Majorana-Klein Hybridization in Topological Superconductor Junctions

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We present a powerful and general approach to describe the coupling of Majorana fermions to external leads, of interacting or noninteracting electrons. Our picture has the Klein factors of bosonization appearing as extra Majorana fermions hybridizing with the physical ones. We demonstrate the power of this approach, analyzing a highly nontrivial SO(M) Kondo problem arising in topological superconductors with M Majorana-lead couplings, allowing for arbitrary M and for conduction electron interactions. Mapping the problem on a quantum Brownian motion model we find robust non-Fermi liquid behavior, even for Fermi liquid leads, and a quantum phase transition between insulating and Kondo regimes when the leads form Luttinger liquids. In particular, for $M = 4$ we find a stable realization of the two-channel Kondo fixed point. Obtaining the linear conductance at low temperatures, we predict transport signatures of this Majorana-Kondo-Luttinger physics.

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One of the most influential recent discoveries in condensed matter physics is that topological phases supporting Majorana fermions can be engineered in heterostructures based on s-wave superconductors and materials (e.g., topological insulators, semiconductor nanowires) with strong spin-orbit coupling [1]. This breakthrough development, transforming so-far elusive ideas [2] for Majorana based fault-tolerant quantum computers into a potentially feasible perspective, drives immense theoretical and experimental activities (see Ref. [3] for reviews).

Majorana fermions are localized, robust, zero-energy excitations. Testing the zero bias anomalies [4,5] arising from electrons tunneling onto them forms one of the main experimental directions with promising results so far [6]. Transport can also inform on the Majoranas' quantum computational potential, if it indicates that the nonlocal "topological qubits" formed of pairs of them obey quantum dynamics [7].

In all transport problems, a key role is played by the coupling of Majorana fermions γ to leads of conduction electrons ψ. Here we introduce a general picture of this coupling, observing that $\psi = \Gamma f$ can be broken into Majorana (Γ) and charge density (f) parts using bosonization. Upon Γ hybridizing with the physical Majorana fermion, the charge sector couplings get organized in a simple structure. Rendering the charge sector transparent is a key virtue of our picture. It allows one to approach qualitatively new problems involving the interplay of Majorana fermions and electron interactions, be those between conduction electrons or due to the charging of the superconductor.

We demonstrate this by solving a highly nontrivial problem that includes both types of interactions. This is the topological Kondo effect [7], which, for M Majorana-lead couplings implements a novel, SO(M) Kondo problem. Figure 1 shows the setup for $M = 5$. In Ref. [7], we could solve the simplest, SO(3) $\sim$ SU(2) case for noninteracting leads. Our picture lets us explore vastly more general settings, allowing for arbitary M and interacting conduction electrons. We find a number of striking features. These include stable, non-Fermi liquid (NFL) behavior, even for Fermi liquid leads, a quantum phase transition between Kondo and insulating regimes due to the competition between the Kondo effect and the suppression of electron tunneling in Luttinger liquids, and a long sought-after, stable realization of the two-channel Kondo fixed point for $M = 4$. Our picture delivers these results with ease through mapping the problem to a quantum Brownian motion (QBM) model.

We begin by setting up our Majorana-lead coupling picture more explicitly. We consider a superconducting structure with M_{tot} localized Majoranas γ_j. Their quasiparticle operators obey [3]

$$\gamma_j = \gamma_j^\dagger, \qquad \{\gamma_j, \gamma_k\} = 2\delta_{jk}. \tag{1}$$

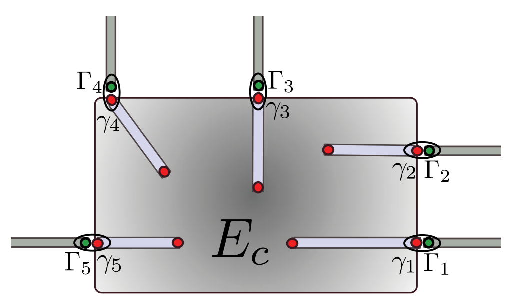

FIG. 1 (color online). Sketch of the topological Kondo setup for $M = 5$. The central rectangle is an s-wave superconductor island (charging energy E_c) with strongly spin-orbit coupled nanowires (light bars) harboring Majorana end states γ_j (red dots). The island is coupled to conduction electron leads (dark bars). The coupling hybridizes γ_j with the Majorana part Γ_j (green dots) of the conduction electron $\psi_j = \Gamma_j f_j$, organizing the charge sector problem involving f_j in a transparent structure.

