEC plays a role in our understanding of the early universe. In cosmology, Eq. (1) is equivalent to $\rho + P \geq 0$, which, combined with the equation for a spatially-flat universe,

$$M_{\rm Pl}^2 \dot{H} = -\frac{1}{2}(\rho + P), \tag{2}$$

forbids a nonsingular bounce from contraction to expansion. This means a contracting universe necessarily ends in a big crunch singularity, and an expanding universe must emerge from a big bang. Violating Eq. (1) is, therefore, central to any alternative to inflation relying either on a contracting phase before the big bang [1–5] or an expanding phase from an asymptotically static past [6,7].

For theories with at most two derivatives, violating the NEC necessarily implies ghosts or gradient instabilities [8]. To evade this, one must, therefore, invoke higher derivatives, as in the *ghost condensate* [9]. Perturbations around the ghost condensate can violate the NEC in a stable manner [10], and this has been used in the new ekpyrotic scenario [11,12]. However, because the scalar field starts out with a wrong-sign kinetic term, the theory is unstable around its Poincaré-invariant vacuum.

Stable NEC violation can also be achieved with *conformal galileons* [13], a class of conformally-invariant scalar field theories with particular higher-derivative interactions. Remarkably, in spite of the fact that there are five independent galileon terms, only the kinetic term contributes to Eq. (1) [14]: violating the NEC requires a wrong-sign kinetic term, just like the ghost condensate. Another issue with conformal galileons is superluminal propagation around slight deformations of the NEC-violating background [7] (though this can be avoided by explicitly breaking special conformal transformations [14]).

In this Letter, we show that the *DBI conformal galileons* [15,16] can also violate the NEC in a stable manner, while avoiding nearly all of the aforementioned issues. Specifically, the coefficients of the five Dirac-Born-Infeld (DBI) galileons can be chosen such that: 1. There exists a stable, Poincaré-invariant vacuum. 2. The $2 \to 2$ scattering amplitude about this vacuum obeys standard analyticity conditions. 3. The theory admits a time-dependent, homogeneous, and isotropic solution which violates the NEC in a stable manner. 4. Perturbations around the NEC-violating background, and around small deformations thereof, propagate subluminally. 5. This solution is stable against radiative corrections.

In other words, starting from a local relativistic quantum field theory defined around a Poincaré-invariant vacuum state, the theory allows consistent, stable, NEC-violating solutions. In fact, this NEC-violating background is an *exact* solution of the effective theory, including all possible higher-dimensional operators consistent with the assumed symmetries.

We will see that the above conditions can be satisfied for a broad region of parameter space. This represents a significant improvement over ghost condensation (which fails to satisfy 1 and 2) and the ordinary conformal galileons (which fail to satisfy 1, 2, and 4). Unfortunately, like conformal galileons, superluminal propagation around deformations of the Poincaré invariant solution is inevitable. As a result, the full S matrix likely fails to be analytic. Additionally, one would like the theory to be consistent with black hole thermodynamics [17]. This is currently under investigation [18].

The geometric origin of the DBI conformal galileon as the theory of a 3-brane moving in an anti–de Sitter (AdS_5) bulk makes contact with stringy scenarios, offering a promising avenue to search for NEC violations in string theory.

The theory.—Consider a 3-brane, with world-volume coordinates x^μ, probing an AdS_5 space-time with coordinates X^A and metric $G_{AB}(X)$ in the Poincaré patch

$$ds^2 = G_{AB}dX^A dX^B = Z^{-2}dZ^2 + Z^2\eta_{\mu\nu}dX^\mu dX^\nu, \quad (3)$$

where $Z \equiv X^5$, $0 < Z < \infty$. The dynamical variables are the embedding functions, $X^\mu(x)$, $Z(x) \equiv \phi(x)$. In unitary gauge, $X^\mu = x^\mu$, the brane induced metric is

$$\mathfrak{g}_{\mu\nu} = G_{AB}\partial_\mu X^A \partial_\nu X^B = \phi^2\eta_{\mu\nu} + \phi^{-2}\partial_\mu\phi\partial_\nu\phi. \quad (4)$$

