Possibility of Direct Observation of Edge Majorana Modes in Quantum Chains

A. A. Zvyagin

Max-Planck-Institut für Physik komplexer Systeme, Noethnitzer Strasse, 38, D-01187, Dresden, Germany
B. I. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, Lenin Avenue, 47, Kharkov, 61103, Ukraine
(Received 15 February 2013; published 21 May 2013)

Several scenarios for the realization of edge Majorana modes in quantum chain systems, spin chains, chains of Josephson junctions, and chains of coupled cavities in quantum optics, are considered. For all these systems excitations can be presented as superpositions of a spinless fermion and a hole, characteristic of a Majorana fermion. We discuss the features of our exact solution with respect to possible experiments, in which edge Majorana fermions can be directly observed when studying magnetic, superconducting, and optical characteristics of such systems.

DOI: 10.1103/PhysRevLett.110.217207 PACS numbers: 75.10.Pq, 42.50.Pq, 74.81.Fa

Majorana fermions (MFs) are particles identical to their own antiparticles. They may appear as elementary neutral particles, or emerge as quasiparticles in many-body systems [1]. During recent years, MFs, in addition to being of fundamental interest on their own, have attracted great attention as the basis for potential application in topological quantum computation [2]. The search for MFs is among the most prominent tasks for modern physicists. During the last few years great progress has been achieved in such a search in condensed matter physics. Obviously, we cannot expect MFs to exist in ordinary metals because excitations, electrons—considered as quasiparticles there—and their counterparts, holes (which linear combination would correspond to the MFs), can destroy each other: they carry opposite charges. Hence, the search in different, nonstandard systems of fermions with special properties, where MFs can exist as emergent nontrivial excitations, is necessary. Superconducting systems seemingly provide a basis for such states, because elementary excitations there are superpositions of electrons and holes. However, for conventional superconductors with, e.g., s-wave pairing, those superpositions of electrons and holes carrying *opposite* spin are different from Majorana's construction. Then it follows that for a system of spinless fermions with pairing, like, e.g., model superconductors with p pairing in one-dimensional (1D) systems [3] or with $(p + ip)$ pairing in 2D systems [4], MFs can emerge. Among the most well-known predicted candidates for MF existence are topological insulators [5] and semiconducting quantum wires [6], where pairing can be achieved by interfacing them with an ordinary superconductor. The modern "state of the art" of theoretical predictions for realizations of such systems has been recently reviewed, e.g., in Ref. [7]. While recent papers [8] claim that they have observed zero bias anomalies in the tunneling conductance of normal conducting and superconducting systems, which can be explained by the presence of zero energy MFs, very recent publications mention that in those experiments the spatial resolution might not be enough to detect MFs and that disorder can result in zero bias features [9] even for nontopological systems (where MFs are absent). That is why proposals for realization of direct observations of MFs are highly desirable.

In this Letter we consider several scenarios for the *direct* observation of edge MFs in quantum chains, which can be realized in quantum magnetic, superconducting, and optical systems. For all these systems, excitations can be presented as superpositions of spinless fermions and holes, the hallmark of MFs. We choose 1D systems because exact theoretical results can be obtained there, which is very important for comparison with experiment, and because of the significant success in fabrication and manipulation of quasi-1D materials in recent years. We propose to use an external parameter, which *directly governs* the behavior of the edge MFs in those quantum chains.

To set the stage, we start with the consideration of the spin-1/2 chain, whose Hamiltonian is

$$\mathcal{H}_0 = -\sum_{n=1}^{N-1}(J_x S_n^x S_{n+1}^x + J_y S_n^y S_{n+1}^y) - J_x' S_0^x S_1^x - J_y' S_0^y S_1^y. \quad (1)$$

