Characterizing Nonclassical Correlations via Local Quantum Uncertainty

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Quantum mechanics predicts that measurements of incompatible observables carry a minimum uncertainty which is independent of technical deficiencies of the measurement apparatus or incomplete knowledge of the state of the system. Nothing yet seems to prevent a single physical quantity, such as one spin component, from being measured with arbitrary precision. Here, we show that an intrinsic quantum uncertainty on a single observable is ineludible in a number of physical situations. When revealed on local observables of a bipartite system, such uncertainty defines an entire class of bona fide measures of nonclassical correlations. For the case of $2 \times d$ systems, we find that a unique measure is defined, which we evaluate in closed form. We then discuss the role that these correlations, which are of the "discord" type, can play in the context of quantum metrology. We show in particular that the amount of discord present in a bipartite mixed probe state guarantees a minimum precision, as quantified by the quantum Fisher information, in the optimal phase estimation protocol.

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Introduction.—In a classical world, error bars are exclusively due to technological limitations, while quantum mechanics entails that two noncommuting observables cannot be jointly measured with arbitrary precision [1], even if one could access a flawless measurement device. The corresponding uncertainty relations have been linked to distinctive quantum features such as nonlocality, entanglement, and data processing inequalities [2–4].

Remarkably, even a single quantum observable may display an intrinsic uncertainty as a result of the probabilistic character of quantum mechanics. Let us consider for instance a composite system prepared in an entangled state [5], say, the Bell state $|\phi^+\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$ of two qubits. This is an eigenstate of the global observable $\sigma_z \otimes \sigma_z$ [$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices], so there is no uncertainty on the result of such a measurement. On the other hand, the measurement of *local* spin observables of the form $\vec{a} \cdot \vec{\sigma} \otimes \mathbb{I}$ (where $\vec{a} \neq 0$ is a real vector) is intrinsically uncertain. Indeed, the state $|\phi^+\rangle\langle\phi^+|$, and in general any entangled state, cannot be eigenstates of a local observable. Only uncorrelated states of the two qubits, e.g., $|00\rangle$, admit at least one completely "certain" local observable.

Extending the argument to mixed states, one needs to filter out the uncertainty due to classical mixing, i.e., lack of knowledge of the state, in order to identify the genuinely quantum one. We say that an observable K on the state ρ is "quantum certain" when the statistical error in its measurement is solely due to classical ignorance. By adopting a meaningful quantitative definition of quantum uncertainty, as detailed later, we find that K is quantum certain if and only if $\rho = \rho_K$, where ρ_K is the density matrix of the state after the measurement of K. It follows that not only entangled states but also almost all (mixed) separable states [6] cannot admit any quantum-certain local observable. The only states left invariant by a local complete measurement are those described within classical probability theory [7], i.e., embeddings of joint probability distributions. These are the states with zero *quantum discord* [8–10]. The quantum uncertainty on local observables is then entwined to the notion of quantum discord (see Fig. 1), a form of nonclassical correlation which reduces to entanglement on pure states, and is currently subject to intense investigations for quantum computation and information processing [11–14]. In the following, an entire class of discordlike measures is defined, interpreted, and analyzed within the framework of local quantum uncertainty.

Skew information and local quantum uncertainty.—There are several ways to quantify the uncertainty on a measurement, and here we aim at extracting the truly quantum share. Entropic quantities or the variance, although employed extensively as indicators of uncertainty [1,3,4], do not fit our purpose, since they are affected by the state mixedness. It has been proposed to isolate the quantum contribution to the total statistical error of a measurement as being due to the noncommutativity between state and observable: this may be reliably quantified via the *skew information* [15,16]

$$\mathcal{I}(\rho, K) = -\frac{1}{2}\,\mathrm{Tr}\{[\rho^{1/2}, K]^2\}, \qquad (1)$$

introduced in [16] and employed for studies on uncertainty relations [15], quantum statistics, and information geometry [15,17–20]. Referring to [16] for the main properties of the skew information, we recall the most relevant ones: it is nonnegative, vanishing if and only if state and observable commute, and is convex, that is, nonincreasing under classical mixing. Moreover, $\mathcal{I}(\rho, K)$ is always smaller than the variance of K, $\mathcal{I}(\rho, K) \leq \mathrm{Var}_\rho(K) \equiv \langle K^2\rangle_\rho - \langle K\rangle_\rho^2$, with