The DBI conformal galileons are five geometric invariants consisting of 4D Lovelock terms ($\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_4$) and the boundary terms of 5D Lovelock terms ($\mathcal{L}_3$ and $\mathcal{L}_5$):

$$\mathcal{L}_1 = -\frac{1}{4}\phi^4, \qquad \mathcal{L}_2 = -\sqrt{-\mathfrak{g}} = -\gamma^{-1}\phi^4, \qquad \mathcal{L}_3 = \sqrt{-\mathfrak{g}}K = -6\phi^4 + \phi[\Phi] + \gamma^2\phi^{-3}(-[\phi^3] + 2\phi^7),$$
$$\mathcal{L}_4 = -\sqrt{-\mathfrak{g}}\mathfrak{R} = 12\gamma^{-1}\phi^4 + \gamma\phi^{-2}\{[\Phi^2] - ([\Phi] - 6\phi^3)([\Phi] - 4\phi^3)\} + 2\gamma^3\phi^{-6}\{-[\phi^4] + [\phi^3]([\Phi] - 5\phi^3)$$
$$- 2[\Phi]\phi^7 + 6\phi^{10}\},$$
$$\mathcal{L}_5 = \frac{3}{2}\sqrt{-\mathfrak{g}}\left(-\frac{K^3}{3} + K^2_{\mu\nu}K - \frac{2}{3}K^3_{\mu\nu} - 2\mathfrak{G}_{\mu\nu}K^{\mu\nu}\right) = 54\phi^4 - 9\phi[\Phi] + \gamma^2\phi^{-5}\{9[\phi^3]\phi^2 + 2[\Phi^3] - 3[\Phi^2][\Phi]$$
$$+ 12[\Phi^2]\phi^3 + [\Phi]^3 - 12[\Phi]^2\phi^3 + 42[\Phi]\phi^6 - 78\phi^4\} + 3\gamma^4\phi^{-9}\{-2[\phi^5] + 2[\phi^4]([\Phi] - 4\phi^3)$$
$$+ [\phi^3]([\Phi^2] - [\Phi]^2 + 8[\Phi]\phi^3 - 14\phi^6) + 2\phi^7([\Phi]^2 - [\Phi^2]) - 8[\Phi]\phi^{10} + 12\phi^{13}\}. \quad (5)$$

Here, $\gamma \equiv 1/\sqrt{1 + (\partial\phi)^2/\phi^4}$ is the Lorentz factor for the brane motion, $\mathcal{L}_1$ measures the proper 5-volume between the brane and some fixed reference brane [15], and $\mathcal{L}_2$ is the world-volume action, i.e., the brane tension [19]. The higher-order terms $\mathcal{L}_3$, $\mathcal{L}_4$, and $\mathcal{L}_5$ are functions of the extrinsic curvature tensor $K_{\mu\nu} = \gamma(-\phi^{-1}\partial_\mu\partial_\nu\phi + \phi^2\eta_{\mu\nu} + 3\phi^{-2}\partial_\mu\phi\partial_\nu\phi)$ and the induced Ricci tensor $\mathfrak{R}_{\mu\nu}$ and scalar $\mathfrak{R}$, with $\mathfrak{G}_{\mu\nu} \equiv \mathfrak{R}_{\mu\nu} - \mathfrak{R}\mathfrak{g}_{\mu\nu}/2$ (and indices raised by $\mathfrak{g}^{\mu\nu}$). Following [13], Φ denotes the matrix of second derivatives $\partial_\mu\partial_\nu\phi$, $[\Phi^n] \equiv \mathrm{Tr}(\Phi^n)$ and $[\phi^n] \equiv \partial\phi\Phi^{n-2}\partial\phi$, with indices raised by $\eta^{\mu\nu}$.