Here $S_n^{x,y}$ are operators of the projections of spin-1/2 at the nth site, and $J_{x,y}$ ($J_{x,y}'$) are coupling constants for the host (impurity situated at the site $n = 0$). To realize the manifestation of edge MFs in observable characteristics, we propose to study the system with the Hamiltonian $\mathcal{H} = \mathcal{H}_0 - hS_0^x$. The local field h, acting at the edge site of the chain, can be realized if the spin chain system neighbors a ferromagnet that is magnetized along the x axis. Let us (formally) add the spin S_{-1} at the left edge of the chain with the coupling $-2hS_0^x S_{-1}^x$, to study the Hamiltonian $\mathcal{H}_M = \mathcal{H}_0 - 2hS_0^x S_{-1}^x$ instead of $\mathcal{H}$ [10,11]. We see that $\mathrm{Tr}_{N+1}(\rho S_0^x) = \mathrm{Tr}_{N+2}[\rho_M S_0^x(1 + 2S_{-1}^x)] \equiv 2\mathrm{Tr}_{N+2}(\rho_M S_0^x S_{-1}^x)$, where ρ (ρ_M) is the density matrix with the Hamiltonian $\mathcal{H}$ ($\mathcal{H}_M$). It means that to obtain the average value of the operator of edge spin projection with $\mathcal{H}$, we can calculate the one for the pair correlation function with $\mathcal{H}_M$. After the Jordan-Wigner transformation with Dirac creation (destruction) fermionic operators $d_m^\dagger$ (d_m) we get

$$\mathcal{H}_M = -\frac{1}{2}\Big[h(d_{-1}^\dagger d_0 + d_{-1}^\dagger d_0^\dagger + \text{H.c.}) + I'(d_1^\dagger d_0 + d_0^\dagger d_1) + J'[d_1^\dagger d_0^\dagger + d_0 d_1] + \sum_{n=1}^{N-1}(I[d_n^\dagger d_{n+1} + d_{n+1}^\dagger d_n] + J[d_n^\dagger d_{n+1}^\dagger + d_{n+1}d_n])\Big], \quad (2)$$

where I, $J = (J_x \pm J_y)/2$, I', $J' = (J'_x \pm J'_y)/2$. In what follows we consider the limit $N \to \infty$ (semi-infinite chain). Equation (2) is, in fact, the Hamiltonian of the inhomogeneous Kitaev toy model [3]. (The Hamiltonian of the homogeneous Kitaev toy model has the same form as the fermionic representation for the Hamiltonian of the XY spin-1/2 chain introduced in Ref. [12].) Here $I \to w$, w is the hopping parameter of spinless electrons, and $J \to |\Delta|$, with Δ the induced superconducting (SC) gap, or the p-wave pairing amplitude of the 1D topological superconductor [5,7], or a quantum wire [6,7] with zero chemical potential of electrons and with inhomogeneities of hopping amplitudes and gaps near the edge of the chain. Zero chemical potential in Kitaev's model permits the topological superconductivity, i.e., the weak pairing regime, in which the size of a Cooper pair is infinite (see below). The model Eq. (2) can also describe the 1D system of coupled cavities with strong in-cavity photon-photon repulsion and nonlinear photon driving [13] in the cavity quantum electrodynamics. There necessary redefinitions are $J \to \hat{\Delta}$, where $\hat{\Delta}$ is the magnitude of the photon driving, and $I \to \hat{J}$, where $\hat{J}$ is the tunneling amplitude for photon hopping between nearest neighbor cavities. The term with h describes the interaction of the edge cavity with the light [13]. Our model is related to photons being in resonance with cavities. It has been also pointed out recently that Kitaev's model can be realized in 1D arrays of Josephson junctions [14], the chain of SC islands coupled via strong Josephson junctions to common ground superconductors. Each island contains a pair of MFs at the endpoints of a semiconductor nanowire. The parameters of our Hamiltonian are related to the one of the *inhomogeneous* array of Josephson junctions as $J^y \to E_M$, where E_M is the tunnel coupling of individual electrons between SC islands, $J^x \to U$, where $U = \Gamma_U \cos(2\pi q/e)$ is the tunneling amplitude due to the Aharonov-Casher interference caused by the effective capacitance coupling between two islands (e is the electron charge and $q = C_g V_g$ is the induced charge, where C_g is the capacitance to a common back gate at voltage V_g with respect to the ground superconductor). Finally, $h \to \tilde{\Delta}$, where $\tilde{\Delta} = \Gamma_\Delta \cos(\pi q/e)$ is the charging energy. and U and $\tilde{\Delta}$ can be tuned through the inhomogeneous gate voltage at each SC island. We can also consider the term with the boundary field h in $\mathcal{H}$ as Andreev's tunneling.