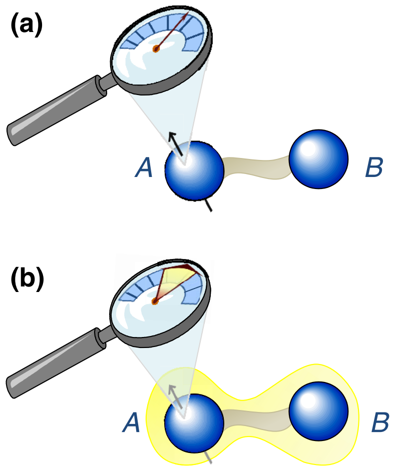

FIG. 1 (color online). Quantum correlations trigger local quantum uncertainty. Let us consider a bipartite state ρ. An observer on subsystem A is equipped with a *quantum meter*, a measurement device whose error bar shows the quantum uncertainty only. (Note that, in order to access such a quantity, the measurement of other observables that are defined on the full bipartite system may be required, in a procedure similar to state tomography) (a) If ρ is uncorrelated or contains only classical correlations [darker (brown) inner shade], i.e., ρ is of the form $\rho = \sum_i p_i |i\rangle\langle i|_A \otimes \sigma_{iB}$ (with $\{|i\rangle\}$ an orthonormal basis for A) [8–10], the observer can measure at least one observable on A without any intrinsic quantum uncertainty. (b) If ρ contains a nonzero amount of quantum correlations [lighter (yellow) outer shade], as quantified by entanglement for pure states [5] and quantum discord in general [10], any local measurement on A is affected by quantum uncertainty. The minimum quantum uncertainty associated to a single measurement on subsystem A can be used to quantify discord in the state ρ, as perceived by the observer on A. In this Letter, we adopt the Wigner-Yanase skew information [16] to measure the quantum uncertainty on local observables.

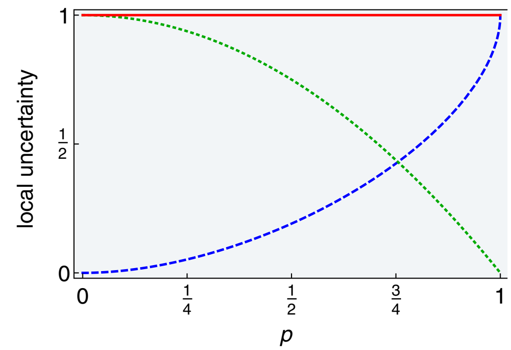

FIG. 2 (color online). The plot shows different contributions to the error bar of spin measurements on subsystem A in a Werner state [5] $\rho = p|\phi^+\rangle\langle\phi^+| + (1-p)\mathbb{I}/4,\ p \in [0, 1]$, of two qubits A and B. The solid red line is the variance $\mathrm{Var}_\rho(\sigma_z^A)$ of the σ_z^A operator, which amounts to the total statistical uncertainty. The dashed blue curve represents the local quantum uncertainty $\mathcal{U}_A(\rho)$, which in this case is $\mathcal{I}(\rho, \sigma_z^A)$ (any local spin direction achieves the minimum for this class of states). The dotted green curve depicts the (normalized) linear entropy $S_L(\rho) = (4/3)(1 - \mathrm{Tr}\{\rho^2\})$ of the global state ρ, which measures its mixedness. Notice that the Werner state is separable for $p \leq 1/3$ but it always contains discord for $p > 0$.

equality reached on pure states, where no classical ignorance occurs (see Fig. 2). Hence, we adopt the skew information as a measure of quantum uncertainty and deliver a theoretical analysis in which we convey and discuss its operational interpretation.