Each $\mathcal{L}$ is invariant up to a total derivative under the $so(4,2)$ conformal algebra, inherited from the isometries of AdS_5. Aside from Poincaré transformations, Eq. (5) is also invariant under dilation, $\delta_D\phi = -(1 + x^\mu\partial_\mu)\phi$, and special conformal transformations, $\delta_{K_\mu}\phi = (-2x_\mu - 2x_\mu x^\nu\partial_\nu + x^2\partial_\mu + \phi^{-2}\partial_\mu)\phi$.

Around the Poincaré invariant vacuum.—Expanding $\mathcal{L} = \sum_{i=1}^5 c_i\mathcal{L}_i$ around a constant field profile, $\bar\phi_0$, up to quartic order in perturbations $\varphi = \phi - \bar\phi_0$, we obtain

$$\mathcal{L} = -\frac{C_2}{2}(\partial\varphi)^2 + \frac{C_3}{12\bar\phi_0^3}(\partial\varphi)^2\Box\varphi + \frac{(3C_2 - C_3)}{24\bar\phi_0^4}(\partial\varphi)^4 - \frac{C_3}{4\bar\phi_0^4}\varphi(\partial\varphi)^2\Box\varphi + \frac{C_4}{24\bar\phi_0^6}(\partial\varphi)^2[(\partial_\mu\partial_\nu\varphi)^2 - (\Box\varphi)^2];$$
$$C_2 \equiv c_2 + 6c_3 + 12c_4 + 6c_5, \qquad C_3 \equiv 6c_3 + 36c_4 + 54c_5, \qquad C_4 \equiv 12c_4 + 48c_5, \qquad C_5 \equiv c_5, \quad (6)$$

where, in order for $\bar\phi_0$ to be a solution, we have imposed that the tadpole term vanish:

$$C_1 \equiv -\frac{1}{4}c_1 - c_2 - 4c_3 + 12c_5 = 0$$
$$\text{(Poincaré solution).} \quad (7)$$

A necessary and sufficient condition for the stability of small fluctuations is

$$C_2 > 0 \quad \text{(stability).} \quad (8)$$

Next, the scattering S matrix derived from Eq. (6) should satisfy standard relativistic dispersion relations. Firstly, the $2 \to 2$ amplitude in the forward limit must display a positive s^2 contribution [20]. Only the $(\partial\varphi)^4$ vertex contributes in the forward limit—its coefficient must be strictly positive [20,21]. There also exist constraints away from the forward limit [22], which involve the $(\partial\varphi)^2\Box\varphi$ and $(\partial\varphi)^2(\partial_\mu\partial_\nu\varphi)^2$ vertices [23]. These analyticity conditions, respectively, impose

$$C_3 < 3C_2; \qquad C_3^2 > 6C_2C_4 \quad \text{(analyticity).} \quad (9)$$

NEC-violating solution.—We seek a time-dependent, isotropic background solution of the form

$$\bar\phi = \frac{\alpha}{(-t)}; \qquad -\infty < t < 0, \quad (10)$$

where α is a constant. This profile, which is central to pseudoconformal [3,4,24] and Galilean Genesis [7] cosmology, spontaneously breaks the $so(4,2)$ algebra down to an $so(4,1)$ subalgebra. Substituting Eq. (10) into the equation of motion for ϕ derived from Eq. (5), we obtain

$$C_2+\frac{1}{2}C_3\beta+\frac{1}{2}C_4\beta^2+6C_5\beta^3=0 \quad (1/t\,\text{solution}), \tag{11}$$

with $\beta \equiv \bar{\gamma}-1>0$, $\bar{\gamma}=1/\sqrt{1-\alpha^{-2}}$. There is a solution for each real, positive root of Eq. (11).