Then we introduce MFs as $c_{B,j} = d_j + d_j^\dagger$, $c_{A,j} = -i(d_j - d_j^\dagger)$, with $c_{\alpha,m}^\dagger = c_{\alpha,m}$, which satisfy anticommutation relations $\{c_{\alpha,n}, c_{\beta,m}\} = 2\delta_{\alpha,\beta}\delta_{m,n}$ $(\alpha, \beta = A, B)$. In MFs Eq. (2) reads

$$\mathcal{H}_M = -\frac{i}{4}\Big[\sum_{n=1}^{N-1}([J+I]c_{B,n}c_{A,n+1} + [J-I]c_{A,n}c_{B,n+1}) + 2hc_{A,-1}c_{B,0} + (J'+I')c_{B,0}c_{A,1} + (J'-I')c_{A,0}c_{B,1}\Big]. \quad (3)$$

Without the interaction with the (artificial) spin at the site $n = -1$ the term in the Hamiltonian $\mathcal{H}$, which describes the action of the edge field h, has the form $-(h/2)c_{B,0}$; i.e., it is *linear* in MF operators. Hence, the parameter h governs the behavior of the edge MFs. The formal introduction of the spin at site $n = -1$ to the Hamiltonian $\mathcal{H}_M$ is related to the addition of the new (artificial) MFs (cf. Refs. [3,7]), interacting with the linear edge MFs. The total term, proportional to h in $\mathcal{H}_M$, becomes quadratic in MFs.

To diagonalize the Hamiltonian $\mathcal{H}_M$ we use the unitary transformation $d_n = \sum_\lambda(u_{n,\lambda}d_\lambda + v_{n,\lambda}d_\lambda^\dagger)$, where λ's are quantum numbers, which parametrize all eigenstates of the diagonalized Hamiltonian. These quantum numbers can describe extended (band) states. Besides, there is a possibility of localized states, caused by $h \neq 0$, $I' \neq I$, and $J' \neq J$. Let us define $P_{n,\lambda}$, $Q_{n,\lambda} = u_{n,\lambda} \pm v_{n,\lambda}$, i.e., the transfer to MFs $d_n = (1/2)\sum_\lambda(P_{n,\lambda}c_{B,\lambda} - iQ_{n,\lambda}c_{A,\lambda})$. We obtain *two* sets of eigenstates. The first set of solutions describes nonzero $P_{n,\lambda}$ for even n and nonzero $Q_{n,\lambda}$ for odd n (all other P's and Q's are zeros). The second set of solutions describes nonzero $P_{n,\lambda}$ for odd n, and nonzero $Q_{n,\lambda}$ for even n (others are zeros). The details of calculations, and the eigenfunctions $P_{n,\lambda}$ and $Q_{n,\lambda}$, of the Hamiltonian are presented in the Supplemental Material [15]. The energies of the extended (band) states for both sets are $\varepsilon_k^2 = I^2\cos^2 k + J^2\sin^2 k$. As for the localized modes, their energies can be written as

$$4\varepsilon_{(1,2)}^2 = I^2[r_{(1,2)} + r_{(1,2)}^{-1}]^2 - J^2[r_{(1,2)} - r_{(1,2)}^{-1}]^2, \quad (4)$$

where $\ln(r_{(1,2)})$ play the role of the localization radii. We get for the localized state of the first set of eigenfunctions

$$r_{(1)}^2 = \frac{(I-J)}{2(I+J)[(I-J)^2 - (I'-J')^2]}(4h^2 + (I'-J')^2 - 2(I^2+J^2) - [(2h-I-J)^2 + (I'-J')^2 - (I-J)^2]^{1/2}[(2h+I+J)^2 + (I'-J')^2 - (I-J)^2]^{1/2}). \quad (5)$$

This state exists if $[(2h-I-J)^2+(I'-J')^2-(I-J)^2]\times[(2h+I+J)^2+(I'-J')^2-(I-J)^2]>0$. Notice that $|r_{(1)}|<1$; i.e., the localized state decays with the distance from the edge of the chain. Even for the homogeneous case $I'=I$, $J'=J$ for $I+3J>0$ such a localized mode exists at $h\neq 0$. For the second set we obtain $r_{(2)}^2=(I^2-J^2)/[(I'+J')^2-(I-J)^2]$. It does not depend on h. It is easy to check that for $I'=I$ and $J'=J$ such a localized state does not exist.