As a central concept in our analysis, we introduce the *local quantum uncertainty* (LQU) as the minimum skew information achievable on a single local measurement. We remark that by "measurement" in the following we always refer to a complete von Neumann measurement. Let $\rho \equiv \rho_{AB}$ be the state of a bipartite system, and let $K^\Lambda = K_A^\Lambda \otimes \mathbb{I}_B$ denote a local observable, with K_A^Λ a Hermitian operator on A with spectrum Λ. We require Λ to be nondegenerate, which corresponds to maximally informative observables on A. The LQU with respect to subsystem A, optimized over all local observables on A with nondegenerate spectrum Λ, is then

$$\mathcal{U}_A^\Lambda(\rho) \equiv \min_{K^\Lambda} \mathcal{I}(\rho, K^\Lambda). \qquad (2)$$

Equation (2) defines a family of Λ-dependent quantities, one for each equivalence class of Λ-spectral local observables over which the minimum skew information is calculated. In practice, to evaluate the minimum in Eq. (2), it can be convenient to parametrize the observables on A as $K_A^\Lambda = V_A \mathrm{diag}(\Lambda) V_A^\dagger$, where V_A is varied over the special unitary group on A. In this representation, the (fixed) spectrum Λ may be interpreted as a standard "ruler," fixing the units as well as the scale of the measurement (that is, the separation between adjacent "ticks"), while V_A defines the measurement basis that can be varied arbitrarily on the Hilbert space of A.

In the following, we prove some general qualitative properties of the Λ-dependent LQUs, which reveal their intrinsic connection with nonclassical correlations.

A class of quantum correlations measures.—What characterizes a discordant state is, as anticipated, the nonexistence of quantum-certain local observables. In fact, we find that each quantity $\mathcal{U}_A^\Lambda(\rho)$ defined in Eq. (2) is not only an indicator but also a full-fledged *measure* of bipartite quantum correlations (see Fig. 1) [21]; i.e., it meets all the known bona fide criteria for a discordlike quantifier [10]. Specifically, in the Supplemental Material [22], we prove that the Λ-dependent LQU (for any nondegenerate Λ) is invariant under local unitary operations, is nonincreasing under local operations on B, vanishes if and only if ρ is a zero discord state with respect to measurements on A, and reduces to an entanglement monotone when ρ is a pure state.

If we now specialize to the case of bipartite $2 \times d$ systems, we further find that quantifying discord via the LQU is very advantageous in practice, compared to all the other measures proposed in the literature (which typically involve formidably hard optimizations not admitting a closed formula even for two-qubit states) [10,23]. Indeed, the minimization in Eq. (2) can be expressed in closed form for arbitrary states ρ_{AB} of a qubit-qudit system defined on $\mathbb{C}^2 \otimes \mathbb{C}^d$, so that $\mathcal{U}_A^\Lambda$ admits a *computable* closed formula. Moreover, notice that, when A is a qubit, all the Λ-dependent measures are equivalent up to a multiplication constant [24]. We thus drop the superscript Λ for brevity and pick nondegenerate observables K_A on the qubit A of the form $K_A = V_A \sigma_{zA} V_A^\dagger = \vec{n} \cdot \vec{\sigma}_A$, with $|\vec{n}| = 1$. This choice corresponds to a LQU normalized to unity for pure, maximally entangled states. Equation (2) can then be rewritten as the minimization of a quadratic form involving the unit vector $\vec{n}$, yielding simply

$$\mathcal{U}_A(\rho_{AB}) = 1 - \lambda_{\max}\{W_{AB}\}, \tag{3}$$

where $\lambda_{\max}$ denotes the maximum eigenvalue and W_{AB} is a 3×3 symmetric matrix whose elements are

$$(W_{AB})_{ij} = \mathrm{Tr}\{\rho_{AB}^{1/2}(\sigma_{iA} \otimes \mathbb{I}_B)\rho_{AB}^{1/2}(\sigma_{jA} \otimes \mathbb{I}_B)\},$$

with $i, j = x, y, z$. It is easy to check that, for a pure state $|\psi_{AB}\rangle\langle\psi_{AB}|$, Eq. (3) reduces to the linear entropy of entanglement $\mathcal{U}_A(|\psi_{AB}\rangle\langle\psi_{AB}|) = 2(1 - \mathrm{Tr}\rho_A^2)$, where ρ_A is the marginal state of subsystem A. Qubit-qudit states represent a relevant class of states for applications in quantum information processing, and we present some pertinent examples in this Letter. The evaluation of the LQU for Werner states of two qubits is displayed in Fig. 2. A case study of the discrete quantum computation with one bit model of quantum computation [25] is reported in the Supplemental Material [22], showing that our measure (evaluated in the one versus n qubit partition) exhibits the same scaling as the canonical entropic measure of discord [8,11]. Beyond the practicality of having a closed formula, the approach adopted in this Letter provides in general a nice physical interpretation of discord as the minimum quantum contribution to the statistical variance associated to the measurement of local observables in correlated quantum systems.