We require this background to be stable against small perturbations. Expanding Eq. (5) to quadratic order in $\varphi \equiv \phi-\bar{\phi}$, we obtain

$$\mathcal{L}_{\text{quad},1/t}=\frac{\mathcal{Z}}{2}\left(\dot{\varphi}^2-\bar{\gamma}^{-2}(\vec{\nabla}\varphi)^2+\frac{6}{t^2}\varphi^2\right), \tag{12}$$

where $\mathcal{Z}\equiv\bar{\gamma}^3(C_2+C_3\beta+3C_4\beta^2/2+24C_5\beta^3)$. Absence of ghosts, therefore, requires

$$C_2+C_3\beta+\frac{3}{2}C_4\beta^2+24C_5\beta^3>0 \quad (\text{stability}). \tag{13}$$

The sound speed is always subluminal, but for small deformations away from the solution to satisfy condition 4, we want the sound speed $c_s=\bar{\gamma}^{-1}$ to be generously less than unity. Thus, we demand

$$\beta \gtrsim 1 \quad (\text{robust subluminality around } 1/t). \tag{14}$$

To check for NEC violation, we calculate the stress tensor $T_{\mu\nu}$ by varying the covariant version of Eq. (5) with respect to the metric. The covariant theory is given uniquely by the brane construction [16], and is given by Eq. (5) with the replacements $\eta_{\mu\nu}\to g_{\mu\nu}$ and $\partial_\mu\to\nabla_\mu$, plus the following nonlinear couplings:

$$\begin{aligned}\delta\mathcal{L}_4&=-\gamma^{-1}R\phi^2+2\gamma\phi^{-2}R^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\\ \delta\mathcal{L}_5&=(3/2)R\phi^{-5}\{\phi^4([\Phi]-4\phi^3)+\gamma^2(-[\phi^3]+2\phi^7)\}\\ &\quad-3\phi^{-1}R^{\mu\nu}\nabla_\mu\nabla_\nu\phi+3\gamma^2\phi^{-5}R^{\mu\nu}\\ &\quad\times[(4\phi^3-[\Phi])\nabla_\mu\phi+\nabla^\kappa\phi\nabla_\kappa\nabla_\mu\phi]\nabla_\nu\phi\\ &\quad+3\gamma^2\phi^{-5}R^{\mu\kappa\nu\lambda}\nabla_\mu\phi\nabla_\nu\phi\nabla_\kappa\nabla_\lambda\phi,\end{aligned} \tag{15}$$

where indices are now raised and lowered with $g_{\mu\nu}$, and we assume an overall $\sqrt{-g}$ factor. Since $\delta\mathcal{L}_{4,5}$ include nonminimal couplings, we must be precise about our definition of $T_{\mu\nu}$ and associated NEC. We couple this theory to Einstein-Hilbert gravity and define $T_{\mu\nu}$ as the source of $G_{\mu\nu}$, i.e., $T_{\mu\nu}\equiv M_{\rm Pl}^2G_{\mu\nu}$. By matching this to a standard, radiation-dominated phase, below, we will unambiguously ascertain whether the NEC violation is "genuine" or simply an artifact of nonminimal couplings.

Varying the action with respect to the metric, and setting $\bar{g}_{\mu\nu}=\eta_{\mu\nu}$ and $\bar{\phi}=\alpha/(-t)$, yields an isotropic $T_{\mu\nu}$, with vanishing energy density and pressure scaling as t^{-4} (as it must by dilation invariance [5,7]),

$$\rho=0; \qquad P=\frac{\alpha^2}{t^4}(C_2-C_4+12C_5), \tag{16}$$

where we have used Eq. (11) to simplify. To violate the NEC, the pressure must be negative,

$$C_2-C_4+12C_5<0 \quad (\text{NEC violation}). \tag{17}$$

Matching to standard cosmology.—Integrating Eq. (2), we obtain a *DBI Genesis* cosmology, describing an expanding universe from an asymptotically static state:

$$H(t)=-(C_2-C_4+12C_5)\frac{\alpha^2}{3M_{\rm Pl}^2(-t)^3}. \tag{18}$$

For this to represent a useful NEC violation, we verify that the DBI Genesis phase matches onto an *expanding* radiation-dominated phase. We remain agnostic about the reheating process; our main concern is whether the universe is expanding after the transition. In theories which admit an Einstein frame, the condition below implies continuity of the Einstein frame H. Because of nonminimal couplings, we instead find that H is discontinuous [14]. Indeed, the pressure is of the form: $P=G(\phi,\dot{\phi})+dF(\phi,\dot{\phi})/dt$. The G term is regular as ϕ is brought instantaneously to a halt, but the F term gives rise to a delta function. Explicitly, we have

$$\begin{aligned}F(t)\equiv\frac{\alpha^2}{6(-t)^3}\bigg[&24C_5-2C_4-(2C_4-60C_5)\beta-18C_5\beta^2\\ &-(C_3-3C_4+90C_5)\left(\frac{\bar{\gamma}\cosh^{-1}\bar{\gamma}}{\sqrt{1+\bar{\gamma}}\sqrt{\beta}}-1\right)\bigg].\end{aligned} \tag{19}$$