The ground state wave function $|\mathrm{GS}\rangle$ ($d_\lambda|\mathrm{GS}\rangle=0$) can be written as $|\mathrm{GS}\rangle\propto\prod_\lambda[1+\varphi_{n,\lambda}^{\mathrm{CP}}d_{-\lambda}^\dagger d_\lambda^\dagger]|0\rangle$, where the wave function of Cooper-like pairs is $\varphi_{n,\lambda}^{\mathrm{CP}}=v_{n,\lambda}/u_{n,\lambda}$. For the considered model(s) we have $\varphi_{n,\lambda}^{\mathrm{CP}}=\mathrm{const}$ (see the Supplemental Material [15]); hence Kitaev's topological arguments [3] are valid for the considered model(s). It means that the models are in the topologically nontrivial weak pairing phase. For extended states MFs are coupled at adjacent sites of the chain (with superscripts B, n and A, $n+1$), and the edge of the chain produces the unpaired MFs. The parameter h helps us to realize such MFs in observable characteristics. It is important that in the case of periodic boundary conditions, e.g., in the 1D topological superconductor ring, such an unpaired MF is combined with the one at the other edge of the chain [3,7,11] into the highly nonlocal Dirac fermion. Equally important, the energy of such isolated MFs can become nonzero, e.g., h dependent. Without inhomogeneities edge MFs become zero modes in the limit $N\to\infty$. The nonzero edge field h, actually, removes the degeneracy of the chain, cf. Ref. [11].

Using the obtained total set of eigenvalues and eigenfunctions (see the Supplemental Material [15]) we can calculate any average characteristic of the considered model. For example, for $m=0,1,\ldots$ we have

$$\langle c_{B,2m}c_{A,2m+1}\rangle = i\sum_\lambda P_{2m,\lambda}Q_{2m+1,\lambda}\tanh\frac{\varepsilon_\lambda}{2T},$$
$$\langle c_{B,2m-1}c_{A,2m}\rangle = i\sum_\lambda Q_{2m-1,\lambda}P_{2m,\lambda}\tanh\frac{\varepsilon_\lambda}{2T}, \quad (6)$$

where the thermal averaging with the density matrix, determined by the Hamiltonian $\mathcal{H}_M$, is performed (T is the temperature). We also get

$$\langle c_{A,2m}c_{B,2m+1}\rangle = i\sum_\lambda Q_{2m,\lambda}P_{2m+1,\lambda}\tanh\frac{\varepsilon_\lambda}{2T},$$
$$\langle c_{A,2m-1}c_{B,2m}\rangle = i\sum_\lambda P_{2m-1,\lambda}Q_{2m,\lambda}\tanh\frac{\varepsilon_\lambda}{2T}; \quad (7)$$

i.e., $\langle c_{B,n}c_{A,n+1}\rangle=4i\langle S_n^xS_{n+1}^x\rangle$ is determined by the first set of eigenstates (because the contribution of the second set is zero), while $\langle c_{A,n}c_{B,n+1}\rangle=-4i\langle S_n^yS_{n+1}^y\rangle$ is determined by the second set of eigenstates (zero contribution from the first set). The average value $\langle c_{B,0}\rangle\equiv 2\langle S_0^x\rangle$ with the Hamiltonian $\mathcal{H}$ is equal to $4\langle S_{-1}^xS_0^x\rangle$ with the Hamiltonian $\mathcal{H}_M$; i.e., in such a way, by observing $\langle S_0^x\rangle$ in the spin chain one can *directly* observe the average value of the MF operator. Notice that $\langle c_{A,0}\rangle=-2i\langle S_0^y\rangle=0$. In addition, we obtain $\langle c_{B,n}c_{A,n}\rangle=0$ valid for any h and T (for zero chemical potential in Kitaev's model). Each of the obtained observables is determined by extended and localized states. These average values can be related to the characteristics of Kitaev's model [3], the chain of coupled cavities with strong in-cavity photon-photon repulsion and nonlinear photon driving [13], and the chain of SC islands coupled via strong Josephson junctions to common ground superconductors [14]. The term, proportional to h in $\mathcal{H}$, i.e., the edge MFs, for Kitaev's model and the model of Josephson junctions is related to the edge charge, caused by the local applied potential, or to Andreev's tunneling. For the quantum optics model the term, proportional to h, describes the state of the cavity at the edge of the chain (e.g., the magnitude of the photon of light, proportional to the light absorption by the edge cavity). In Table I we list possible realizations of edge MFs in considered systems. There $e\langle n_0\rangle$ is the charge of an edge SC island [14], and b_0 ($b_0^\dagger$) are the destruction (creation) operators for the photon in the edge cavity [13].