Ordinary fermionic modes $c_{jk} = (1/2)(\gamma_j + i\gamma_k)$ arise from pairs of Majoranas; in a fermionic system $M_{\rm tot}$ is thus even.

We will work with half infinite ($x \geq 0$), single-channel leads furnishing effectively spinless conduction electrons. This can be achieved in several Majorana realizations [1], including the nanowire based setups of recent experiments [6]. Bosonization [8] transforms the Hamiltonian of lead j into a quadratic problem,

$$H_0(\varphi_j, \theta_j) = \frac{\hbar u}{8\pi}\int dx K(\partial_x\theta_j)^2 + K^{-1}(\partial_x\varphi_j)^2, \qquad (2)$$

even if interactions are present. Here, θ_j, φ_j are bosonic fields encoding the charge density $\rho_j = (\partial_x\varphi_j)/(2\pi)$. They obey

$$[\varphi(x), \theta(y)] = 4\pi i\Theta(x-y), \qquad (3)$$

where $\Theta(x)$ is the Heaviside function, and satisfy [9] $\varphi(0) = (\partial_x\theta)(0) = 0$ at the end point of the lead. Electron interactions enter Eq. (2) through the Luttinger parameter K and through renormalizing the velocity u. We have $K < 1$ ($K > 1$) in the repulsive (attractive) and $K = 1$ in the noninteracting regimes.

Working at energies much below the gap, M leads couple to the superconductor through M Majorana fermions [10]:

$$H_t = \sum_{j=1}^{M} t_j\gamma_j\psi_j e^{i\hat{\chi}/2} + \text{H.c.}, \qquad (4)$$

where t_j is the tunneling amplitude, ψ_j annihilates electrons at the end of lead j, and the phase exponential $e^{\pm i\hat{\chi}/2}$ changes the number N of electrons on the superconductor by ± 1.

Bosonization fractionalizes the electron operator as $\psi_j(x) = \Gamma_j f_j(x)$, where $f_j = (i/\sqrt{a})e^{i\theta_j/2}$ at the endpoint [9], up to a numerical factor to be absorbed in t_j and with being a the short distance cutoff. The Klein factors Γ_j are often omitted but for us they are crucial. They obey

$$\Gamma_j = \Gamma_j^\dagger, \qquad \{\Gamma_j, \Gamma_k\} = 2\delta_{jk}, \qquad \{\Gamma_j, \gamma_l\} = 2\delta_{jl}. \quad (5)$$

The first two are standard relations in bosonization; the anticommutator, in particular, ensures $\{\psi_j, \psi_{k\neq j}\} = 0$. The third relation is less usual: it is introduced to ensure $\{\psi_j, \gamma_l\} = 0$. Equation (5) extends Eq. (1). One can thus view Γ_j as additional Majorana fermions which can, for example, form ordinary fermion modes $d_j = (1/2)(\gamma_j + i\Gamma_j)$ with the physical Majorana fermions. Equation (4) now becomes

$$H_t = \sum_{j=1}^{M}\frac{it_j}{\sqrt{a}}(\gamma_j\Gamma_j)(e^{i\theta_j/2}e^{i\hat{\chi}/2}) + \text{H.c.} \qquad (6)$$

The coupling terms factorize into Majorana-Majorana and charge sector parts. This is our main observation.

In the simplest setting [5] of a (grounded) superconductor coupled to a Luttinger liquid lead ($M = 1$, $E_c \to 0$ in Fig. 1), Eq. (6) immediately recovers a number of results which previously were accessible only after an elaborate Jordan-Wigner procedure [11]. Our picture can, however, be also used to tackle more complex problems. To illustrate this, we now turn to the topological Kondo effect [7]. In this Kondo effect strong correlations emerge, even for noninteracting leads, from the interplay of the charging energy E_c of the superconductor island (connected to ground by a capacitor) and the ground state degeneracy [3] associated with the Majorana fermions [12]. The former enters through the charging term, $H_c(N) = E_c[N - (q/e)]^2$, where q is a background charge (set by the voltage across the capacitor). The ground state degeneracy comes from the fermion modes c_{jk}: they have zero energy (up to exponentially small corrections in the Majoranas' separation) which leads to a $2^{M_{\rm tot}/2-1}$ ground states. The (-1) in the exponent is because of the constraint $(-1)^N = (-1)^{\sum c_{jk}^\dagger c_{jk}}$ [10].