Integrating Eq. (2) around the delta-function singularity, we discover that $H+F/2M_{\rm Pl}^2$ matches continuously at the transition. Hence, we obtain the matching condition:

$$H_{\text{Genesis}}+\frac{F}{2M_{\rm Pl}^2}=H_{\text{rad.-dom.}}. \tag{20}$$

Combining Eqs. (18) and (19), we find that the universe will be expanding in the radiation-dominated phase if

$$\begin{aligned}&2C_2+(2C_4-60C_5)\beta+18C_5\beta^2\\ &\quad+(C_3-3C_4+90C_5)\left(\frac{\bar{\gamma}\cosh^{-1}\bar{\gamma}}{\sqrt{1+\bar{\gamma}}\sqrt{\beta}}-1\right)<0\\ &\quad(\text{matching}).\end{aligned} \tag{21}$$

Summary of conditions.—We started out with five coefficients, $C_1,\ldots,C_5$. Stability of the Poincaré-invariant vacuum sets $C_1=0$ and (without loss of generality) $C_2=1$. This leaves us with three coefficients, C_3, C_4, and C_5, which must be chosen such that the cubic equation Eq. (11) has a real root with $\beta\gtrsim1$ [per Eq. (14)], and which must satisfy the inequalities Eqs. (9), (13), (17), and (21).

All these conditions can be satisfied even with $C_5=0$. With $C_2=1$, the first inequality in Eq. (9) gives $C_3<3$, while Eq. (17) simplifies to $C_4>1$. The equation of motion Eq. (11) reduces to a quadratic equation, with roots $\beta_\pm=(\pm\sqrt{C_3^2-8C_4}-C_3)/2C_4$. It is easy to check that

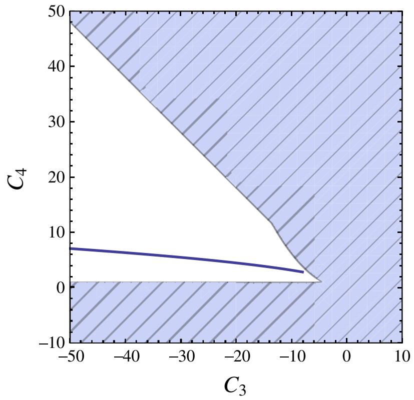

FIG. 1 (color online). Allowed (white) region of (C_3, C_4) parameter space satisfying all of our conditions, with $C_1 = C_5 = 0$ and $C_2 = 1$. In the allowed region, $\beta \simeq -C_3/C_4$ for $|C_3| \gg 1$. On the solid curve, β grows without bound as $C_3 \to -\infty$, showing that all constraints can be satisfied for arbitrarily large β.

only β_+ can lead to a stable $1/t$ solution. In order for β_+ to be real and $\gtrsim 1$, we must require $C_3^2 > 8C_4$ and $C_3 \lesssim -(2 + C_4)$. With these conditions, Eq. (13) and the second inequality of Eq. (9) are automatically satisfied. The only remaining constraint is Eq. (21). Figure 1 shows (in white) the allowed region of (C_3, C_4) parameter space satisfying all of our constraints. Generalizing the analysis to $C_5 \neq 0$ only widens the allowed region.