So, the presence of the edge MFs can be seen from the features of temperature- and h-dependent behavior of $M\equiv(1/2)\langle c_{B,0}\rangle$. In fact, we see that the parameter h governs the behavior of the edge MFs. For $h=0$ we have $\langle c_{B,0}\rangle=0$ as it must be. For $J'=J$ and $I'=I$ the localized state exists due to nonzero h. Figure 1 shows the behavior of $M(h)$. The latter is the average value of the edge MF operator for the chain of Josephson junctions as a function of the strength of the local applied voltage, and for the chain of cavities in quantum optics as a function of tunneling or pumping. For the spin chain, $M(h)$ describes the local magnetic moment at the edge of the chain as a function of the local field. At small h the average value is determined by the contribution from the extended (band) states, while at large h it is determined by the localized excitation. The edge MFs (as well as the localized state) exist even for $J=J'=0$ for $h\neq 0$ (i.e., for Kitaev's model in the absence of pairing, $\Delta=0$), due to the pairing caused by h itself. For $J=J'=0$ at small values of h the average value of the local MF operator shows $M\sim(h/I)|\ln(h/I)|$ behavior. For smaller values of I' the region of h appears, in which the contribution of the localized mode is zero. Similar features can be also seen in the behavior of the local susceptibility with respect to h, $\chi=\partial M/\partial h$. For instance, temperature dependences of the local susceptibility for several values of the strength of the local applied

TABLE I. Edge Majorana modes (EMM) in quantum chains

Quantum chain	Spins-1/2	SC islands	Cavity QED
observable EMM	spin projection $\langle S_0^x\rangle$	charge $e\langle n_0\rangle$	light absorption $\langle b_0+b_0^\dagger\rangle$

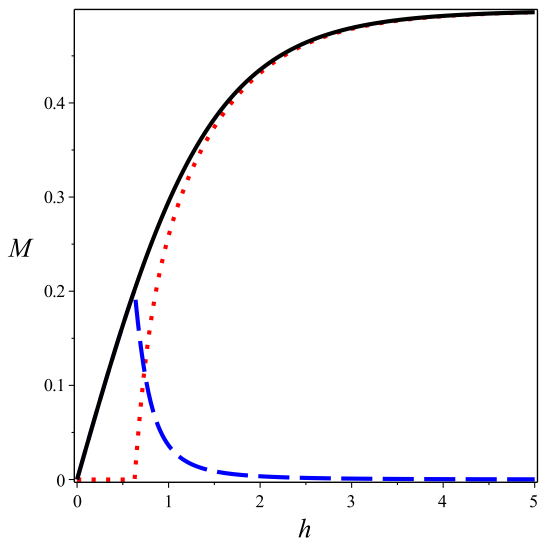

FIG. 1 (color online). The average value of the edge MFs as the function of the applied local voltage (magnetic field, tunneling) for $I = 1$, $J' = J = 0$, and $I' = 1.1$ at $T = 0.7$. The dashed (blue) line shows the contribution from extended states, the dotted (red) line describes the contribution from the localized mode, and the solid (black) line is the total value.