The fermion modes c_{jk} provide the topological qubits and the Kondo effect arises from coupling these to conduction electrons via the Majorana fermions. For low temperatures, voltages and weak coupling, T, V, $|t_j| \ll E_c$, the physics is dominated by virtual transitions connecting the lowest energy charge state of the island to the neighboring ones with ± 1 extra electrons. These are captured by the effective Hamiltonian

$$H_{\rm eff} = \sum_{j\neq k=1}^{M}\lambda_{jk}^+\gamma_j\gamma_k\psi_k^\dagger\psi_j - \sum_{j=1}^{M}\lambda_{jj}^-\psi_j^\dagger\psi_j, \qquad (7)$$

where $\lambda_{jk}^\pm \sim t_jt_k/E_c$, $\lambda_{jk}^+ > 0$. Equation (7) is obtained by a Schrieffer-Wolf transformation keeping the leading order terms in t_j/E_c. As shown in Ref. [7], the nontrivial lead-qubit couplings of the first term implement an SO(M) Kondo problem for $M \geq 3$ (with a spinor "impurity spin" through $\gamma_j\gamma_k$) [14]. In Ref. [7], the problem in Eq. (7) was solved for the minimal SO(3) $\sim$ SU(2) case applying the Affleck-Ludwig conformal field theory method [15]. This is, however, unsuited for interacting leads and becomes complicated for $M > 3$. As we now show, our picture handles both challenges with ease.

Substituting $\psi_j = \Gamma_j f_j$ into Eq. (7), the Majoranas again enter only through $\gamma_j\Gamma_j$ [16]. Importantly, $\gamma_j\Gamma_j$ commute with each other and thus can be diagonalized simultaneously, $\gamma_j\Gamma_j = \pm i$, where we can absorb the sign in θ_j. We get

$$H_{\rm eff} = -\sum_{j\neq k}\lambda_{jk}^+\frac{e^{-i\theta_k/2}e^{i\theta_j/2}}{a} - \sum_j\frac{\lambda_{jj}^-}{2\pi}\partial_x\varphi_j. \qquad (8)$$

Equation (8) is the central formula underlying all our subsequent analysis. We reduced the Kondo problem into one for the charge densities, eliminating the Majorana degrees of freedom through the $\gamma_j\Gamma_j$ hybridizations.

For $\lambda^- = 0$, Eq. (8) is known to map to QBM which, for $\lambda^+_{jk} = \lambda^+ < 0$, $K > 1$, is related to the $M \geq 3$ channel Kondo model [9,17,18]. Our SO(M) problem is in the $\lambda^+ > 0$ sector, and we have $K \leq 1$ in its lead-Majorana implementation. Eq. (8), for $M = 3$, $\lambda^+_{jk} = \lambda^+$, $\lambda^-_{jj} = 0$, also appeared as an unphysical description of quantum wire trijunctions, used for illustrating the dangers of omitting Klein factors [9]. For us Eq. (8) arises precisely from using Klein factors correctly.

In what follows, we apply the renormalization group to our full problem, focusing on $K \leq 1$ and allowing for the anisotropy ($\lambda^+_{jk} \neq \lambda^+$) and λ^-_{jj} terms arising in a physical realization. The key aspects of the physics will be shown to come from the λ^+_{jk} term, extending the relation to QBM to this more general case. The results will be used to obtain the low temperature behavior of the (zero bias) Kubo conductance G_{kl} between leads k, l. The transport setup (for $M = 5$) is sketched in the inset of Fig. 2.

The weak $\lambda^\pm_{jk}$ flow under rescaling $a \to ae^l$, obtained from the operator product expansion, is

$$\frac{d\lambda^+_{jk}}{dl} = (1 - K^{-1})\lambda^+_{jk} + 2\nu \sum_{m\neq j,k} \lambda^+_{jm}\lambda^+_{mk}. \qquad (9)$$

Here, ν is the density of states of the leads. The couplings λ^-_{jj} do not renormalize. As the λ^+_{jk} terms transfer charge between the leads [see Eq. (7)], the first term in Eq. (9) promotes a suppression, the second term an enhancement of tunneling processes. This corresponds to the competition of two distinct mechanisms: the suppression of electron tunneling in Luttinger liquids [19], and the enhancement of the coupling due to the Kondo effect. Depending on which of these wins one will find a markedly different low temperature behavior, described by the decoupled lead and the Kondo fixed points.