Quantum stability.—We now argue that the NEC-violating solution is robust against other allowed terms in the effective theory, i.e., all diffeomorphism invariants of the induced metric and extrinsic curvature. Using the Gauss-Codazzi relation $\mathfrak{R}_{\mu\nu\rho\sigma} = 2/3(\mathfrak{g}_{\mu\rho}\mathfrak{g}_{\nu\sigma} - \mathfrak{g}_{\mu\sigma}\mathfrak{g}_{\nu\rho}) + K_{\mu\rho}K_{\nu\sigma} - K_{\mu\sigma}K_{\nu\rho}$ to eliminate all instances of $\mathfrak{R}_{\mu\nu\rho\sigma}$ in favor of $K_{\mu\nu}$, we see that the DBI galileons are particular polynomials in $K_{\mu\nu}$. As argued in the appendix of [25], however, any polynomial in $K_{\mu\nu}$ can be brought to the galileon form through field redefinitions.

It remains to consider terms with covariant derivatives acting on $K_{\mu\nu}$, such as $K_{\mu\nu}\Box K^{\mu\nu}$. Since $\bar{K}_{\mu\nu} = -\bar{\gamma}\bar{\mathfrak{g}}_{\mu\nu}$ on the $1/t$ background, it is annihilated by ∇, so these higher-derivative terms do not contribute to the equation of motion for the $1/t$ ansatz. Hence, the $1/t$ solution is an *exact* solution, including all possible higher-derivative terms in the effective theory.

These higher-derivative terms *do* contribute to perturbations, but it is technically natural to set their coefficients to zero if there is a hierarchy, $C_3 \sim \beta$, $C_2 \sim C_4 \sim \mathcal{O}(1)$, $C_5 \sim 1/\beta$, where $\beta \gg 1$ ($\alpha \simeq 1$). This corresponds to relativistic brane motion. The solid curve in Fig. 1, corresponding to $C_4 \simeq -C_3/\beta$ for $\beta \gg 1$, shows that all of our constraints can be satisfied for arbitrarily large β. In the limit of large $|t|$, the theory of perturbations is approximately the same as that about a constant background. Consequently, the fluctuation Lagrangian takes the form Eq. (6), where now $\bar{\phi}_0$ is Eq. (10), except that every spatial gradient is multiplied by a factor of the sound speed, $1/\bar{\gamma} \simeq 1/\beta$. A computation shows that the coefficient of an $\mathcal{O}(\varphi^n)$ term scales as β^{2n+1}. The (ordinary) galileon terms are suppressed by the lowest scale in the theory

$$\Lambda_s \equiv \beta^{1/6}t^{-1} \simeq \beta^{1/6}\bar{\phi}(t), \tag{22}$$

which we identify as the strong coupling scale. We now study the limit $\beta \to \infty$, $|t| \to \infty$, keeping Λ_s fixed. Only the ordinary galileon terms [13] survive, with spatial gradients suppressed by γ, so we scale them in taking the limit so that the limiting theory looks Lorentz invariant. Because of the galileon nonrenormalization theorem [26–28], it follows that if we work at finite β, radiative corrections to $C_1, \ldots, C_5$ must be suppressed by powers of $1/\beta$, so the hierarchy we have set up is stable. Loop corrections also produce higher-derivative terms suppressed by Λ_s, but these are consistently small at low energy so we have a derivative expansion in ∂/Λ_s.

Finally, we discuss the issue of superluminality around the Poincaré-invariant vacuum $\phi = \bar{\phi}_0$. With $C_3 \neq 0$, weak deformations of this background exhibit superluminal propagation [23]. (Our conditions cannot be simultaneously satisfied with $C_3 = 0$.) Following the arguments of [23], superluminal effects can be consistently ignored in the effective theory if the cutoff is sufficiently low: $\Lambda_0 \lesssim \bar{\phi}_0/\sqrt{|C_3|} \sim \bar{\phi}_0/\sqrt{\beta}$. By relativistic and conformal invariance, the cutoff around any background scales as $\Lambda \sim \phi/\gamma$. For consistency of our analysis, the lowest allowed cutoff around the NEC-violating solution is set by the mass of φ, namely $1/|t|$. This implies $\Lambda_0 \sim \beta\bar{\phi}_0$; hence, superluminal effects lie within the effective theory.

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