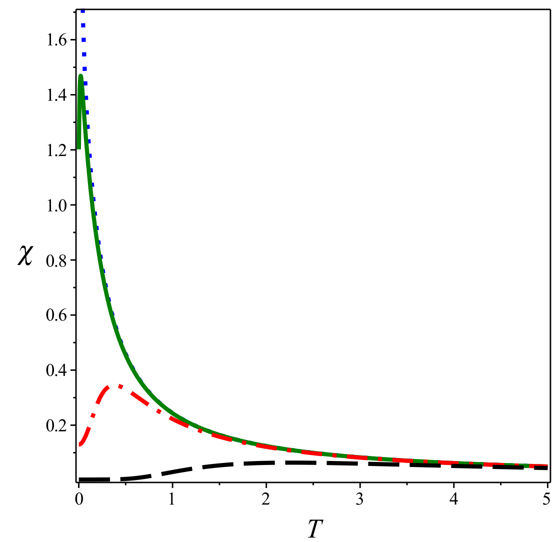

FIG. 2 (color online). The susceptibility of the edge MFs χ as the function of the temperature for $I = 1$, $J' = J = 0$, and $I' = 1.1$. The dotted (blue) line corresponds to the strength of the applied local voltage (magnetic field, tunneling) $h = 0$, the solid (green) line shows $h = 0.1$, the dash-dotted (red) line describes $h = 0.635$ (where the contribution from the localized state appears; see Fig. 1), and the dashed (black) line shows $h = 3.5$.

potential (magnetic field, tunneling) are shown in Fig. 2. At $h = 0$ the local susceptibility diverges (for $J = J' = 0$), while at nonzero h it manifests nonmonotonic temperature behavior: First it grows with T at low temperatures, gets the maximum value (which becomes lower with the growth of h), and then decays with temperature. Such behavior of the edge MFs can be observed in a spin chain with the help of, e.g., nuclear magnetic resonance (NMR). In NMR experiments with spin chains the shift of the resonance position is proportional to the local susceptibility [16]. We expect similar results to persist in the case of any spin-$1/2$ antiferromagnetic chain with the "easy-plane" magnetic anisotropy (with or without in-plane anisotropy, which is important for experimental realization in spin chain materials) with the local magnetic field applied in plane. For example, spin chain materials with magnetic ions Cu^{2+} or V^{4+} (spin-$1/2$) often exhibit magnetic anisotropy about 5%–10%, and finite spin chains can be realized via substitution of nonmagnetic ions instead of magnetic ones [17]. Single crystals of quasi-1D magnetic materials are necessary for the realization of the effect because, in powders, spin chains can be directed randomly. The local field can be caused by the proximity effect of a ferromagnet, neighboring to the spin chain, with the value of h governed by the distance to that ferromagnet. One can realize in-plane direction of h by rotation of the ferromagnet. Then the local magnetic susceptibility at the edge of the spin chain can be measured via the NMR shift. It is worth noting that the Luttinger liquid approach cannot in principle describe localized states, which affect the behavior of edge MFs; however, it can describe the low-h behavior, determined by extended states of the chain. For the chain of Josephson junctions such a characteristic can be observed when studying the charge of the edge island as a function of the voltage, applied locally to the edge of the chain [14], and temperature, or the tunneling Andreev conductance. Finally, in quantum optics the edge MFs can be detected by measuring the state of the probe cavity (or the edge cavity) as a function of the tunneling amplitude [13]. We expect similar effects for the edge MFs on the opposite side of the finite chain. For the extended states of the latter one can replace $k \rightarrow \pi q/N + 2$ with integer q.

In summary, we have proposed a way of direct observation of the edge MFs in several realizations in quantum chains, where excitations can be presented as superpositions of spinless fermions and holes, the necessary condition for MFs. These three realizations are (1) in "easy-plane" spin-$1/2$ chains with in-plane polarized magnetic field, applied to the edge of the chain, (2) in the chain of Josephson junctions, and (3) in the chain of cavities in quantum optics with the tunneling of photons to the edge cavity. As we have shown, such edge MFs can be observed at nonzero temperatures in experiments on dc or ac Josephson currents in chains of superconducting islands, nonlinear quantum optics, and quantum spin chain materials, as the local characteristic of the edge under the action of the governing parameter, h, which *directly* affects the edge MFs.