The transition between the two cases is governed by a repulsive, isotropic ($\lambda^+_{jk} = \lambda^+$) intermediate fixed point at $\nu\lambda^{+\star} = (1 - K)/(2K(M - 2))$, with exponent $(1/K) - 1$ in the relevant, isotropic direction. (The fixed point is attractive from the orthogonal directions.) This is consistent with the weak coupling regime for $K \lesssim 1$, but we believe that the existence of an isotropic fixed point governing the transition remains true in general.

For $\bar{\lambda} < \lambda^{+\star}$, where $\bar{\lambda}$ is the typical bare value of λ^+_{jk}, the low temperature behavior is dictated by the decoupled fixed point. In the opposite case, λ^+_{jk} flow to large values while becoming more and more isotropic (consistently with the flow around $\lambda^{+\star}$). The crossover to strong coupling is at the scale of the Kondo temperature, $T_{\rm K} \sim E_c \exp[1/2(2 - M)\nu\bar{\lambda}]$. The analysis of the strong coupling regime becomes transparent after rotating $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_M)$, $\boldsymbol{\varphi} = (\varphi_1, \ldots, \varphi_M)$ to decompose them to $R_0 = \mathbf{v}_0 \cdot \boldsymbol{\theta}$, $K_0 = \mathbf{v}_0 \cdot \boldsymbol{\varphi}$ [with $\mathbf{v}_0 = (1/\sqrt{M})(1, \ldots, 1)$] and to $M - 1$ components r_j, k_j along the directions orthogonal to $\mathbf{v}_0$. This canonically decouples the K_0, R_0 sector, while the $\mathbf{r}$, $\mathbf{k}$ sector has $\sum_j H_0(k_j, r_j)$ perturbed by

$$H_{\rm eff} = -\sum_{j\neq k} \lambda^+_{jk} \frac{e^{i(\mathbf{w}_k - \mathbf{w}_j)\cdot\mathbf{r}/2}}{a} - \sum_j \frac{\lambda^-_{jj}}{2\pi} \mathbf{w}_j \cdot \partial_x \mathbf{k}, \qquad (10)$$

where $\mathbf{w}_j \cdot \mathbf{w}_l = \delta_{jl} - (1/M)$, implying that the minima of the first term form a (hyper) triangular lattice. Near the Kondo fixed point $\lambda^+ \to \infty$, $\mathbf{r}$ tends to be pinned to one of these minima. This, at the same time, suppresses [9] $\partial_x \mathbf{k}$, thus the λ^-_{jj} term can be neglected in this regime. The λ^-_{jj} couplings, therefore, only give small, marginal perturbations (through the R_0, K_0 sector) again, showing that the QBM model indeed captures the essential physics in both the weak and strong coupling regimes.

Using QBM results [17] we immediately find that the leading perturbations at the Kondo fixed point, given by $\mathbf{r}$ tunneling between the adjacent minima, have dimension $\Delta = 2K(M - 1)/M$. The Kondo fixed point is thus stable as long as $K > M/(2M - 2) > 1/2$; it is robust against weak conduction electron interactions and its robustness is enhanced as M increases. Because of their noninteger dimension, these charge conserving processes do not admit a free fermion description. The SO(M) Kondo problems thus give rise to NFL behavior even with Fermi liquid ($K = 1$) leads. Such stable NFL fixed points, arising without fine tuning of tunnel couplings or $K \neq 1$ leads, are absent from conventional Kondo systems based on spin degeneracy [13,20,21]. Our method allows us to prove that this remarkable feature is a generic property of the topological Kondo effect, vastly generalizing the $M = 3$, $K = 1$ result of Ref. [7].