Interestingly, the LQU in a general state ρ_{AB} of a $\mathbb{C}^2 \otimes \mathbb{C}^d$ system can be reinterpreted geometrically as the minimum squared Hellinger distance between ρ_{AB} and the state after a least disturbing root-of-unity local unitary operation applied on the qubit A, in a spirit close to that adopted to define "geometric discords" based on other metrics [10,23,26–28]. Let us recall that the squared Hellinger distance between density matrices ρ and χ is defined as $D_H^2(\rho, \chi) = (1/2)\mathrm{Tr}\{(\sqrt{\rho} - \sqrt{\chi})^2\}$ [29,30]. Observing that, for qubit A, any generic nondegenerate Hermitian observable $K_A = \vec{n} \cdot \vec{\sigma}_A$ is a root-of-unity unitary operation, which implies $K_A f(\rho_{AB}) K_A = f(K_A \rho_{AB} K_A)$ for any function f, we have $\mathcal{I}(\rho_{AB}, K^A) = 1 - \mathrm{Tr}\{\rho_{AB}^{1/2} K^A \rho_{AB}^{1/2} K^A\} = 1 - \mathrm{Tr}\{\rho_{AB}^{1/2}(K^A \rho_{AB} K^A)^{1/2}\} = D_H(\rho_{AB}, K^A \rho_{AB} K^A)$; therefore, minimizing over the local observables $K^A = K_A \otimes \mathbb{I}_B$ yields the geometric interpretation of the LQU, analytically computed in Eq. (3), in terms of Hellinger distance. The study of further connections between uncertainty on a single local observable and geometric approaches to nonclassicality of correlations, possibly in larger and multipartite systems, opens an avenue for future investigations.

Applications to quantum metrology.—We now discuss the operative role that discord, as quantified by the LQU, can play in the paradigmatic scenario of phase estimation in quantum metrology [31]. We focus here on an "interferometric" setup employing bipartite probe states, as sketched in Fig. 3.

Given a (generally mixed) bipartite state ρ used as a probe, subsystem A undergoes a unitary transformation (specifically, a phase shift) so that the global state changes to $\rho_\varphi = U_\varphi \rho U_\varphi^\dagger$, where $U_\varphi = e^{-i\varphi H_A}$, with H_A a local Hamiltonian on A, which we assume to have a nondegenerate spectrum $\bar{\Lambda}$. The goal is to estimate the unobservable parameter φ. The protocol, which has wide-reaching applications, from gravitometry to sensing technologies [31,32], can be optimized by picking the best probe state ρ and the most informative measurement at the output. It is known that the latter optimization can be solved in general by choosing, for any probe state ρ, the measurement strategy which saturates asymptotically the quantum Cramér-Rao bound, $\mathrm{Var}(\tilde{\varphi}) \geq 1/[\nu \mathcal{F}(\rho_\varphi)]$ [33], where the quantum Fisher information (QFI) $\mathcal{F}(\rho_\varphi)$ sets then the precision