Support from the Institute for Chemistry of the V. N. Karasin Kharkov National University is acknowledged.

[1] F. Wilczek, Nat. Phys. **5**, 614 (2009).
[2] D. A. Ivanov, Phys. Rev. Lett. **86**, 268 (2001); S. Das Sarma, M. Freedman, and C. Nayak, *ibid.* **94**, 166802 (2005); A. Yu. Kitaev, Ann. Phys. (N.Y.) **303**, 2 (2003).
[3] A. Yu. Kitaev, Phys. Usp. **44**, 131 (2001).
[4] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000); N. B. Kopnin and M. M. Salomaa, Phys. Rev. B **44**, 9667 (1991).
[5] L. Fu and C. L. Kane, Phys. Rev. Lett. **100**, 096407 (2008); Phys. Rev. B **79**, 161408(R) (2009).
[6] R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Phys. Rev. Lett. **105**, 077001 (2010); Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. **105**, 177002 (2010).
[7] J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012); C. W. J. Beenakker, Annu. Rev. Condens. Matter Phys. **4**, 113 (2013); T. D. Stanescu and S. Tewari, arXiv:1302.5433 [J. Phys. Condens. Matter (to be published)].
[8] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Science **336**, 1003 (2012); A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, Nat. Phys. **8**, 887 (2012); M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, Nano Lett. **12**, 6414 (2012); L. P. Rokhinson, X. Liu, and J. F. Furdyna, Nat. Phys. **8**, 795 (2012); J. G. Rodrigo, V. Crespo, H. Suderow, S. Vieira, and F. Guinea, arXiv:1302.0598 [New J. Phys. (to be published)].
[9] J. Liu, A. C. Potter, K. T. Law, and P. A. Lee, Phys. Rev. Lett. **109**, 267002 (2012); E. J. H. Lee, X. Jiang, R. Aguado, G. Katsaros, C. M. Lieber, and S. De Franceschi, Phys. Rev. Lett. **109**, 186802 (2012); D. I. Pikulin, J. P. Dahlhaus, M. Wimmer, H. Schomerus, and C. W. J. Beenakker, New J. Phys. **14**, 125011 (2012).
[10] V. Z. Kleiner and V. M. Tsukernik, Fiz. Nizk. Temp. **6**, 332 (1980) [Sov. J. Low Temp. Phys. **6**, 158 (1980)]; V. Z. Kleiner, Ph.D. thesis, ILTPE, Kharkov, 1982.
[11] See, e.g., A. A. Zvyagin, *Finite-Size Effects in Correlated Electron Systems: Exact Results* (Imperial College, London, 2005), and references therein.
[12] E. H. Lieb, T. D. Schultz, and D. C. Mattis, Ann. Phys. (N.Y.) **16**, 407 (1961).
[13] C.-E. Bardyn and A. Imamoglu, Phys. Rev. Lett. **109**, 253606 (2012); I. Carusotto, D. Gerace, H. Tureci, S. De Liberato, C. Ciuti, and A. Imamolu, Phys. Rev. Lett. **103**, 033601 (2009).
[14] B. van Heck, F. Hassler, A. R. Akhmerov, and C. W. J. Beenakker, Phys. Rev. B **84**, 180502(R) (2011); B. van Heck, A. R. Akhmerov, F. Hassler, M. Burrello, and C. W. J. Beenakker, New J. Phys. **14**, 035019 (2012); F. Hassler and D. Schuricht, New J. Phys. **14**, 125018 (2012).
[15] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.110.217207 for details of calculations of eigenvalues and eigenfunctions.
[16] See, e.g., A. A. Zvyagin, Phys. Rev. B **85**, 134435 (2012) and references therein.
[17] See, e.g., V. Kataev, K.-Y. Choi, M. Grüninger, U. Ammerahl, B. Büchner, A. Freimuth, and A. Revcolevschi, Phys. Rev. Lett. **86**, 2882 (2001); K. M. Kojima *et al.*, Phys. Rev. B **70**, 094402 (2004); R. Klingeler, N. Tristan, B. Büchner, M. Hücker, U. Ammerahl, and A. Revcolevschi, Phys. Rev. B **72**, 184406 (2005).