In particular, for $M = 4$, $K = 1$ we find $\Delta = 3/2$, as for the two-channel Kondo fixed point [15]. Through relabeling ψ_j one can indeed map our problem to the two-channel Kondo model as $\lambda^+_{jk} \to \lambda^+$, $\lambda^-_{jj} \to 0$. The topological Kondo effect thus provides a long sought-after stable realization of this fixed point that does not hinge on (but is robust against) having NFL leads, in contrast to earlier proposals [22]. Placing the two channel Kondo in the QBM context, we also find a new theoretical perspective, alternative to Refs. [15,23].

We can now apply our findings to the Kubo conductance G_{kl}. The phase diagram in terms of $\bar{\lambda}$ and K is sketched in Fig. 2. Its topology is dictated by the QBM [17], but the physical meaning of the phases is specific to the topological Kondo effect. Tuning $\bar{\lambda}$ or K, the system undergoes a quantum phase transition, switching $G_{k\neq l}$ from 0 to $(2K/M)(e^2/h)$ at $T \to 0$. The transition using $\bar{\lambda}$ is especially appealing, as $\bar{\lambda}$ is gate tunable, in principle, in the nanowire realizations of recent experiments [6].

Near the decoupled lead fixed point, G_{kl} vanishes as $G_{kl} \sim T^{2/K-2}$ as $T \to 0$. This is the known suppression

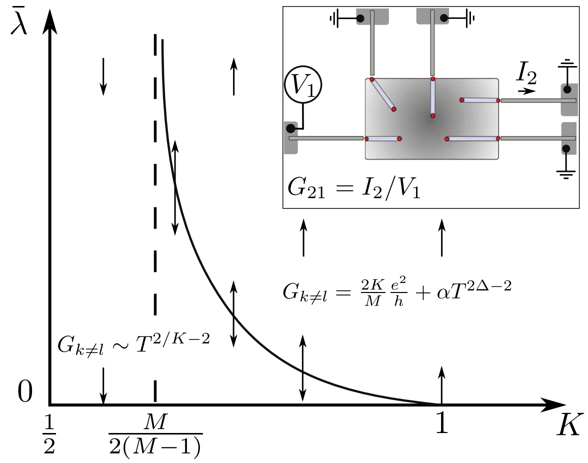

FIG. 2 (color online). The phase diagram of the topological Kondo effect in terms of the typical bare λ^+ coupling $\bar{\lambda}$ and the Luttinger parameter K. The arrows indicate the flow of the couplings under the renormalization group, and $G_{k\neq l}$ is the low temperature Kubo conductance. The curved line separates insulating ($\lambda^+ \to 0$) and Kondo ($\lambda^+ \to \infty$) phases. The inset shows the transport setup for $M = 5$.

of electron tunneling between Luttinger liquid leads [19], with the exponent coming from the first term of Eq. (9).

For $T \ll T_K$ near the Kondo fixed point, we have $G_{kl} = (2Ke^2/h)(\delta_{kl} - (1/M)) + \alpha_{kl}T^{2\Delta-2}$, where α_{kl} are nonuniversal constants. The fixed point ($T \to 0$) value follows from the emergent boundary conditions $\mathbf{r}$ = fixed, $\partial_x \mathbf{k} = 0$ at $\lambda^+ \to \infty$, and can be obtained from an immediate generalization of the calculations of Ref. [9]. Note that $G_{ll} = (e^2/h)\Delta$. A stable Kondo fixed point ($\Delta > 1$) thus comes with G_{ll} violating the e^2/h limit for single-channel normal conduction. This is due to the emergent boundary conditions translating into correlated, multiparticle Andreev processes by which holes, not only electrons, can be backscattered [9]. The $T^{2\Delta-2}$ dependence is due to second order corrections in processes tunneling $\mathbf{r}$ between the minima in Eq. (10). (The first order corrections of the current-current correlators underlying G_{kl} vanish [24].) The convergence to an enhanced conductance through such nontrivial power laws gives a clear signature of the NFL Kondo physics.

In conclusion, we have introduced a bosonization based picture for Majorana-lead couplings relevant to ongoing transport experiments. The key feature is the appearance of Klein factors Γ_j, virtually extending the number of Majoranas in the system. We have illustrated the utility of our approach by providing a QBM description of the topological Kondo problem with an arbitrary number of leads of possibly interacting electrons. We expect that our picture will be a useful starting point for a number of new problems exploring the interplay of Majorana fermions and strong correlations.

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to this case too due to the reducibility of spinors for $M_{\rm tot} > M = 4$.

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