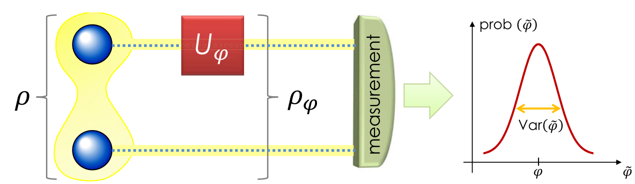

FIG. 3 (color online). Quantum correlations-assisted parameter estimation. A probe state ρ of a bipartite system AB is prepared, and a local unitary transformation depending on an unobservable parameter φ acts on subsystem A, transforming the global state into ρ_φ. By means of a suitable measurement at the output, one can construct an (unbiased) estimator $\tilde{\varphi}$ for φ. The quality of the estimation strategy is benchmarked by the variance of the estimator. For a given probe state ρ, the optimal measurement at the output returns an estimator $\tilde{\varphi}_{\rm best}$ for φ with the minimum allowed variance given by the inverse of the QFI $\mathcal{F}(\rho_\varphi)$, according to the quantum Cramér-Rao bound [33]. In the prototypical case of optical phase estimation, the present scheme corresponds to a Mach-Zender interferometer. Restricting to pure inputs, research in quantum metrology [31] has shown that in this case, entangled probes allow us to beat the shot noise limit $\mathcal{F} \propto n$ (n being the input mean photon number) and reach ideally the Heisenberg scaling $\mathcal{F} \propto n^2$. However, recent investigations have revealed how in the presence of realistic imperfections, the achieved precision quickly degrades to the shot noise level [35–37]. For mixed bipartite probes, we show that the QFI is bounded from below by the amount of quantum correlations in the probe state ρ as quantified by the LQU.

of the optimal estimation, and ν denotes the number of times the experiment is repeated ($\nu \gg 1$ is assumed). We will denote by $\tilde{\varphi}_{\text{best}}$ the estimator obtained from the optimal measurement strategy, so that $\text{Var}(\tilde{\varphi}_{\text{best}}) = 1/[\nu \mathcal{F}(\rho_\varphi)]$. Recall that the QFI can be written as [32,34] $\mathcal{F}(\rho_\varphi) = \text{Tr}\{\rho_\varphi L_\varphi^2\}$, with L_φ being the symmetric logarithmic derivative defined implicitly by $2\partial_\varphi \rho_\varphi = L_\varphi \rho_\varphi + \rho_\varphi L_\varphi$.

We focus therefore on the optimization of the input state. In practical conditions, e.g., when the engineering of the probe states occurs within a thermal environment or with a reduced degree of control, it may not be possible to avoid some degree of mixing in the prepared probe states. It is then of fundamental and practical importance to investigate the achievable precision when the phase estimation is performed within specific noisy settings [35–37]. Here, we assess whether and how quantum correlations in the (generally mixed) state ρ play a role in determining the sensitivity of the estimation. Notice that the remaining steps of the estimation process are assumed to be noiseless (the unknown transformation U_φ is unitary, and the output measurement is the ideal one defined above). The key observation stems from the relation between the Wigner-Yanase and the Fisher metrics [34], which implies that the skew information of the Hamiltonian is majorized by the QFI [15,38]. As H_A is not necessarily the most certain local observable with spectrum $\bar{\Lambda}$, the $\bar{\Lambda}$ LQU itself fixes a lower bound to the QFI:

$$\mathcal{U}_A^{\bar{\Lambda}}(\rho) \leq \mathcal{I}(\rho, H_A) = \mathcal{I}(\rho_\varphi, H_A) \leq \frac{1}{4}\mathcal{F}(\rho_\varphi). \tag{4}$$

Then, for probe states with any nonzero amount of discord, and for $\nu \gg 1$ repetitions of the experiment, the optimal detection strategy which asymptotically saturates the quantum Cramér-Rao bound produces an estimator $\tilde{\varphi}_{\text{best}}$ with necessarily limited variance, scaling as

$$\text{Var}(\tilde{\varphi}_{\text{best}}) = \frac{1}{\nu \mathcal{F}(\rho_\varphi)} \leq \frac{1}{4\nu \mathcal{U}_A^{\bar{\Lambda}}(\rho)}. \tag{5}$$

Hence, we established on rigorous footings that the quantum correlations measured by LQU, although not necessary [39–41], are a sufficient resource to ensure a guaranteed upper bound on the smallest possible variance with which a phase φ can be measured with mixed probes.

We now provide a simple example to clarify the above general discussion. Suppose system A is a spin-j particle undergoing a phase rotation $U_\varphi = \exp(-i\varphi J_z)$, where J_z is the third spin component, and φ the phase to be estimated. In this case, the estimation precision is bounded by the so-called *Heisenberg limit* $\mathcal{F}_{\max} = 4j^2$ [31,42]. A typical scheme achieving this limit can be outlined as follows. Assume that system B is simply a qubit with states $|0\rangle_B$ and $|1\rangle_B$. The AB system is initially prepared in the product state $|j\rangle_A|+\rangle_B$, where $|m\rangle_A$ are the eigenstates of J_z with eigenvalues $m = -j, -j+1, \ldots, j$, and $|\pm\rangle_B = (1/\sqrt{2})(|0\rangle_B \pm |1\rangle_B)$. Then, a "control-flip" operation $\propto \exp(i\pi J_{xA}|1\rangle\langle 1|_B)$ is applied, so that the system evolves to $|\psi\rangle_{AB} = (1/\sqrt{2})(|j\rangle_A|0\rangle_B + |-j\rangle_A|1\rangle_B)$. One can see that the entangled state $|\psi\rangle_{AB}$ used as a probe achieves the Heisenberg limit. Our general treatment allows us to study quantitatively the effect of noise on the estimation power of the bipartite state $|\psi\rangle_{AB}$. Suppose now that the probe state, ideally $|\psi\rangle_{AB}$, is prepared in a noisy environment, which induces partial dephasing in the basis $|m\rangle_A$. Then, our probe state is in general given by $\rho_{AB} = (1/2)[|j,0\rangle\langle j,0| + |-j,1\rangle\langle -j,1| + r(|j,0\rangle\langle -j,1| + \text{H.c.})]$, where $0 \leq r \leq 1$ quantifies the degree of residual coherence, and $|m,\phi\rangle \equiv |m\rangle_A|\phi\rangle_B$. As this is effectively a two-qubit state, we can restrict our analysis to a truncated 2×2 Hilbert space. Here, the restriction of J_z has the spectrum $\bar{\Lambda} = (-j, j)$. We can thus calculate the $\bar{\Lambda}$ LQU in this effective 2×2 Hilbert space, obtaining $\mathcal{U}_A^{\bar{\Lambda}} = j^2(1-\sqrt{1-r^2})$. For any j, notice that the discord is a monotonically increasing function of the coherence r. Hence, from Eq. (4), one has $\mathcal{F}(\rho_{AB}^\varphi) \geq 4\mathcal{U}_A^{\bar{\Lambda}}(\rho_{AB}) = 4j^2(1-\sqrt{1-r^2})$. As the spin number j is increased, this guarantees that the classical scaling $\mathcal{F} \sim 2j$ (i.e., the so-called *shot noise limit* [31]) can still be beaten, provided that $r \gtrsim 1/\sqrt{j}$.

The connection between the LQU and the sensitivity of parameter estimation can also be appreciated in more abstract geometrical terms, without the need for invoking the Fisher information. As shown by Brody [19], the skew information $\mathcal{I}(\rho_\varphi, H_A)$ of the Hamiltonian H_A determines the squared speed of evolution of the density matrix ρ under the unitary $U_\varphi = e^{-i\varphi H_A}$. This provides another geometric interpretation for the LQU: The observable K_A which achieves the minimum in Eq. (2) is the local observable with the property that the resulting local unitary operation $e^{-i\varphi K_A}$ makes the given state ρ of the whole system evolve as slowly as possible (the observable K_A is the least disturbing in this specific sense). Since a higher speed of state evolution under a change in the parameter φ means a higher sensitivity of the given probe state to the estimation of the parameter, our result can be interpreted as follows: The amount of discord (LQU) in a mixed correlated probe state ρ used for estimation of a parameter φ bounds from below the squared speed of evolution of the state under any local Hamiltonian evolution $e^{-i\varphi H_A}$, hence the sensitivity of the given probe state ρ to a variation of φ, which is a general measure of precision for the considered metrological task.

Conclusions.—In this Letter, we studied the quantum uncertainty on single observables. The exploration of this concept allowed us to define and investigate a class of measures of bipartite quantum correlations of the discord type [10], which are physically insightful and mathematically rigorous. In particular, for qubit-qudit states, a unique measure is defined (up to normalization), and it is computable in closed form. Quantum correlations, in the form known as quantum discord [8,9], manifest in the fact that any single local observable displays an intrinsic quantum uncertainty. Discord in mixed probe states, measured by the local quantum uncertainty, is further proven to guarantee a minimum sensitivity in the protocol of optimal phase estimation [31]. We believe it worthwhile to substantiate in future work the promising uncovered connections between quantum mechanics, information geometry, and complexity science [20,33,43] by addressing the role of quantum uncertainty, in particular induced by quantum correlations, in such contexts.

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Note added in proof.—Very recently, an alternative measure of discord based on the quantum Hellinger distance has been proposed in Ref. [44].

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